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The (Strong) Rainbow Connection Number of Join of Ladder and Trivial Graph

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ABSTRACT

Let $G = (V, E)$ be a nontrivial, finite, and connected graph. A function c from E to $\{1, 2, \dots, k\}$, $k \in \mathbb{N}$, can be considered as a rainbow k-coloring if every two vertices x and y in G has an $x - y$ path. Therefore, no two path's edges receive the same color; this condition is called a "rainbow path". The smallest positive integer k, designated by $rc(G)$, is the G rainbow connection number. Thus, G has a rainbow k-coloring. Meanwhile, the c function is considered as a strong rainbow k-coloring within the condition for every two vertices x and y in G have an $x - y$ rainbow path whose length is the distance between x and y. The smallest positive integer k, such as G, has a strong rainbow k-coloring; such a condition is called a strong rainbow connection number of G, denoted by $src(G)$. In this research, the rainbow connection number and strong rainbow connection number are determined from the graph resulting from the join operation between the ladder graph and the trivial graph, denoted by $rc(L_n \vee K_1)$ and $src(L_n \vee K_1)$ respectively. So, $rc\ (L_n \vee K_1) = src\ (L_n \vee K_1) = 2$, for $3 \le n \le 4$ and $rc\ (L_n \vee K_1) = 3$, while $src(L_n \vee K_1) = \left\lceil \frac{n}{2} \right\rceil$, for $n \ge 5$.

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A. INTRODUCTION

In the context of graphs, the idea of "rainbow connection" was initially proposed by Chartrand et al. (2008). Let $G = (V, E)$ be a graph with vertex set V and edge set E. A coloring $c: E \to \{1, 2, \dots, k\}$, $k \in \mathbb{N}$, thus, the adjacent edges can share an identical color. Let x and y be in V. An $x - y$ path in G is determined as a rainbow path if each path edge has a different color. Meanwhile, G is determined as a rainbow connected if every two vertices x and y has a rainbow path. Rainbow coloring is the term used to describe an edge color on G that connects to the G rainbow. If the c function employs k colors, it constitutes the rainbow k −coloring. The smallest positive integer k, designated by rc(G), is the rainbow connection number of G. Thus, G has a rainbow k-coloring. Meanwhile, the $x - y$ rainbow path is considered as the x − y rainbow geodesic if the length of the path constitutes the distance between x and y. G becomes a strong rainbow connected if every two vertices x and y have an $x - y$ rainbow geodesic. Strong rainbow coloring is edge coloring on G that gives G a strong rainbow connection. If the c function uses k colors, it is said to have a strong rainbow k −coloring. The lowest positive integer k is the strong rainbow connection number of G, indicated by $src(G)$. As a result, G has a strong rainbow k-coloring, and $rc(G) \leq src(G)$ for any connected graph G.

On condition that G is a rainbow connection, the least diam(G) colors are necessary; the diam(G) refers to the G's diameter. On the other hand, rainbow coloring is defined by G if each of its edges is colored differently. Hence, the formula is as follows.

$$\text{diam}\,(G) \leq rc(G) \leq src(G) \leq m. \tag{1}$$

Several previous studies have investigated both the rainbow connection number and strong rainbow connections number. Chartrand et al. (2008) have determined some $rc(G)$ and $src(G)$ of connected G graphs, as follows.

Proposition 1. Let G be a nontrivial connected graphs of size m. Then

1. $src(G) = 1$ if and only if G is a complete graph
2. $rc(G) = 2$ if only if $src(G) = 2$
3. $rc(G) = m$ if only if G is a tree.

The strong rainbow connection number of stellar graphs, which is a corona product of a trivial graph and an m-copies ladder graph, was discovered by Shulhany and Salman back in 2016 (Shulhany & Salman, 2016). On the other hand, Fitrianda et al. (2018) have determined a generalized triangular ladder graph's rainbow connection number and strong rainbow connection number. Meanwhile, (L. Chen et al., 2018) present some results of the six rainbow connection parameters. Other previous studies have found other results of (strong) rainbow connection of graph (H. Li et al., 2011; Schiermeyer, 2011).

The concept of rainbow connectivity can apply to data security. Confidential information should be protected from being transferred from one party to another. The security system must be able to prohibit not just unauthorized users from accessing the system, but also users who are already signed in from performing actions that they are not permitted to perform (Morris & Thompson, 1979). A security cracking method known as a rainbow table employs a precalculated table of inverted password hashes to decipher database passwords. The user is verified whether the values match. The rainbow table database is utilized to decrypt the password hash and get authentication (Zhang, Tan & Yu, 2013). To minimize data leakage from such confidential information, each agent should have a different password when transferring information. As a result, a lot of passwords are needed. Fortunately, the rainbow connection concept requires a minimum number of passwords so that two agents can exchange information with different passwords.

Apart from this motivation, the concept of rainbow connection is interesting to study. Many researchers have developed the concept of rainbow connectivity by applying it to various types of graphs. One of them is by performing operations on several graphs and specifying $rc(G)$ and $src(G)$ on those graphs. Li and Sun (2012) have obtained several results of the rainbow connection number of graph products, comprising lexicographic products, join, cartesian products, etc. Meanwhile, Resty and Salman (2015) have determined the rainbow connection number of the corona product of the n-crossed prism graph with the trivial graph. Septyanto and Sugeng (2017) give lower and upper limits for rainbow connectison numbers and strong rainbow connection numbers by joining two graphs based on individual graph parameters: the number of vertices of degree 0, maximum degree, independent dominance number, clique number, and independent number.

Li and Ma (2017) have done related study that calculated the rainbow connection number of operating graphs by using several ways such as a) includes a union of graphs, b) adds and removes edges, and c) adds vertices. Strong and accurate values of rainbow connection numbers, as well as their upper bounds of comb product, path, triangular book, circular, or fan graphs are discussed by Dafik et al. (2018). In addition, several previous studies focus on examining rainbow connection numbers from operating graphs (Basavaraju et al., 2014; X. Chen et al., 2019; Fitriani & Salman, 2016; Gembong & Agustin, 2017; Gologranc et al., 2014; Liu, 2014; Maulani et al., 2019; Doan, Ha, and Schiermeyer, 2022).

To two agents to share information while using different passwords, the rainbow connection idea allows for the requirement of a minimum number of passwords. The concept of the rainbow link is intriguing to examine even without this reason. Research on rainbow connection numbers from the join operation of ladder and trivial graphs has never been done. (Kartika, 2020) assert that the ladder graph, denoted by L_n, is obtained from two duplicates of the path P_n, becoming $P_{n_1}$and P_{n_2}. The vertex of v_i on P_{n_1} is connected to the vertex of w_i on P_{n_2} by an edge, with $i = 1, 2, \dots, n$. The trivial graph refers to a graph with only one vertex, denoted K_1 (Diestel, 2005). Meanwhile, let G_1 and G_2 connected graphs. The process of joining two disconnected graphs G_1 and G_2, denoted by $G_1 \vee G_2$, is the graph with vertex set $V(G_1) \cup V(G_2)$, and edge set of $V(G_1) \cup V(G_2) \cup \{uv | u \in V(G_1), v \in V(G_2)\}$ (Li & Sun, 2012). This research determined the (strong) rainbow connection number from the graphs resulting from the joint operation of ladder graphs and trivial graphs.

B. METHODS

This research is a literature study. The purpose of this study is to determine the rainbow connection number and strong rainbow connection number from graphs resulting from the join operations of ladder and trivial graphs. The steps in this research are as follows:

1. Defining the problem to be discussed
2. Doing a literature study on rainbow connected numbers and strong rainbow connected numbers
3. Describing a graph resulting from the join operation of a ladder graph and a trivial graph, denoted by $L_n \vee K_1$. The vertex set and edge set of $L_n \vee K_1$ are defined as follows.
   Definition 1. *Let $n \geq 3$ be an integer. The graph resulting from the join operation of a ladder graph and a trivial graph is a graph with*
   $V = \{u\} \cup \{v_i | i \in \{1, 2, \dots, n\}\} \cup \{w_i | i \in \{1, 2, \dots, n\}\}$
   $E = \{uv_i |\, i \in \{1, 2, \dots, n\}\} \cup \{uw_i |\, i \in \{1, 2, \dots, n\}\} \cup \{v_i w_i | i \in \{1, 2, \dots, n\}\} \cup$
   $\{v_i v_{i+1} |\, i \in \{1, 2, \dots, n-1\}\} \cup \{w_i w_{i+1} |\, i \in \{1, 2, \dots, n-1\}\}$

   As an illustration, the graph $L_n \vee K_1$ as shown in Figure 1.

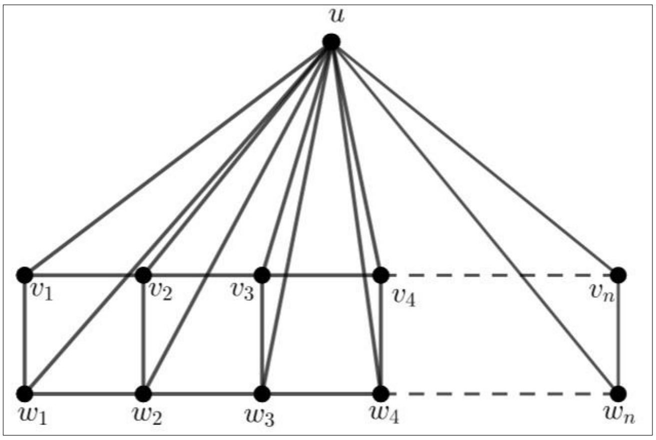

Figure 1. Graph $L_n \vee K_1$

Based on the picture above, it can be seen that $\text{diam}(L_n \vee K_1) = 2$.

4. Finding a rainbow coloring pattern for $L_n \vee K_1$ and determining the accurate values of $rc(L_n \vee K_1)$ and $src(L_n \vee K_1)$.
5. Proving the rainbow connection number and the strong rainbow connection number of $L_n \vee K_1$. If $rc(L_n \vee K_1) = k$, $rc(L_n \vee K_1) \geq k$ and $rc(L_n \vee K_1) \leq k$ should be done. To prove lower bound of $rc(L_n \vee K_1) \geq k$, it is necessary to show a reason for the absence of rainbow coloring with $k-1$ color or less. Proving upper bound of $rc(L_n \vee K_1) \leq k$ by constructing a rainbow coloring in $L_n \vee K_1$ using k colors. The same thing is also done for the $src(L_n \vee K_1)$.
6. Formulating conclusions based on the results of the theorem analysis that has been proven.

C. RESULTS AND DISCUSSION

In this section, we calculate the rainbow connection number and the strong rainbow connection number of the following: the joined ladder graph and trivial graph; according to Theorem 1 and Theorem 2, respectively.

1. The Rainbow Connection Number of $L_n \vee K_1$

Theorem 1. *Let n be a positive integer with an $n \geq 3$. The rainbow connection number of $L_n \vee K_1$ is as follows.*

$$rc(L_n \vee K_1) = \begin{cases} 2, & \text{for } 3 \leq n \leq 4; \\ 3, & \text{for } n \geq 5. \end{cases}$$

Proof.

First, we prove a lower bound of $rc(L_n \vee K_1)$.

For $3 \leq n \leq 4$ is determined using the inequality (1), and we obtain $rc(L_n \vee K_1) \geq 2$.

For $n \geq 5$, suppose that $rc(L_n \vee K_1) \leq 2$.

Furthermore, there is a rainbow 2-coloring c^* for $(L_n \vee K_1)$. With no loss of generality, we assume that $c^*(uv_1) = 1$ and consider v_1 and v_5. Meanwhile, $d(v_1v_5) = 2$ and the only path from v_1 to v_5 with length 2 is $v_{1,}u, v_5$ it follows that $c^*(uv_5) = 2$. Then, we consider v_1 and w_3. $d(v_1w_3) = 2$ and the only path from v_1 to w_3 with length 2 is $v_{1,}u, w_3$, it follows that

$c^*(u, w_3) = 2$. Then we consider w_3 and v_5. Meanwhile, $c^*(uv_5) = 2$, $c^*(uw_3) = 2$, and the only path from w_3 to v_5 with length 2 is $w_{3,}u, v_5$ prove that $w_3 - v_5$ rainbow path was not found. We get a contradiction with $rc(L_n \vee K_1) \geq 3$. Next, we prove an upper bound of $rc(L_n \vee K_1)$ by dividing the proof into two cases based on the value of n.

Case 1. $3 \leq n \leq 4$

The 2-coloring, $c : E(Pd_n) \rightarrow [1,2]$ is defined using the following formula.

$$c(e) = \begin{cases} 1, & \text{if } e \in \{uv_i, uw_i\} \text{ for } i \in \{1,2\} \text{ and } e \in \{v_iv_{i+1}, w_iw_{i+1}\} \text{ for } i \in [1,3]; \\ 2, & \text{if } e \in \{uv_i, uw_i\} \text{ for } i \in \{3,4\} \text{ and } e \in \{v_iw_i\} \text{ for } i \in [1,4]. \end{cases}$$

Case 2. $n \geq 5$

The 3-coloring, $c : E(Pd_n) \rightarrow [1,3]$ is defined using the following formula.

$$c(e) = \begin{cases} 1, & \text{if } e \in \{uv_i, uw_i\} \text{ for odd } i \in [1,n] \text{ and } e \in \{v_iw_i\} \text{ for } i \in [1,\mathrm{n}]; \\ 2, & \text{if } e = uv_i \text{ where } i \in [1,n] \text{ is even and } e = w_iw_{i+1} \text{ for } i \in [1,\mathrm{n}-1]; \\ 3, & \text{if } e = uw_i \text{ where } i \in [1,n] \text{ is even and } e = v_iv_{i+1} \text{ for } i \in [1,\mathrm{n}-1]. \end{cases}$$

The formulas show that every two adjacent vertices x and y in $V(L_n \vee K_1)$ have an $x - y$ rainbow path. Meanwhile, every x and y in $V(L_n \vee K_1)$ with $d(x,y) = 2$ for $3 \leq n \leq 4$ has an $x - y$ rainbow path (see Table 1). Every x and y in $V(L_n \vee K_1)$ with $d(x,y) = 2$ for $n \geq 5$ has an $x - y$ rainbow path (see Table 2). The proves of a lower bound and an upper bound of $rc(L_n \vee K_1)$ complete the proof, as shown in Table 1 and Table 2. ■

Table 1. An $x - y$ rainbow path on $L_n \vee K_1$ for $3 \leq n \leq 4$

x	y	Condition (i and j in $[1,n]$)	$x - y$ rainbow path
v_i	v_j	-	v_i, u, v_j
w_i	w_j	-	w_i, u, w_j
v_i	w_j	$j - i = 1$	v_i, w_i, w_j
		$i - j = 1$	v_i, w_{j+1}, w_j
		$\lvert j - i \rvert \neq 1$	v_i, u, w_j

Table 2. An $x - y$ rainbow path on $L_n \vee K_1$ for $n \geq 5$

x	y	Condition (i and j in $[1,n]$)	$x - y$ rainbow path
v_i	v_j	i and j have equal standing with $i < j$.	v_i, u, v_{j-1}, v_j
		i and j have varying parities.	v_i, u, v_j
w_i	w_j	i and j have equal standing with $i < j$.	w_i, u, w_{j-1}, w_j
		i and j have varying parities.	v_i, u, v_j
v_i	w_j	i and j have equal standing with $i < j$.	v_i, u, w_{j-1}, w_j
		i and j have varying parities.	v_i, u, v_j

The rainbow of 2-coloring of $L_4 \vee K_1$ and the rainbow of 3-coloring of $L_6 \vee K_1$ are illustrated in Figure 2.

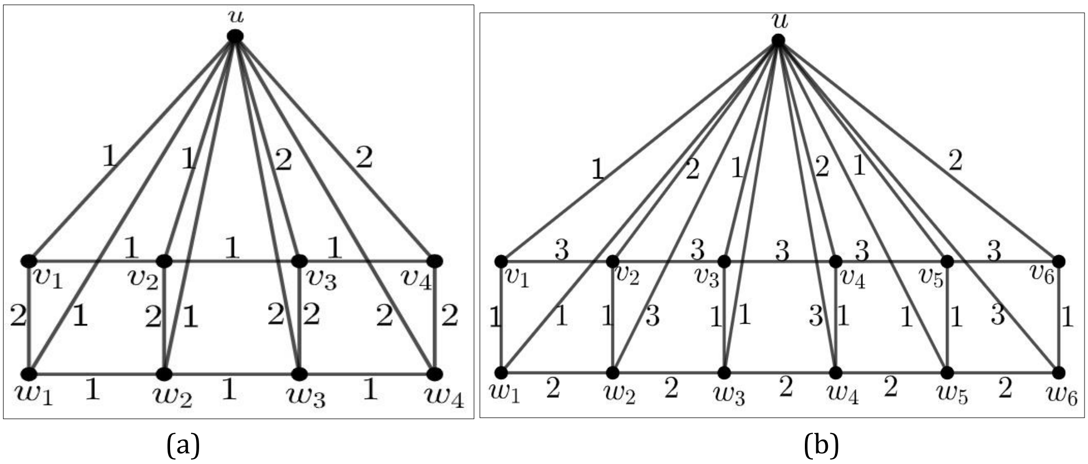

Figure 2. (a) a rainbow of 2-coloring of $L_4 \vee K_1$; (b) a rainbow of 3-coloring of $L_6 \vee K_1$

Figure 2(a) shows the 2-coloring rainbow on graph $L_4 \vee K_1$. For example, vertex v_1 and w_4 are selected. Based on Table 1, the rainbow path that connect them is v_1, u, w_4. Additionally, suppose v_4 and w_3 are selected. In the same way, the rainbow path that connects these points is v_4, w_4, w_3. Meanwhile, Figure 2(b) shows the rainbow 3-coloring on graph $L_6 \vee K_1$. It has been explained in the proof of Theorem 1 that for $n \geq 5$ it takes 3 colors to color the vertices on graph $L_6 \vee K_1$ so that every two vertices have a rainbow path. For example, vertex v_1 and w_5 are selected. Based on Table 2, the rainbow path is v_1, u, w_4, w_5.

2. The Strong Rainbow Connection of $L_n \vee K_1$

Theorem 2. *Let n be a positive integer with an $n \geq 3$. The strong rainbow connection number of $L_n \vee K_1$ is calculated using the following formula.*

$$src(L_n \vee K_1) = \left\lceil \frac{n}{2} \right\rceil, \text{for } n \geq 3$$

Proof.

By using Proposition 1. b) and Theorem 1, we obtain $src(L_n \vee K_1) = 2$ for $3 \leq n \leq 4$.

Let $k = \left\lceil \frac{n}{2} \right\rceil$. For $n \geq 5$, we discover that $src(L_n \vee K_1) \geq k$. If $src(L_n \vee K_1)$ is $\leq k-1$, a strong rainbow $k-1$ -coloring c^* exists. $L_n \vee K_1$ consists of a ladder graph $L_n \coloneqq v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n$ and u are adjacent to v_i and w_i for every $i = 1, 2, \dots, n$. An integer k also exists with $2k - 1 \leq n \leq 2k$. Since $d(u) = 2n > 2(k-2)$, $A \subseteq V(L_n)$ exists so that $|A| = 3$ and all edges in $\{ut: t \in A\}$ have the same color. There are at least two vertices $t', t'' \in A$ with $d_{L_n}(t', t'') \geq 3$ and $d_{L_n \vee K_1}(t', t'') = 2$. Since t', u, t'' is the only geodesic in $L_n \vee K_1$, $t' - t''$ rainbow geodesic in $L_n \vee K_1$, does not exist; this is a contradiction. Thus, $src(L_n \vee K_1) \geq k$.

Afterward, we show $src(L_n \vee K_1) \leq k$.

The a k-coloring, $c\,; E(L_n \vee K_1) \to \{1, 2, \dots, k\}$ is defined using the following formula.

$$c(e) = \begin{cases} 1, & \text{if } e \in \{v_i v_{i+1}, w_i w_{i+1}\} \text{ and odd } i \in [1, n-1]; \\ 2, & \text{if } e \in \{v_i v_{i+1}, w_i w_{i+1}\} \text{ and even } i \in [1, n-1]; \\ 3, & \text{if } e = v_i w_i \text{ and } i \in [1, n]; \\ j+1, & \text{if } e \in \{uv_i, uw_i\} \text{ and } i \in \{2j+1, 2j+2\} \text{ for } 0 \leq j \leq k-1. \end{cases}$$

Every x and y in $V(L_n \vee K_1)$ for $n \geq 5$ has an $x - y$ rainbow geodesic, as shown in Table 3. The results of the calculation complete the proof. ■

Table 3. An $x - y$ rainbow geodesic on $L_n \vee K_1$ for $n \geq 5$

x	y	**Conditions (i and j in $[1, n]$)**	**$x - y$ rainbow geodesic**
v_i	v_j	-	v_i, u, v_j
w_i	w_j	-	w_i, u, w_j
v_i	w_j	$j - i = 1$ $i - j = 1$ $\lvert j - i \rvert \neq 1$	u_i, w_i, w_j v_i, w_{j+1}, w_j v_i, u, w_j

A strong rainbow of 4-coloring of $L_8 \vee K_1$is illustrated in Figure 3.

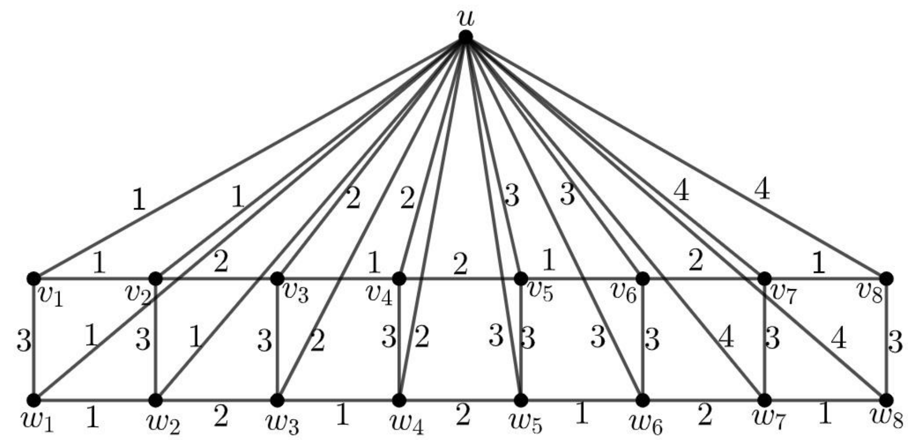

Figure 3. a strong rainbow of 4-coloring of $L_8 \vee K_1$

Figure 3 shows the strong rainbow 4-coloring on graph $L_8 \vee K_1$. Unlike the rainbow coloring, the rainbow path on the strong rainbow coloring that connects two vertices on the graph must be the same size as the distance between the two vertices. Therefore, in a graph $L_n \vee K_1$ the more the number of vertices, the more colors it takes to become a strong rainbow coloring. Suppose that points v_i and w_j are chosen with $|i - j| \geq 2$, then the rainbow path must pass through point u. This is because the only paths of length 2 are v_i, u, w_j; or what is known as rainbow geodesic.

As explained earlier, this rainbow coloring can be applied to security issues. If it is associated with the join operation of a ladder graph with a trivial graph in this study, it can be likened to the point u being the center of information. Meanwhile, points v_i and w_j are information agents. To be able to exchange information between one agent and another, you must first pass through the information center. Of course, the concept of strong rainbow coloring provides benefits so that the transfer of information is safer.

D. CONCLUSION AND SUGGESTIONS

This study has determined the exact value of (strong) rainbow connection number of joined ladder graph and trivial graph using the following formulas: $rc(L_n \vee K_1) = src(L_n \vee K_1) = 2$, for $3 \leq n \leq 4$ and $rc(L_n \vee K_1) = 3$, for $n \geq 5$. This study has discovered $src(L_n \vee K_1) = \left\lceil \frac{n}{2} \right\rceil$, for $n \geq 5$. Data security can benefit from the rainbow connectivity concept. Transferring confidential information from one party to another should be prevented. Each agent should use a distinct password when sharing information to reduce data leaking. With the rainbow connection concept, two agents can share information while using separate passwords because it enables the requirement of a minimum number of passwords.

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