

# The Compound Area of Quadrilaterals and Triangles: A Worked Example Based Learning Design

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## ABSTRACT

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Complex yet hierarchical mathematical material in fact does not mean that the process of schema acquisition and accommodation in long-term memory is easy. For an instance, year seven students learn to solve problems related to the compound area of quadrilaterals and triangles. This problem may be categorized as a well-structured problem, therefore, it might be procedurally obvious to reach the solution, for those who have possessed sufficient prior knowledge of the aspects of the shapes. This study aims to discuss instructional strategy for novices when learning to solve problems like these. Based on a cognitive load theory, the strategy of worked example could be applied effectively for novices. The design of the worked example must be presented in such a way it could minimize extraneous cognitive load. The composite shapes of quadrilateral and triangles might be challenging to be described in an integrated format to avoid the split attention effect. This paper shows how this can be done by using techniques, such as (1) number sequences of the solution steps; (2) different colors for prompting attention resource; (3) less wording, more procedure application; and (4) consistency of layout within overall material. In order to apply these, the instructional designers should follow ADD steps, (1) Analyze the learning content, arrange them accordingly; (2) Design one worked example, evaluate its accuracy and niche; and (3) Develop into several worked example pairs. Two worked example with paired problems have been successfully developed and declare valid. Eventually, it might be suggested that comprehensive consideration when designing worked example that contain pictures should reflect how students use their cognitive resource during learning.



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## A. INTRODUCTION

Mathematics consists of hierarchical concepts that implies the schema structure stored in long-term memory. Schema may represent a collection of knowledge constructed in accord to how the learner perceives or understands the knowledge. Schema construction determines how new information is then learned as well as how complex problem solving is dealt with. Therefore, the goal of learning is to assist students with schema acquisition and automation (Kalyuga, 2010; Wirzberger et al., 2018). Schema automation occurs when learners can use them to learn new information effortlessly. This can reduce cognitive processes during learning. Having sufficient schemes make information processing more efficient, and hence reduce the amount of new information required to be processed in working memory. This schema automation allows more cognitive resources to be used to construct new schema

(Paas & van Merriënboer, 2020; Sweller, 2011). In consequence, learning mathematics should be directed to construct well-structured and automated schemas.

Kalyuga (2010) pointed out that schema theory is helpful in explaining how a person processes more complex information. Schema construction is considered an important factor for improving problem-solving skills (Kim, 2013) and generating new solutions to a problem (Wirzberger et al., 2018). When the schema is already owned by the student, solving the problem becomes automatic so less effort is needed to process it in working memory, thus leaving more memory capacity (Kalyuga, 2010). This is why automated schemes are considered necessary to improve problem-solving skills (Van Merriënboer & Sweller, 2005). Students can use existing schemas as general procedures to recognize to-be-learned problem solving by mapping them into problem situations to generate solutions (Kalyuga, 2010).

When students do not have relevant schemas in long-term memory, cognitive load in working memory increases since more efforts are required to identify and organize the to-be-learned material (Van Merriënboer & Sweller, 2005). In fact, working memory has a very limited capacity, and is only able to process some chunk of information once at a time (Paas et al., 2010; Sweller, 2019). Due to the limitations of working memory to process new information, receiving excessive unfamiliar information will cause cognitive overload (Chen et al., 2018). This means the learning process will be clumsy and ineffective.

Cognitive load may be defined as the total amount of mental energy imposed on working memory when accomplishing a learning task simultaneously (Chen & Kalyuga, 2020). Cognitive load is categorized as intrinsic, extraneous and germane cognitive load (Chen & Kalyuga, 2020; Sweller, 2019; Sweller et al., 2019a). The cognitive load categories may be used to explain which source working memory might be exposed during learning (Paas et al., 2010; Sweller et al., 2011). Cognitive load theory is an instructional design theory that focuses on activities of schema acquisition and automation. The theory suggests that an instruction must be designed in such a way as not to exceed the capacity of working memory. The main concern of cognitive load theory is how to increase germane cognitive load that is the process of constructing schemas, and facilitating schema automation.

Paas & Van Merriënboer (2020) elaborate these cognitive loads. First intrinsic cognitive load is determined by the amount of elements of interactivity where a complex task might force working memory to work. Second, extraneous cognitive load is related to difficulty level caused by the instructional design used by the teacher. This might also be influenced by a student's prior knowledge, prior thoughts, affections, and disturbing information during the learning process in the classroom. Third, the germane cognitive load is resulted when the extraneous cognitive load could be kept at a minimum level since it might assist students in focusing their working memory on the essential part of the learning.

A worked example can be used as a technique to reduce extraneous cognitive load (Sweller, 2019) when it is designed by paying attention to the cognitive capacity of students (Sweller et al., 2011). An effective worked example basically contains a problem specification accompanied by a successful solution that leads to easy understanding. It is argued that example-based learning is a fast study strategy for novices (Renkl, 2014; Widyastuti & Retnowati, 2021). Although problem based learning with minimum guidance is widely suggested, there is numerous evidence showing that novice learners learn more from

studying with worked examples than from just trying solving problems themselves (Manson & Ayres, 2021). Renkl & Atkinson (2010) reviewed many studies that provide evidence of the effectiveness of using worked example compared to learning through problem-solving. More recent studies, such as those conducted by Cardellini (2014) and van Gog et al., (2011) also confirm the former. When learning a new task with a problem-solving approach, students might be exploited to use a large part of their working memory for trial-and-errors, resulting in the unavailability of cognitive resources for learning and constructing schemas. On the other hand, when learning a new task is guided by worked examples, learning is assisted in efficient manners. However, not all worked examples may be good (Sweller, 2019). It was found that some designs of worked examples might simulate split attention or redundancy effect, and hence be confusing (Retnowati & Marissa, 2018; Sweller, 2019).

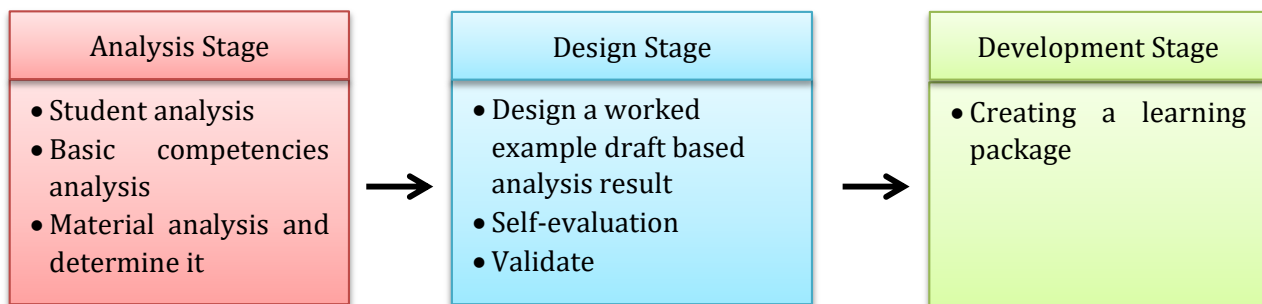
More studies on designs of worked examples are needed therefore the theory can be practically implemented by teachers. Retnowati & Marissa (2018) designed a worked example in tangent lines to circle learning while Azizah & Retnowati (2017) created a worked example in learning angles and lines. Likewise, Rodiawati & Retnowati (2019) showed a worked example in learning patterns. These discuss how to design the worked examples. A mathematics learning material of two-dimensional shapes of quadrilateral and triangle problems is intended to be the focus of the current article. Problem solving in this topic requires at least spatial ability and strategy. This article also attempt to explain the use of worked example learning that can be applied by teachers in the classroom, so that the main goal in using worked example remains on track.

## **B. METHODS**

This type of study is design and development research. This study primarily used the three steps to conceptualize the design of the intended worked example. ADD is used which means analysis, design and development, it is often used as a guidance to design an educational product. Analyze stage is a pre-planned development of a worked example. Begins with analyzing students, i.e. who will use by breaking down the objective of studying the worked example. Followed by exploring the basic competencies used in the curriculum. Subject matter analysis was carried out, so can assume the prior knowledge owned by students who are taking the worked examples, the essential knowledge to deliver and the depth of the content on the worked example. This is where the intrinsic cognitive load of the topic studied is managed.

Design stage is writing down a draft of a worked example in the topic anticipating all the analysis results. Another important thing is to explore the criteria for effective worked examples, namely avoiding split attention and redundancy. This cause might be sources from the wording, graph, positioning, segmentation, or the step-by-step instruction of the task. Before being validated, the researcher independently evaluates and improves the draft prepared. Validation was carried out by three experts in their field, this was reviewed from the characteristics of the content, construct, and language. Panel discussions were held with lectured and doctoral student. The discussion results are used to improve the draft. This is where the extraneous cognitive load of the topic studied is reduced. The development stage is creating a learning package consisting of worked example pairs based on the selected design

and how it is used. Figure 1 describes the stages of developing a worked example, as shown in Figure 1.



**Figure 1.** The stage of study

### C. RESULT AND DISCUSSION

This study resulted in a design of a worked example in learning composites of quadrilaterals and triangles. The following discussion will elaborate the results of each step during the development of the design.

#### 1. Analysis

Sweller (2019) that the worked example is recommended as an instructional design for novice learners. Therefore, the selected topic should consist of high element interactivity learning material. The main objective of learning is “calculating the area of two-dimensional”. Since students have prior knowledge on the type of quadrilaterals, the selected topic should be the composite shapes. Students may have learned calculating the area of each quadrilateral, such as square, rectangle, trapezoid, parallelogram, rhombus, and kite, in their previous grade. The essential knowledge to deliver is the Pythagorean theorems as new knowledge and is used in solving the compound shape problem. The depth of content is related to how students analyze quadrilaterals based on their prior knowledge, so that students can make auxiliary lines, followed by calculating the area of geometrical shape.

#### 2. Design

According to Sweller, Ayres & Kalyuga (2011) several effects like expertise reversal, split-attention and redundancy, and pairing problems must be considered when designing worked example for learning so that they do not become an extraneous cognitive load. Expertise reversal which is related to instructional design will be effective to teach students who have low initial abilities. However, it is not efficient for students who have high initial abilities (Sweller et al., 2019a). Split attention occurs when information is presented separately and is not integrated, thus making the readers have to divide their attention while integrating various sources of information at once (Ayres & Sweller, 2012; Sweller, 2011; Sweller et al., 2019b) which causes the extraneous cognitive load to increase. This condition can be overcome by integrating problem-solving steps in one adjacent room and/or using arrows, bold lines, and different colors. This can help students stay focused and not divide their attention. Redundancy occurs when the information presented is repetitive or overlapping because excess information that can be understood separately is presented simultaneously (Retnowati, 2014). It can be overcome by designing questions that do not contain

unnecessary information. The design of the worked example was validated by three lecturers and one doctoral student who was an expert in his field. The first design is presented in Figure 2.

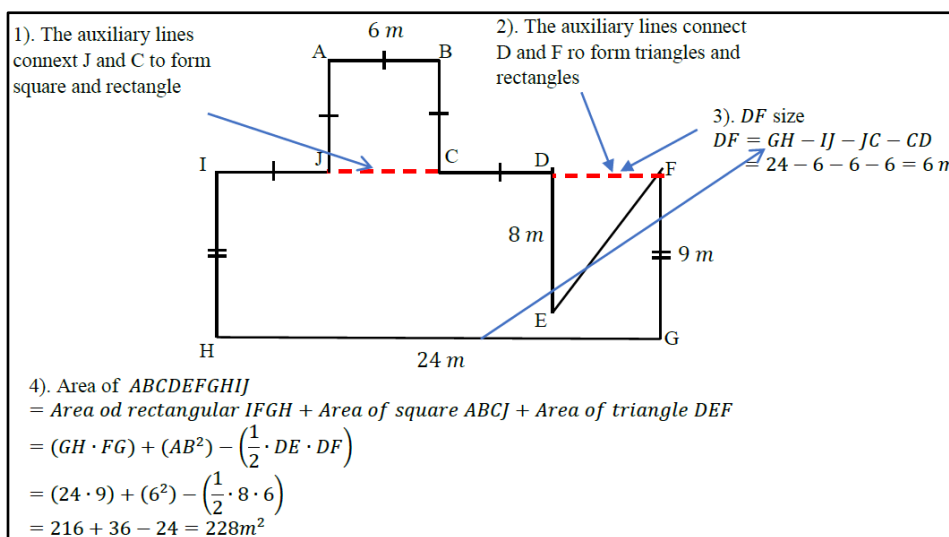


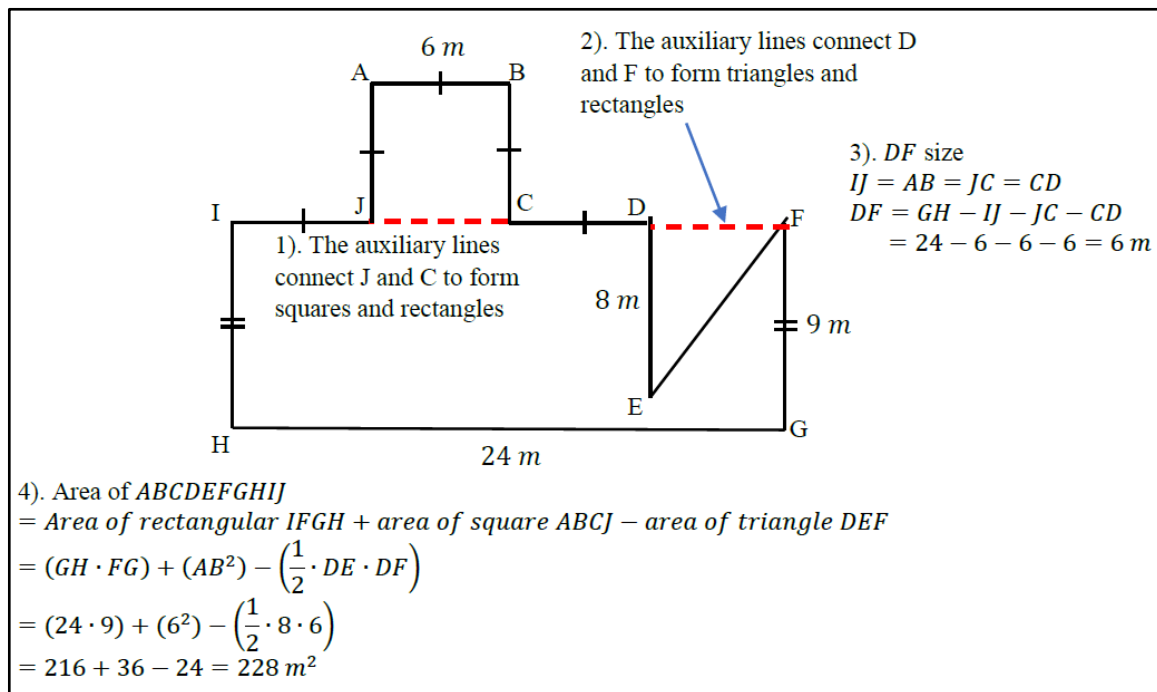
Figure 2. The draft or worked example

The results of the validation provided by the expert are presented in Table 1.

Tabel 1. Expert Review Results

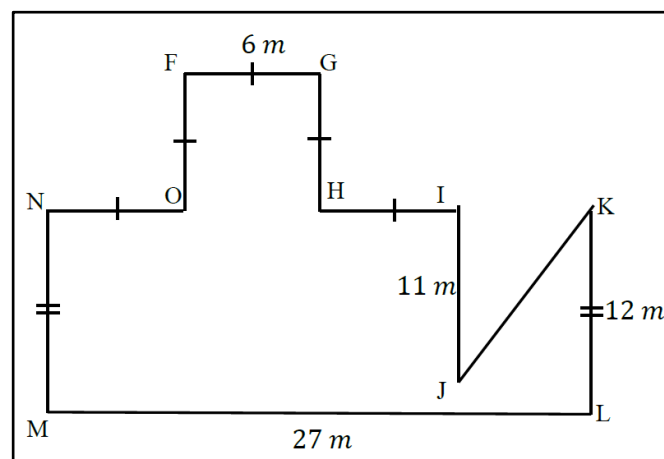
Validator	Suggestion and comments
Lecturers and doctoral students	<ul style="list-style-type: none"> <li>To many arrows, explaining unnecessary things</li> <li>Redundancy</li> <li>Wrong symbol in the formula for finding the are of a shape</li> <li>Stage 1) solutions can be integrated</li> </ul>

The results of suggestions and comments are used to improve the worked example design. The design is revised according to Table 1 and declared valid. Pairing the problem is done to present questions in the form of isomorphism with the previously given worked example that was solved without looking at the example. This is done to see students' understanding after studying the previous worked example. Students will also have the opportunity to automate what they have just learned from the example and be familiarized with remembering the concepts learned to be stored in long-term memory. The following is a worked example followed by a problem to be solved by students after being validated by experts and ready to be used in learning. The first worked example 1 is a compound shape of a rectangle, square and triangle shape shown in Figure 3. The Instruction: Read the following example, pay attention and understanding to steps 1, 2, 3, and 4. The worked example 1: Find the area of the two dimensional  $ABCDEFGHI!$ , as shown in Figure 3.



**Figure 3.** Example 1: Area of Quadrilateral and triangle

Figure 3 presents a worked example 1 by finding the area of two-dimensional. Students are asked to read the instruction, and the completion of the example is considered according to the steps. This is so that it is easy to understand, and not to think about matching and confirming. The next page presents the paired problems of worked example 1, this is presented in the form of isomorphism, the use of which has been explained previously. The paired problems of worked example 1 shown in Figure 4. The paired problem 1. Find the area two dimensional  $FGHIJKLMN$ !, as shown in Figure 4.



**Figure 4.** The Paired Problem of Worked Example 1

After students have finished studying and understanding worked example 1 (Figure 3), students are asked to solve problem 1 (Figure 4), and make sure students do not see the previous example. It is presented on a different page. Second, worked example 2 is a compound shape of a rhombus, trapezoid, and triangle shape shown in Figure 5. The Instruction: Read the

following example, pay attention and understanding to steps 1, 2, 3, 4 and 5. The worked example 2: Find the area of the two-dimensional  $ABCDEF$ GH!, as shown in Figure 5.

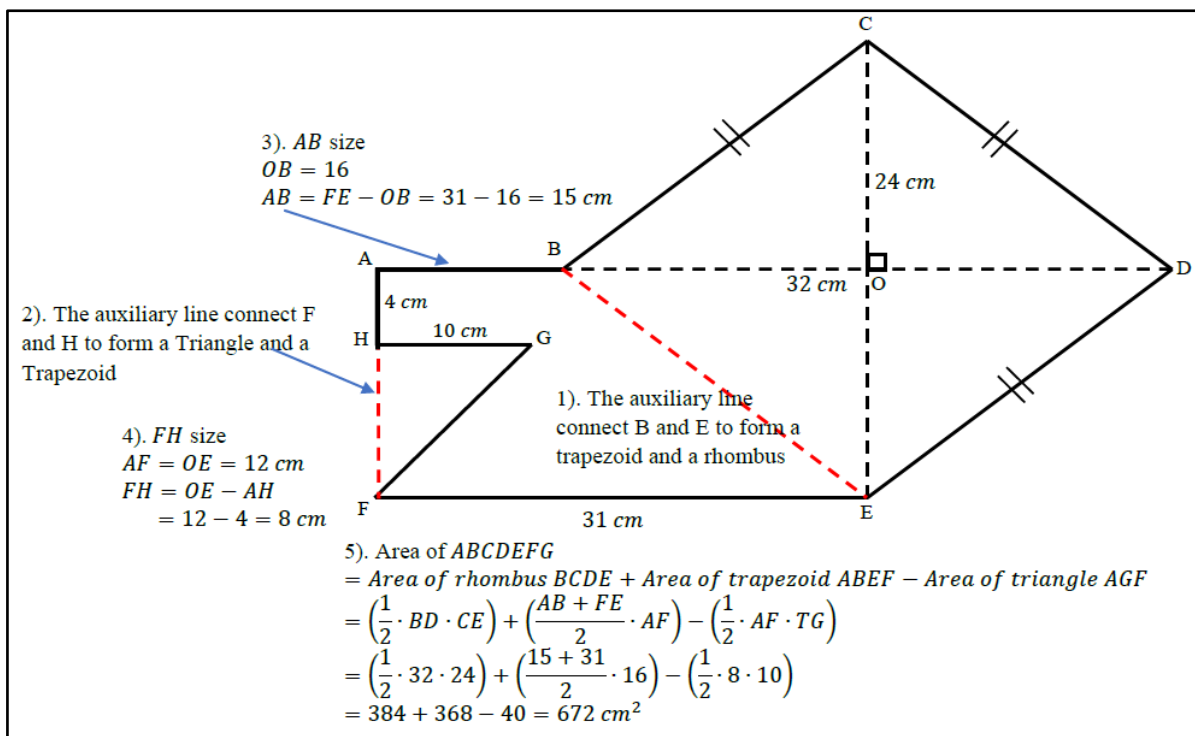


Figure 5. Example 2: Area of Quadrilateral and Triangle

The worked example 2 is presented in Figure 5 by finding the area of two-dimensional. Instructions for using sample solutions will always be noticed. The next page presents the paired problems of worked example 2. The paired problems of worked example 1 shown in Figure 6. The paired problem 2. Find the area two-dimensional  $FGHIJKLMN$ !, as shown in Figure 6.

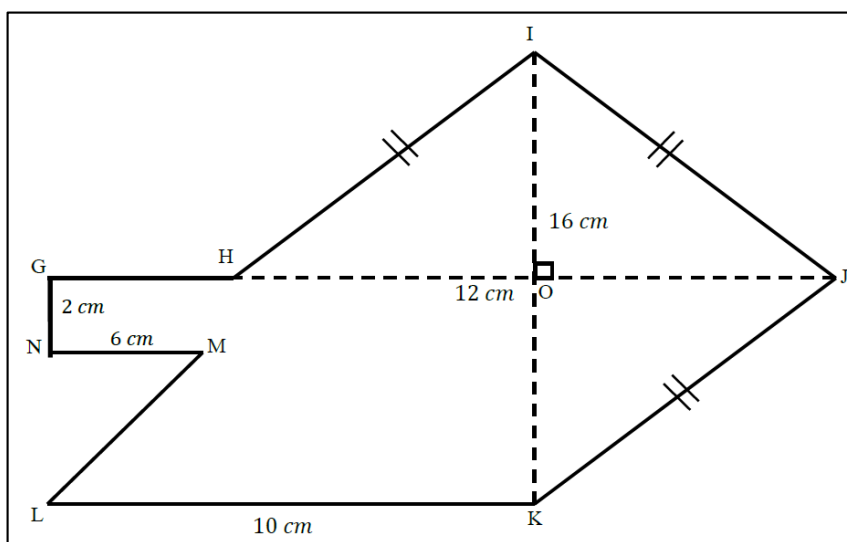
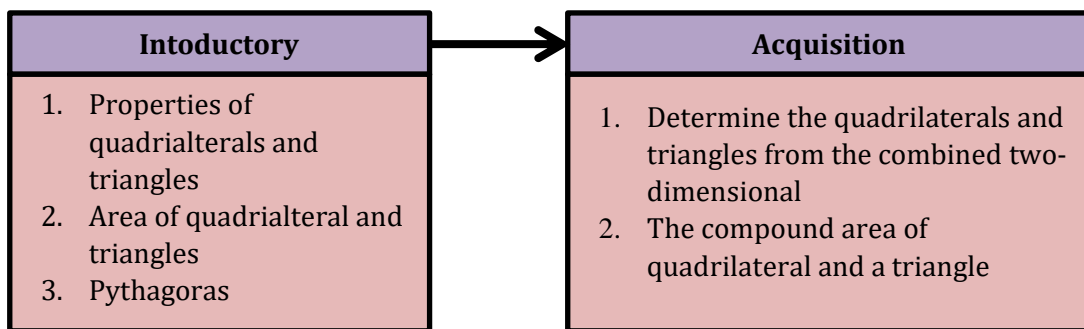


Figure 6. The Paired Problem of Worked Example 2

Worked example 2 (Figure 5) has been studied and understood. Students can shift to the next page to solve the paired problems of worked example 2 (Figure 6). It will always be solve without seeing the worked example.

## 2. Development

Sweller, Ayres & Kalyuga (2011) describe the phase of using worked example which consists of an introductory and acquisition phase. The first phase, introductory, is the learning phase where students learn prerequisite material that must be mastered before learning the new ones. The goal is to generate prior knowledge possessed by students to be integrated into new problems. Meanwhile, the acquisition phase is the learning phase where a worked example is used. Students are given examples that must be studied carefully by students. By giving some example, students will get an idea of the strategies that can be used to solve problems. Teacher play an important role in motivating students to understand the examples given, by reminding students to study the example thoroughly and not just memorize them. The following is an illustration of how learning is delivered using a worked example is shown in Figure 5.



**Figure 5.** Schematics Relevant to the Material of Quadrilaterals and Triangles

After students understand the example given, questions in the form of isomorphism are given for students to complete without seeing the previous examples. Then, students learn a worked example adapted from Retnowati et al (2010) and Rohman & Retnowati (2018) where students present the results of their work in solving problems and conclude the learning outcomes. Giving answer keys may be deemed necessary for students to validate the results of their work.

Evidence of the effectiveness of worked example based instruction has been demonstrated by cognitive load theorists. The use of worked example is suitable for presenting complex material and novice students (Rodiawati & Retnowati, 2019). Learning with worked example for this material is considered important, learn with guided instruction is more effective than conventional problem-solving to teach novice students (Kalyuga, 2011; Sweller et al., 2011). It is possible that when teaching doing learning using worked example in the classroom, the teacher considers which one is more suitable for the students to be completed as an individual or in group assignment. Regarding the learning phase using the worked example described above, when students are given questions to solve without seeing examples, this is an opportunity for students to implement their understanding into the questions independently. Research related to how worked example task should be given as individual or group worked has been carried out previously by several expert, for example research by Retnowati, Ayres & Sweller (2010). The study showed that worked example learning with high test complexity is better studied individually than in groups.

Moreover, further studies are suggested to be able to carry out all stages of ADDIE development from the worked examples that have been developed. Researchers also suggest



that the results of this study can be used in the classroom as needed. Eventually, it might be suggested that comprehensive consideration when designing worked example that contain pictures is required.

#### D. CONCLUSION AND SUGGESTIONS

Material on composites of quadrilaterals and triangles are more complex, which students might not be provided an explicit shape of two-dimensional quadrilaterals and triangles. This paper shows that the worked examples to study this material. Designing effective worked example can pay attention to split-attention and redundancy effect. Using integration, minimized wording and more procedure implementation is the solution. Reducing search and match procedures is important, so as not to use up working memory capacity for irrelevant things. Arranging sequential solutions, signaling different colors and consistency of layout within overall material can promote mindfulness resources. This has generated a worked example with paired problems, which are declared valid. The design and development of worked example have resulted in instructional designs that manage intrinsic cognitive load, reduce extraneous cognitive load, and facilitate germane cognitive load. It might be suggested that comprehensive consideration when designing worked example that contain pictures is required. A follow up inquiry should be done by implementing this design in real a classroom to draw empirical data that inform the effectiveness of the development.

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