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Mathematical Modeling of Foot and Mouth Disease Spread on Livestock using Saturated Incidence Rate

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ABSTRACT

Foot and Mouth Disease (FMD) is an acute infectious disease that attacks livestock, thus threatening the availability of food and the husbandry industry. This paper discusses the formulation of a mathematical model for the spread of FMD in livestock with a saturated incidence rate. The research method used is quantitative mathematical modeling with simulation, with stages including problem identification, determining assumptions, model formulation, analysis and model simulation. The discussion results obtained two equilibrium points, namely the non-endemic equilibrium point and the endemic equilibrium point, and then analyzed for stability. Numerical simulation is presented using Runge-Kutta approximation with MATLAB. Furthermore, after a sensitivity analysis, the parameters that greatly influenced the spread of FMD were direct or indirect contact (which led to the entry of the FMD virus) and the supporting capacity of livestock. Then the most influential parameter in reducing the spread of FMD is the application of culling on exposed animals and infected animals. The FMD modeling is a form of mathematical application to simulate the spread of disease on livestock.

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A. INTRODUCTION

Foot and Mouth Disease (FMD) is an acute infectious disease that attacks livestock, so it becomes a serious threat to the husbandry industry and food stability (Bahiru & Assefa, 2022) (G et al., 2021). FMD is caused by type A virus from the family Picornaviridae, genus Apthovirus (Belsham, 2020). Even though the fatality is low, this disease causes a decrease in body weight and milk production of livestock, thereby lowering the selling price. Cattle, sheep, goats, deer and pigs are very susceptible to FMD, with clinical symptoms seen after an incubation period of 2 to 14 days, even longer, especially for sheep and goats (Lazarus et al., 2021).

FMD transmission occurs when there is direct or indirect contact with infected animals (Mohr et al., 2018) (Schnell et al., 2019). Livestock are susceptible to infection through aerosols, inhalation, ingestion, and through natural or artificial mating (Arzt et al., 2018). Clinical symptoms in livestock infected with FMD include pyrexia, anorexia, excessive mucus discharge from the mouth and foaming, sores such as canker sores in the oral cavity and tongue, sores on the feet and loose nails, limping, shaking, and drastic drop in milk production (Dubie & Amare, 2020). FMD has seven different variants and more than 60 viral subtypes (Mushayabasa et al., 2011). In general, there is no universal vaccine against this disease, vaccines for FMD must be compatible with the types and subtypes present in the affected area. Some of the symptoms of FMD can be seen in Figure 1.

Figure 1. The Symptoms of FMD in Livestock

Animals that infected by FMD cannot be treated. The efforts that can be done are increasing the immunity and body resistance of animals through supportive therapy by providing vitamin and feed supplements. Then for symptoms, appropriate therapy can be carried out, such as: giving fever reducers, pain relievers, and antibiotics to prevent infection (Cabezas et al., 2018).

The efforts to eradicate FMD are focused on preventive control. To increase the immunity of animals that are susceptible to contracting FMD, it is necessary to intensify the mass vaccination program Ringa & Bauch (2014) and carry out mitigation program in areas that have not been infected these efforts can be in the form of surveillance and the formation of early awareness and implementing disease resilience (Elnekave et al., 2016). This needs to be done in order to obtain a disease spread map as a basis for determining control measures, in addition to communicating, providing information and educating the farming community. In this article, in addition to using vaccination, researchers also involve the culling of infected animals as an effort to control the spread of FMD.

Mathematical models have been widely used to describe the problem of infectious diseases spread (Fahcruddin, 2019). Research on the spread of FMD has been carried out by several researchers such as (Mushayabasa et al., 2011), (Belayneh et al., 2020), (Gashirai et al., 2020), (Chanchaidechachai et al., 2021), and (Sseguya et al., 2021). (Mushayabasa et al., 2011) have studied the spread of FMD by paying attention the bilinear incidence rate. Then, (Belayneh et al., 2020) conducted studies on the spread of FMD using the SIR model in Ethiopia. (Gashirai et al., 2020) studied the spread of FMD by focused to vaccine failures and environmental transmission. Furthermore, (Chanchaidechachai et al., 2021) conducted a study on the spread of FMD using a spatial model in Thailand. Next, (Sseguya et al., 2022) conducted research on the spread of FMD in two locations that were close to each other. The four studies used the incidence rate form in formulating mathematical models. The mathematical model of the spread of FMD on livestock in this article refers to the model used by (Mushayabasa et al., 2011). The model has been modified, by changing the form of the incidence rate which was originally a bilinear incidence rate (βIS) to a saturated incidence rate $\frac{\beta IS}{1+a_c I}$. Farther, the growth of livestock is also developed in the FMD distribution model using a logistic model $r_h S\left(1-\frac{S}{k_1}\right)$. The main contribution in this article is that by applying the saturated incidence rate and logistic growth to the FMD model in livestock, the mathematical model used is more applicable, especially in limited and specific areas.

B. METHODS

The type of research used is quantitative, with the following stages:

1. Problem identification is carried out to understand the problem to be formulated so that it can be notated into mathematical symbols.
2. Determine assumptions. The number of factors that influence the observed events needs to be simplified by assuming a simple relationship between variables. The assumptions here are divided into several categories, namely:
   a. Variable classification. Things that affect the behaviour of observations in step 1 are identified as variables, both in the form of independent variables and dependent variables.
   b. Determine the interrelation between the selected variables to be studied. The steps taken are to create a sub-model according to the assumptions made in the main model, then study separately on one or more independent variables.

3. The formulation of the model can be done either through functional relationships by creating compartment diagrams, mathematical equations, or with the help of a software or by analytical means.
4. Analysing the model. After the model is obtained then it is solved mathematically, in this case, the model is a system of differential equations and the analysis includes the equilibrium point, stability analysis, and sensitivity analysis.
5. Simulate model. Simulations are carried out to see whether the interpretation of the model made is rational or not, using MATLAB or MAPLE software. If the interpretation results meet the requirements and are rational, then the results can be accepted, only then can the implementation of the obtained model be carried out. The steps involved are presented in Figure 2.

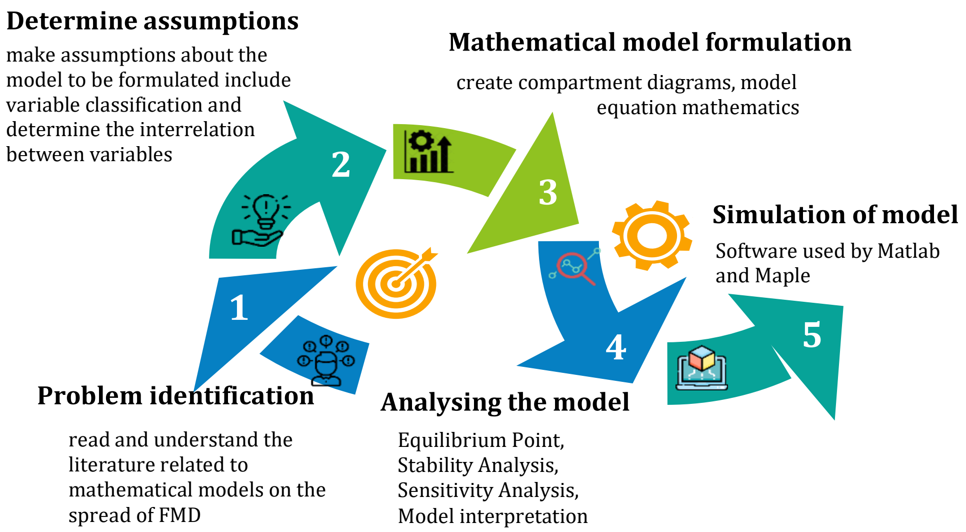

Figure 2. The Stages of Mathematical Modelling for the Spread of on Livestocks.

The FMD epidemic model was constructed by consider the saturated infection. $N(t)$ expressed the total population of animals, which is classified into five classes: susceptible animal $S(t)$, vaccinated animals $V(t)$, exposed animals $E(t)$, and infected animals $I(t)$, where:

$$N(t) = S(t) + V(t) + E(t) + I(t)$$

The assumptions used in the formation of mathematical models for spread of disease are birth rate of livestock fulfills the logistic function. Furthermore, the vaccinated livestock become susceptible again after the vaccine immunity is lost. Then, susceptible livestock become latent if there is direct or indirect contact with livestock infected with Apthovirus; according to the saturated incidence rate. Livestock that recover from infection become susceptible. Then, the following assumptions are taken in deriving the model (1):

1. Assumption 1. The population of susceptible animals increases due to growth that fulfills the logistic function $r_h S(t)\left(1 - \frac{S(t)}{k_1}\right)$, and reduced immunity in rapidly vaccinated animals $\emptyset > 0$.
2. Assumption 2. Vulnerable livestock will become latent if there is direct or indirect contact with infected animals with a saturated incidence rate $\frac{\beta IS}{1+a_c I}$ with maximum contact rate $\beta > 0$ and intervention level $a_c > 0$. Then, latent animals will become infected and able to transmit the virus to susceptible animals at a rapid rate $\gamma > 0$.
3. Assumption 3. The efforts to control the spread of FMD are carried out by vaccinating susceptible animals at a rate $u_1 > 0$, then extermination of infected animals and latent animals at a rate $u_2 > 0$.
4. Assumption 4. Natural mortality of animals at a rate $\mu > 0$, while the mortality due to FMD in infected animals at a rate $d > 0$. Then, birth rate of animal more than natural mortality rate $r_h > \mu$.

The following is an illustration of the FMD transmission scheme, as shown in Figure 3.

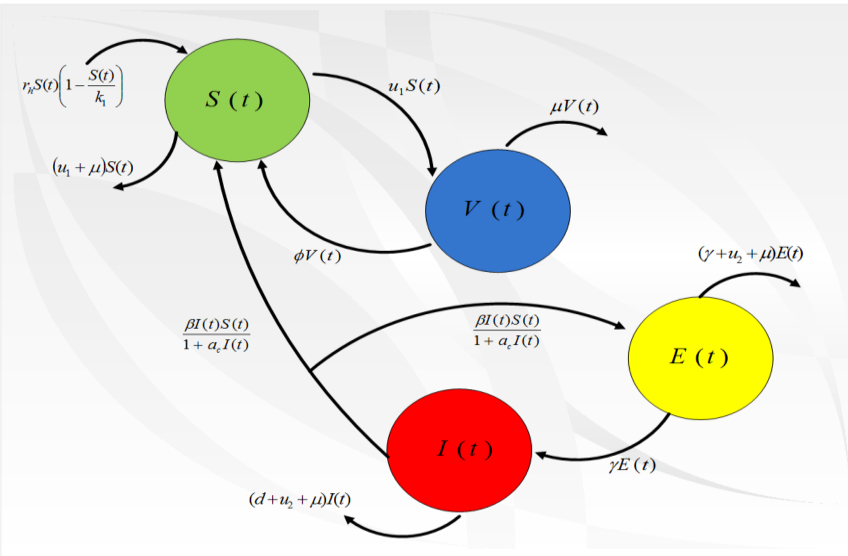

Figure 3. FMD Model Flow Diagram

The description of the parameters used in the mathematical modelling of FMD on livestock is presented in Table 1 below.

Table 1. Parameter Description

Notations	Description	Units
r_h	Birth rate of animal	1/time unit
k_1	Animal support capacity	Individual
$\emptyset$	Waning rate	1/ individual
β	Transmissibility	1/ (individual x time unit)
a_c	Saturation level	1/ individual
γ	Incubation period	1/ individual
u_1	Rate of susceptible animal are vaccinated	1/ individual
u_2	Rate of latent and infected animal are culled	1/ individual
μ	Natural mortality rate	1/ individual
d	Disease related mortality	1/ individual

Therefore, a mathematical model of FMD transmission can be constructed based on the interaction diagram in Figure 1:

$$\left.\begin{aligned}\frac{dS(t)}{dt} &= r_h S(t)\left(1-\frac{S(t)}{k_1}\right)+\emptyset V-\frac{\beta I(t)S(t)}{1+a_c I(t)}-(u_1+\mu)S(t)\\ \frac{dV(t)}{dt} &= u_1 S(t)-(\emptyset+\mu)V(t)\\ \frac{dE(t)}{dt} &= \frac{\beta I(t)S(t)}{1+a_c I(t)}-(\gamma+u_2+\mu)E(t)\\ \frac{dI(t)}{dt} &= \gamma E(t)-(d+u_2+\mu)I(t)\end{aligned}\right\} \quad (1)$$

with initial condition is $S(0)>0, V(0)>0, E(0)>0, I(0)>0$.

C. RESULT AND DISCUSSION

1. Non-Endemic Equilibrium Point

By setting the derivatives of each equation in system (1) to zero, i.e., $\frac{dS(t)}{dt}=0, \frac{dV(t)}{dt}=0, \frac{dE(t)}{dt}=0, \frac{dI(t)}{dt}=0$, next substitute $I(t)=0$, $E(t)=0$, so that the non-endemic equilibrium point (P^0) is obtained as:

$$P^0=(S^0,V^0,E^0,I^0)=\left(k_1+\frac{u_1k_1}{(\emptyset+\mu)r_h}-\frac{(u_1+\mu)k_1}{r_h},\frac{u_1}{(\emptyset+\mu)}\left(k_1+\frac{u_1k_1}{(\emptyset+\mu)r_h}-\frac{(u_1+\mu)k_1}{r_h}\right),0,0\right).$$

2. Basic Reproduction Number

The basic reproduction number $(\mathcal{R}_0)$ is the average number of newly generated infected livestock by a single infected livestock. Using the next generation matrix [12], the non-negative matrix F, and the non-singular matrix V, from system (1), we obtained:

$$\mathcal{F}=\begin{bmatrix}\frac{\beta IS}{1+a_c I}\\ 0\end{bmatrix} \text{ and } \mathcal{V}=\begin{bmatrix}(\gamma+u_2+\mu)E\\ (d+u_2+\mu)I-\gamma E\end{bmatrix}.$$

Next,

$$F = \begin{bmatrix} 0 & \beta\left(k_1 + \frac{u_1 k_1}{(\emptyset+\mu)r_h} - \frac{(u_1+\mu)k_1}{r_h})\right) \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \mu + u_2 + \gamma & 0 \\ -\gamma & d + u_2 + \mu \end{bmatrix}.$$

Furthermore, the eigenvalues of FV^{-1} are:

$$\lambda_1 = \frac{\beta\gamma}{(\mu + u_2 + \gamma)(d + u_2 + \mu)}\left(k_1 + \frac{u_1 k_1}{(\emptyset + \mu)r_h} - \frac{(u_1+\mu)k_1}{r_h})\right), \qquad \lambda_2 = 0.$$

The spectral radius of matrix FV^{-1} is $\rho(FV^{-1}) = \mathcal{R}_0$, where:

$$\mathcal{R}_0 = \frac{\beta\gamma}{(\mu + u_2 + \gamma)(d + u_2 + \mu)}\left(k_1 + \frac{u_1 k_1}{(\emptyset + \mu)r_h} - \frac{(u_1+\mu)k_1}{r_h})\right). \tag{2}$$

3. Endemic Equilibrium Point

All derivatives of all equations in (1) are consistently set to zero, in order to determine the endemic point. Excluding for the fourth equation, by calculating all these equations, we can get:

$$P^* = (S^*, V^*, E^*, I^*),$$

with

$$S^* = \frac{(\beta + u_2 + \gamma)^2}{\gamma\beta}(1 + a_c I^*)$$

$$V^* = \frac{u_1(\mu + u_2 + \gamma)^2}{(\emptyset + \mu)\gamma\beta}(1 + a_c I^*)$$

$$E^* = \frac{(d + u_2 + \mu)}{\gamma}(I^*)$$

By substituting S^*, V^*, E^* to the four equations of (1) and set $\frac{dS(t)}{dt} = 0$, we get I^*, it is positive root of equation below:

$$A_1(I^*)^2 + A_2 I^* + A_3 = 0,$$

where:

$$A_1 = \frac{a_c^2 r_h(\beta + u_2 + \gamma)^2}{k_1\gamma\beta},$$

$$A_2 = \beta + \frac{a_c^2 r_h(\beta + u_2 + \gamma)^2}{k_1\gamma\beta} - a_c\left(\frac{\emptyset u_1}{\emptyset + \mu} - \left(u_1 + \mu - r_h - \frac{r_h(\beta + u_2 + \gamma)^2}{k_1\gamma\beta}\right)\right),$$

$$A_3 = \frac{r_h(\beta + u_2 + \gamma)^2}{k_1\gamma\beta} + (u_1 + \mu - r_h) - \left(\frac{\emptyset u_1}{\emptyset + \mu}\right).$$

4. Local Stability of Equilibrium Point

We determine the Jacobian matrix and analyze its eigenvalues for all points, to investigate the local stability around each equilibrium point. The Jacobian matrix for system (1) is

$$J = \begin{bmatrix} r_h - \frac{2r_h S}{k_1} - \frac{\beta I}{1 + a_c I} - (u_1 + \mu) & \theta & 0 & -\frac{\beta S}{(1 + a_c I)^2} \\ u & -(\theta + \mu) & 0 & 0 \\ \frac{\beta I}{1 + a_c I} & 0 & -(\gamma + u_1 + \mu) & \frac{\beta S}{(1 + a_c I)^2} \\ 0 & 0 & \gamma & -(d + u_2 + \mu) \end{bmatrix}$$

Theorem 1. *A non-endemic point P^0 of the system (1) is locally asymptotically stable for $\frac{k_1 m}{2} < \mathcal{R}_0 < 1$, and it is unstable for $\mathcal{R}_0 > 1$* with

$$m = \frac{\beta\gamma}{(\mu + u_2 + \gamma)(d + u_2 + \mu)}.$$

Proof.

The following is the Jacobian System (1) matrix that is evaluated on P^0

$$J(P^0) = \begin{bmatrix} r_h - \frac{2r_h\mathcal{R}_0}{mk_1} - (u_1 + \mu) & \theta & 0 & -\frac{\beta\mathcal{R}_0}{m} \\ u & -(\theta + \mu) & 0 & 0 \\ 0 & 0 & -(\gamma + u_1 + \mu) & \frac{\beta\mathcal{R}_0}{m} \\ 0 & 0 & \gamma & -(d + u_2 + \mu) \end{bmatrix}$$

So that the characteristic polynomial of $J(P^0)$ is obtained as follows

$$P(\lambda) = (\lambda^2 + a_1\lambda + a_0)(\lambda^2 + b_1\lambda + b_0)$$

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$$m = \frac{\beta\gamma}{(\mu + u_2 + \gamma)(d + u_2 + \mu)} > 0$$

$$a_1 = \mu + \theta + u_1 + \frac{2}{m}\left(\mathcal{R}_0 - \frac{m}{2}\right)$$

$$a_0 = \mu^2 + \theta\mu + u_1\mu + \frac{2r_h\mu}{m}\left(\mathcal{R}_0 - \frac{m}{2}\right) + \frac{2\theta r_h}{m}\left(\mathcal{R}_0 - \frac{k_1 m}{2}\right)$$

$$b_1 = d + 2u_2 + 2\mu + \gamma > 0$$

$$b_0 = (\mu + u_2 + \gamma)(d + u_2 + \mu)(1 - \mathcal{R}_0)$$

Based on the Routh-Hurtwitz criteria, all real parts of the root $P(\lambda)$ will be negative if $a_1, a_0, b_1, b_0 > 0$ are satisfied. It is clear that $m > 0$ and $b_1 > 0$ because all parameters are positive. Coefficient $a_1 > 0$ if $\mathcal{R}_0 > \frac{m}{2}$ and a_0 is positive if $\mathcal{R}_0 > \frac{k_1 m}{2}$ with $\frac{k_1 m}{2} > \frac{m}{2}$ because $k_1 > 0$. While the coefficient $b_0 > 0$ if $\mathcal{R}_0 < 1$. So, if $\frac{k_1 m}{2} < \mathcal{R}_0 < 1$ then all eigenvalues $J(P^0)$ are negative, so it is evident that the disease-free equilibrium point P^0 is locally asymptotically stable. Otherwise, if $\mathcal{R}_0 > 1$ then $b_0 < 0$ as a result there is an eigenvalue $J(P^0)$ which is positive, so that the non-endemic equilibrium point P^0 is unstable.

5. Sensitivity Analysis of the Reproductive Number

Sensitivity analysis is used to determine the most influential parameter in a model (Hurint et al., 2017). In this case, the sensitivity index will be determined for each parameter involved in the basic reproduction number ($\mathcal{R}_0$) from the FMD model with saturated incidence rate. The value of $\mathcal{R}_0$ that will be used in the sensitivity analysis of FMD model with saturated incidence rate as follows:

$$\mathcal{R}_0 = \frac{\beta\gamma}{(\mu + u_2 + \gamma)(d + u_2 + \mu)}\left(k_1 + \frac{u_1 k_1}{(\emptyset + \mu)r_h} - \frac{(u_1 + \mu)k_1}{r_h})\right).$$

The parameter sensitivity index is formulated as follows:

$$idm = \frac{\partial \mathcal{R}_0}{\partial p} \cdot \frac{p}{\mathcal{R}_0}$$

where:

idm = parameter sensitivity index p
p = analyzed parameters.

There are 8 parameters whose sensitivity index will be calculated, namely, $\beta, \gamma, u_2, d, k_1, u_1, \emptyset, r_h$. The results of the sensitivity index of the FMD model parameter with the saturated incidence rate can be seen in the following equation:

$$\left.\begin{aligned}
\frac{\partial \mathcal{R}_0}{\partial \beta} \cdot \frac{\beta}{\mathcal{R}_0} &= 1 \\
\frac{\partial \mathcal{R}_0}{\partial \gamma} \cdot \frac{\gamma}{\mathcal{R}_0} &= \frac{\mu + u_2}{(\mu + u_2 + \gamma)} \\
\frac{\partial \mathcal{R}_0}{\partial u_2} \cdot \frac{u_2}{\mathcal{R}_0} &= -\frac{(d + 2u_2 + 2\mu + \gamma)u_2}{(d + u_2 + \mu)(\mu + u_2 + \gamma)} \\
\frac{\partial \mathcal{R}_0}{\partial d} \cdot \frac{d}{\mathcal{R}_0} &= -\frac{d}{(d + u_2 + \mu)} \\
\frac{\partial \mathcal{R}_0}{\partial k_1} \cdot \frac{k_1}{\mathcal{R}_0} &= 1 \\
\frac{\partial \mathcal{R}_0}{\partial u_1} \cdot \frac{u_1}{\mathcal{R}_0} &= \frac{u_1(\mu + \emptyset - 1)}{\mu^2 + \emptyset\mu + u_1\mu + u_1\emptyset - u_1 - r_h\mu - r_h\emptyset} \\
\frac{\partial \mathcal{R}_0}{\partial \emptyset} \cdot \frac{\emptyset}{\mathcal{R}_0} &= -\frac{u_1\emptyset}{\left(1 + \frac{u_1}{(\emptyset + \mu)r_h} - \frac{(u_1 + \mu)}{r_h})\right) r_h (\emptyset + \mu)^2} \\
\frac{\partial \mathcal{R}_0}{\partial r_h} \cdot \frac{r_h}{\mathcal{R}_0} &= -\frac{\mu^2 + \emptyset\mu + u_1\mu + u_1\emptyset - u_1}{\mu^2 + \emptyset\mu + u_1\mu + u_1\emptyset - u_1 - r_h\mu - r_h\emptyset}
\end{aligned}\right\} \quad (3)$$

From the Equation 3, it can be seen that the results of the sensitivity analysis to the parameters, $\beta,\ \gamma, k_1$ are always positive, so that they are directly proportional to $\mathcal{R}_0$, while the sensitivity analysis to the parameters $u_1,\ u_2,\ d,\ \emptyset,$ and r_h can be negative, so that it is inversely proportional to the $\mathcal{R}_0$. Furthermore, the most influential parameters in the spread of FMD on livestock with saturated incidence rate are β and k_1. Then, the parameter that greatly influences the decrease in the spread of FMD on livestock is u_2.

6. Numerical Simulation

This section presents a simulation of the FMD model on livestock using saturated incidence rate. The simulations carried out are classified into two conditions, namely conditions without spread of FMD and conditions when FMD spread occurs. The simulations were carried out using MATLAB software by entering initial values for each population. The condition without FMD occurs when there are no FMD spreading animals, in other words there is no FMD spread on livestock ($\mathcal{R}_0 < 1$). The simulation was carried out for 7 days. The parameter values used in the simulation are presented in the Table 2.

Tabel 2. The Parameters values used in the simulation FMD model

Notations	Value		Value	
	Case $\mathcal{R}_0 < 1$	Source	Case $\mathcal{R}_0 > 1$	Source
r_h	200 day^{-1}	(Mushayabasa et al., 2011)	230 day^{-1}	Assumed
k_1	250	(Widya & Alfiniyah, 2020)	120	Assumed
$\emptyset$	0.001 day^{-1}	(Widya & Alfiniyah, 2020)	0.001 day^{-1}	(Widya & Alfiniyah, 2020)
β	0.004	(Mushayabasa et al., 2011)	3.33	Assumed
a_c	0,004	(Widya & Alfiniyah, 2020)	0,00001914	(Mushayabasa et al., 2011)
γ	0.26 day^{-1}	(Widya & Alfiniyah, 2020)	3.26 day^{-1}	Assumed
μ	20 day^{-1}	Assumed	20 day^{-1}	Assumed
d	0.001 day^{-1}	(Mushayabasa et al., 2011)	0.001 day^{-1}	(Mushayabasa et al., 2011)
u_1	0.5	(Mushayabasa et al., 2011)	0.16	Assumed
u_2	0.2	(Mushayabasa et al., 2011)	0.2	Assumed

Based on the parameter values in Table 2, it is obtained that $\mathcal{R}_0 = 0.0037 < 1$ and $P^0 = (225, 6, 0, 0)$. The following is a numerical simulation result when there is no FMD spread, as shown in Figure 4.

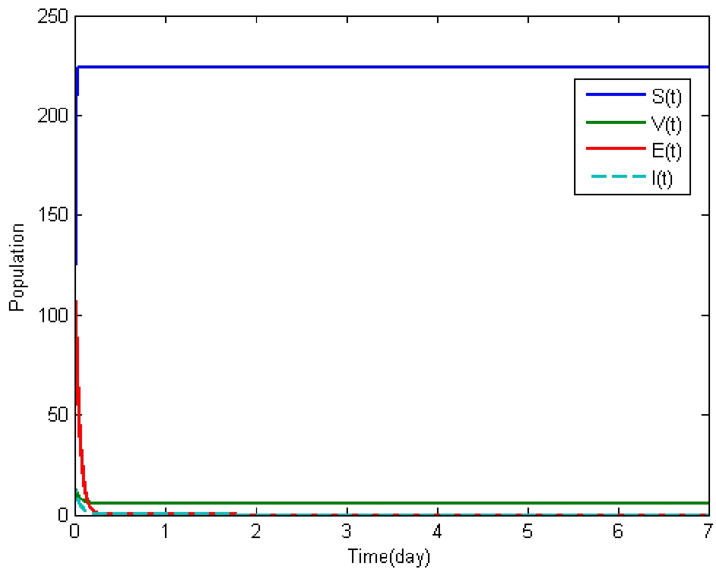

Figure 4. The dynamics of all population for $\mathcal{R}_0 < 1$.
The non-endemic point Po is asymptotically stable.

From Figure 4 it can be seen that all populations produce graphs that tend to converge to the non-endemic equilibrium point of FMD model. Furthermore, the population of susceptible and vaccinated animals tends to increase because there is no spread of FMD on animals. Then, the population of exposed animals and infected animals in conditions free from the spread of FMD will tend to decrease until finally exhausted. The spread of FMD occurs when there are animals that are infected and then infect FMD to other animals $(\mathcal{R}_0 > 1)$. Based on the parameter values in Table 2, it is obtained that $\mathcal{R}_0 = 2.508 > 1$ and $P^* = (44, 0, 235, 38)$. The following is the result of a numerical simulation on the spread of FMD with a saturated incidence rate, as shown in Figure 5.

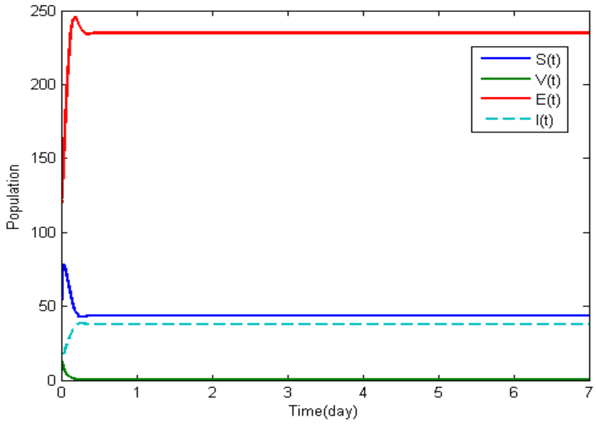

Figure 5. The dynamics of all population for $\mathcal{R}_0 > 1$.
The non-endemic point P* is asymptotically stable.

Figure 5 it can be seen that all populations produce graphs that tend to converge to the endemic equilibrium point of FMD model. Furthermore, the population of exposed animals and infected animals tends to increase before finally being in a constant condition. The results of the analysis sensitivity simulation when FMD spread occurs are presented in Table 3.

Table 3. The result of parameter index calculation for FMD spread condition

Parameter	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{u_2}$	$\boldsymbol{d}$	$\boldsymbol{k_1}$	$\boldsymbol{u_1}$	$\emptyset$	$\boldsymbol{r_h}$
Sensitivity Index	1	0.861	-0.0184	-0.000049	1	-0.00072	-0.0000000019	0.96

Base on Table 3, several conclusions were obtained:

a. The sensitivity index of parameter β is 1, it means if the rate of direct contact with infected animals increases by 10%, then the value of $\mathcal{R}_0$ will increase by 10%.
b. The sensitivity index of parameter γ is 0.861, it means if the incubation rate on animals increases by 10%, then the value of $\mathcal{R}_0$ will increase by 8.6%.
c. The sensitivity index of parameter u_2 is -0.0184, it means if the culling rate on animals increases by 10%, then the value of $\mathcal{R}_0$ will decrease by 0.18%.
d. The sensitivity index of parameter d is -0.000049, it means if the mortality rate due to FMD on animals increases by 10%, then the value of $\mathcal{R}_0$ will decrease by 0.00049%.
e. The sensitivity index of parameter k_1 is 1, it means if the total supporting capacity of the animal population increases by 10%, then the value of $\mathcal{R}_0$ will increase by 10%.
f. The sensitivity index of parameter u_1 is -0.00072, it means if the vaccination rate of animals increases by 10%, then the value of $\mathcal{R}_0$ will decrease by 0.0072%.
g. The sensitivity index of parameter $\emptyset$ is -0.0000000019, it means if the waning rate of animals increases by 10%, then the value of $\mathcal{R}_0$ will decrease by 0.000000019%.
h. The sensitivity index of parameter r_h is 0.96, it means if the birth rate of animals increases by 10%, then the value of $\mathcal{R}_0$ will increase by 9.6%.

Based on the explanation above, there are 8 parameters that affect the value of $\mathcal{R}_0$. Furthermore, the most influential parameters in the spread of FMD on animals with a saturated incidence rate are β and k_1 with sensitivity index of 1. Then the parameter that is very influential in reducing the spread of FMD on animals is u_2 with a sensitivity index of -0.0184. The impact of vaccination and animals culling on the spread of FMD is presented in Figure 6.

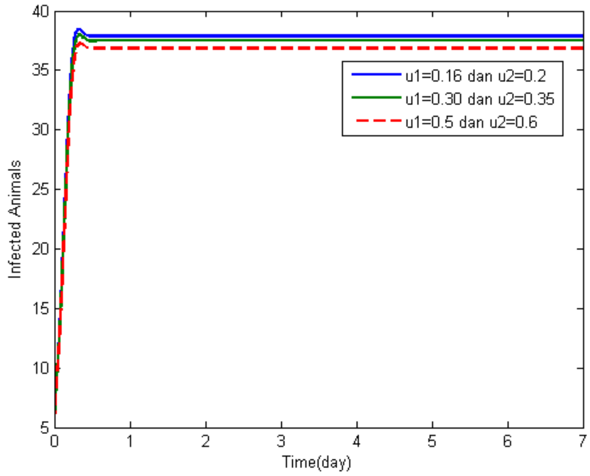

Figure 6. The dynamics of all population for $\mathcal{R}_0 > 1$.
The non-endemic point P^* is asymptotically stable

Figure 6 shows the estimated population of animals infected with FMD for Ro > 1 with different control applications. For cases $u_1 = 0.16$ and $u_2 = 0.2$, the infected animal population converges to 38, then for cases $u_1 = 0.30$ and $u_2 = 0.35$, the infected animal population is 37. Furthermore, for $u_1 = 0.5$ and $u_2 = 0.6$, it is obtained that infected by 36. Thus, from the simulation, it can be concluded that efforts to reduce FMD infected animals can be carried out maximally if large numbers of livestock are vaccinated and large numbers of infected animals are exterminated.

D. CONCLUSION AND SUGGESTIONS

In this paper discussed the model for foot-and-mouth disease (FMD) considering the saturated incidence rate. By determining the number base re-production ($\mathcal{R}_0$), the existence and the stability of equilibrium point of FMD model can be analyzed. Furthermore, the number of exposed animals and the number of infected animals in conditions without the spread of FMD, will tend to decrease until they are finally exhausted. It is different when there is a spread of FMD, the number of exposed animals and the number of infected animals tends to increase before finally being in a constant condition. Furthermore, the impact of vaccination and culling on livestock can reduce the number of animals infected by FMD. The PMK model in this article uses a system of differential equations. However, the cost analysis in implementing vaccination and culling on livestock has not been discussed in this article. So that further research is needed for the application of optimal control involving the cost function. In addition, it is necessary to review the development of PMK distribution models such as more complex discrete models.

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REFERENCES

Arzt, J., Belsham, G. J., Lohse, L., Bøtner, A., & Stenfeldt, C. (2018). Transmission of Foot-and-Mouth Disease from Persistently Infected Carrier Cattle to Naive Cattle via Transfer of Oropharyngeal Fluid. *MSphere*, *3*(5), 1–12. https://doi.org/10.1128/msphere.00365-18

Atuman, Y. J., Kudi, C. A., Abdu, P. A., Okubanjo, O. O., Abubakar, A., Wungak, Y., & Ularamu, H. G. (2020). Seroprevalence of Foot and Mouth Disease Virus Infection in Some Wildlife and Cattle in Bauchi State, Nigeria. *Veterinary Medicine International*, *2020*. https://doi.org/10.1155/2020/3642793

Bahiru, A., & Assefa, A. (2022). Seroepidemiological investigation of Foot and Mouth Disease (FMD) in Northern Amhara, Ethiopia. *Scientific African*, *16*, e01267. https://doi.org/10.1016/j.sciaf.2022.e01267

Belayneh, N., Molla, W., Mesfine, M., & Jemberu, W. T. (2020). Modeling the transmission dynamics of foot and mouth disease in Amhara region, Ethiopia. *Preventive Veterinary Medicine*, *181*(December 2018). https://doi.org/10.1016/j.prevetmed.2019.04.002

Belsham, G. J. (2020). Towards improvements in foot-and-mouth disease vaccine performance. *Acta Veterinaria Scandinavica*, *62*(1), 1–12. https://doi.org/10.1186/s13028-020-00519-1

Cabezas, A. H., Sanderson, M. W., Jaberi-Douraki, M., & Volkova, V. V. (2018). Clinical and infection dynamics of foot-and-mouth disease in beef feedlot cattle: An expert survey. *Preventive Veterinary Medicine*, *158*, 160–168. https://doi.org/10.1016/j.prevetmed.2018.08.007

Chanchaidechachai, T., de Jong, M. C. M., & Fischer, E. A. J. (2021). Spatial model of foot-and-mouth disease outbreak in an endemic area of Thailand. *Preventive Veterinary Medicine*, *195*(June), 105468. https://doi.org/10.1016/j.prevetmed.2021.105468

Dubie, T., & Amare, T. (2020). Isolation, Serotyping, and Molecular Detection of Bovine FMD Virus from Outbreak Cases in Abaala District of Afar Region, Ethiopia. *Veterinary Medicine International*, *2020*. https://doi.org/10.1155/2020/8847728

Elnekave, E., van Maanen, K., Shilo, H., Gelman, B., Storm, N., Berdenstain, S., Berke, O., & Klement, E. (2016). Prevalence and risk factors for foot and mouth disease infection in small ruminants in Israel. *Preventive Veterinary Medicine*, *125*, 82–88. https://doi.org/10.1016/j.prevetmed.2015.12.019

Fahcruddin, I. (2019). Optimal Control For Malaria Epidemic Model With Vaccinating, Human Treatment And Mosquitos Spraying. *SELECCIONES MATEMÁTICAS*, *6*(2), 189. https://doi.org/https://doi.org/10.17268/sel.mat.2019.02.05

G, G., B, G. K., A, K., Hegde, R., Kumar, N., Prabhakaran, K., Wadhwan, V. M., Kakker, N., Lokhande, T., Sharma, K., Kanani, A., Limaye, K, N., PN, A., De, A. K., Khan, T. A., Misri, J., Dash, B. B., Pattnaik, B., & Habibur, R. (2021). Foot and Mouth Disease (FMD) incidence in cattle and buffaloes and its associated farm-level economic costs in endemic India. *Preventive Veterinary Medicine*, *190*(August 2020), 105318. https://doi.org/10.1016/j.prevetmed.2021.105318

Gashirai, T. B., Musekwa-Hove, S. D., Lolika, P. O., & Mushayabasa, S. (2020). Global stability and optimal control analysis of a foot-and-mouth disease model with vaccine failure and environmental transmission. *Chaos, Solitons and Fractals*, *132*. https://doi.org/10.1016/j.chaos.2019.109568

Hurint, R. U., Ndii, M. Z., & Lobo, M. (2017). Analisis Sensitivitas Model Epidemi SEIR. *Natural Science: Journal of Science and Technology*, *6*(1), 22–28. https://doi.org/10.22487/25411969.2017.v6.i1.8076

Lazarus, D. D., Opperman, P. A., Sirdar, M. M., Wolf, T. E., van Wyk, I., Rikhotso, O. B., & Fosgate, G. T. (2021). Improving foot-and-mouth disease control through the evaluation of goat movement patterns within the FMD protection zone of South Africa. *Small Ruminant Research*, *201*(June), 106448. https://doi.org/10.1016/j.smallrumres.2021.106448

Mohr, S., Deason, M., Churakov, M., Doherty, T., & Kao, R. R. (2018). Manipulation of contact network structure and the impact on foot-and-mouth disease transmission. *Preventive Veterinary Medicine*, *157* (August 2017), 8–18. https://doi.org/10.1016/j.prevetmed.2018.05.006

Mushayabasa, S., Bhunu, C. P., & Dhlamini, M. (2011). Impact of Vaccination and Culling on Controlling Foot and Mouth Disease: A Mathematical Modelling Approach. *World Journal of Vaccines*, *01*(04), 156–161. https://doi.org/10.4236/wjv.2011.14016

Ringa, N., & Bauch, C. T. (2014). Impacts of constrained culling and vaccination on control of foot and mouth disease in near-endemic settings: A pair approximation model. *Epidemics*, *9*, 18–30.

https://doi.org/10.1016/j.epidem.2014.09.008

Schnell, P. M., Shao, Y., Pomeroy, L. W., Tien, J. H., Moritz, M., & Garabed, R. (2019). Modeling the role of carrier and mobile herds on foot-and-mouth disease virus endemicity in the Far North Region of Cameroon. *Epidemics*, *29*(November 2018), 100355. https://doi.org/10.1016/j.epidem.2019.100355

Sseguya, I., Mugisha, J. Y. T., & Nannyonga, B. (2022). Outbreak and control of Foot and Mouth Disease within and across adjacent districts—A mathematical perspective. *Results in Control and Optimization*, *6*(April 2021), 100074. https://doi.org/10.1016/j.rico.2021.100074

Van Den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, *180*(1–2), 29–48. https://doi.org/10.1016/S0025-5564(02)00108-6

Widya, E., & Alfiniyah, C. (2020). Analisis Kestabilan Model Matematika Penyebaran Penyakit Schistosomiasis dengan Saturated Incidence Rate. In *Contemporary Mathematics and Applications* (Vol. 2, Issue 2).