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Learning Trajectory in Problem-Based Mathematics Learning with Literacy and Numeracy Reinforcement: An Implementation of Lesson Study at Junior High School

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ABSTRACT

This study aims to explore the learning trajectory of problem-based learning with literacy and numeracy reinforcement. This study was conducted at Junior High School and involved 32 subjects from class VIII. This study employed Gravemeijer and Cobb's design research comprising three stages: preparing the experiment of hypothetical learning trajectory (HLT), designing the experiment by implementing the designed HLT-based learning, and conducting retrospective analysis by comparing HLT with the implemented learning. The learning trajectory in this study consisted of three components: learning objectives, a series of tasks and learning activities, and students' techniques to think and learn. This research was conducted based on lesson study activities to create a lesson study for the learning community. Design research stages corresponded to lesson study stages and included studying curriculum, formulating goals and plans, conducting research lessons, and reflecting the lesson. This study has revealed that students' actual learning trajectory has four levels of modeling: situational, referential, general, and formal levels. At the situational level, non-mathematical literacy becomes the initial reference for students to collaborate and communicate with others; thus, mathematical concepts are created. The referential level leads students to represent real problems in real mathematical concepts. The representation process aims to understand the concept in different situations. Finally, the students analogize the acquired understanding and bring it to the formal level to get a full understanding of the concept of one point position to another in Cartesian coordinates.

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A. INTRODUCTION

Careful learning planning should be implemented by a teacher to produce quality learning. When designing a lesson plan, a teacher will certainly think about how to present didactic situations that can challenge students, what are the students' conjecture responses, and how to create alternative assistance; these aspects should be prepared by teachers to predict whether the students face learning obstacles (Wake et al., 2016). The challenge for the students in question is to give assignments both in the form of sharing tasks and jumping tasks (Fauziyah et al., 2021b).

A lesson plan can show students' conjecture of learning trajectory in a class; such a condition is called a hypothetical learning trajectory. The hypothetical learning trajectory will produce the actual learning trajectory when the hypothetical learning trajectory is already piloted in the classroom. This actual learning trajectory is the real learning scheme. The actual learning trajectory, or simply called learning trajectory, is crucial for teachers to connect student-centered instruction with students' understanding and thinking. Therefore, the learning trajectory should be adequate and appropriate to support students' conceptual understanding (Nickerson & Whitacre, 2010).

Learning trajectory consists of three components: learning objectives, a series of tasks and learning activities, and students' strategies to think and learn (Clements & Sarama, 2014; Simon, 2020). The intended learning objective allows students to understand and achieve the expected concepts. Intended tasks and activities are a series of challenging tasks undertaken by students to achieve learning objectives. Meanwhile, students' intended ways of thinking refer to their thinking flow to understand the learning concept. Learning trajectory can predict the didactic situation in the learning process (Kenedi et al., 2019).

The method to develop a hypothetical learning trajectory is design research. Gravemeijer and Cobb (Van den Akker et al., 2006) propose that the design research method includes three stages: preparing the experiment, experimenting in the classroom, and conducting retrospective analysis. At the experiment preparation stage, the literature review is conducted to formulate the hypothetical learning trajectory based on the teacher's teaching experience. At the teaching experiment stage, in-class trials are conducted to investigate the formulated hypothetical learning trajectory. This stage requires in-depth analysis to obtain valid data. Each thinking stage of the students should be recorded in the form of a transcript. The final stage is retrospective analysis to scrutinize the data obtained from the teaching experiment. The analyzed data are obtained from observations, interviews, and transcripts and will reveal the actual students' learning scheme, or the so-called actual learning trajectory (Risdiyanti & Indra Prahmana, 2020) (Nur et al., 2022) .

Teachers can implement the resulting actual learning trajectory as input materials to prepare a hypothetical learning trajectory in the next material and cycle (Fonger et al., 2020). This activity is carried out collaboratively and emphasizes collegiality in the mathematics teacher group. Moreover, this activity is conducted based on lesson study activities to create a lesson study for the learning community. Design research stages correspond to lesson study stages and consist of studying curriculum, formulating goals and plans, conducting research lessons, and reflecting the lessons. These stages are presented in Figure 1. The stage of experiment preparation is conducted when studying the curriculum and formulating goals and plans. Meanwhile, the stage of the teaching experiment is conducted while conducting the research lesson. Finally, the stage of the retrospective analysis is conducted in the reflection stage (Lewis et al., 2011).

21^{st}-century skills are categorized into three (Ataizi & Donmez, 2020). The first is the learning and innovation skills, which consist of critical thinking and problem solving, communication and collaboration, and creativity and innovation. The second is literacy skills. The third is life skills that consist of flexibility and adaptability, initiative and self-direction, and social interaction. These classifications show that learning mathematics is crucial to cultivate these skills. Therefore, the selected model is a problem-based learning model with literacy and numeracy enforcement, as shown in Figure 1.

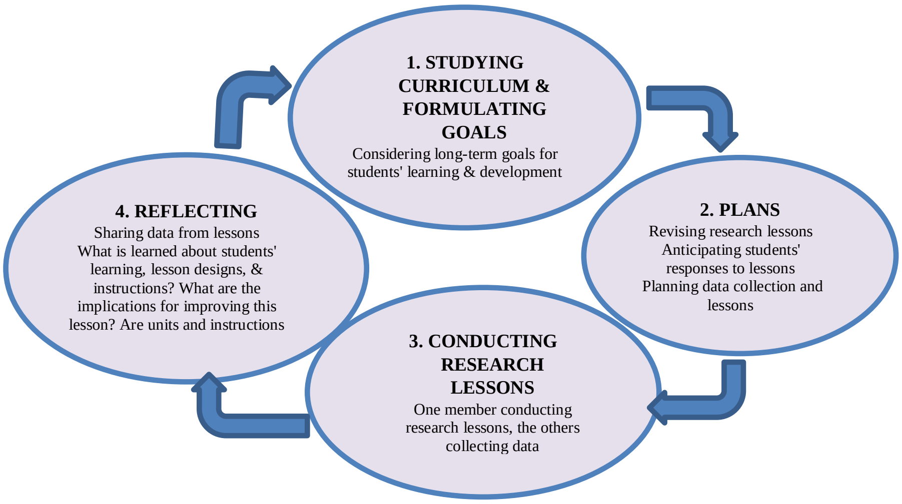

Figure 1. Lesson Study Flowchart

Problem-based learning is a learning model that applies real problems and enables students to acquire knowledge (Sari et al., 2021). Math problems in learning could challenge students (Fauziyah et al., 2019). The basic outline of the problem-based learning process is as follows: initially facing the problem, solving the problem with critical reasoning skills, identifying learning needs in an interactive process, conducting self-learning, applying newly acquired knowledge to the problem, and summarizing the learned material (Albanese & Dast, 2013). Math problems solved by students are derived from reading relevant literacy texts. In other words, problem-based learning strengthens literacy and numeracy (O'Donoghue, 2021).

Numeracy, also called numeracy literacy and mathematical literacy, is defined as the ability to apply mathematical concepts and skills to solve practical problems (Chan & Scalise, 2022). The results of this analysis are interpreted to predict and make decisions. In addition, numeracy includes the ability to analyze and interpret quantitative information around us to display various forms, such as graphs, tables, charts, and diagrams (Begum et al., 2021).

Numeracy problems derived from literacy are presented in two types. The first is task-sharing, which is equal to problems in textbooks that must be understood by all students. The second is task-jumping to provide challenges or exceed the levels of problems in the textbook. Task-sharing can foster students' collaboration and communication skills during the learning process. Meanwhile, task-jumping facilitates students with a level of thinking that goes beyond the specified learning achievement (Fauziyah et al., 2021a).

In this study the problems raised are everyday problems that are known to students in their environment. The problem used is a contextual problem with the aim that students can easily understand the context of the problem (Kolar & Hodnik, 2021). Furthermore, the problem is given literacy reinforcement which is different from the usual problems solved by students. Through strengthening literacy, at the beginning of learning students do not realize that they will learn mathematical concepts. This is different from previous research which explored more about problem-based mathematics learning without providing literacy reinforcement.

Based on the aforementioned background, the researchers are interested in analyzing the learning trajectory in problem-based learning with literacy and numeracy reinforcement and employed three stages of design research through lesson study activities performed for eight cycles. The novelty in this study is the analysis of learning trajectories that have been carried out in lesson study activity-based learning. Lesson study activities are carried out through the stages of collaborative lesson study between lecturers and teachers in the partnership program.

B. METHODS

This study employed Gravemeijer and Cobb's design research Plomp & Nieveen (2013) Van den Akker et al. (2006) that comprises three stages: preparing the experiment of hypothetical learning trajectory, designing the experiment by implementing the designed hypothetical learning trajectory, and conducting retrospective analysis by comparing HLT with the implemented learning, as shown in Figure 2.

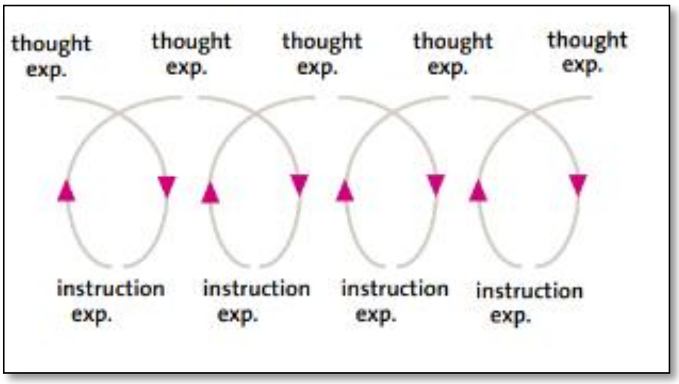

Figure 2. Design Research Flowchart

The research procedure included several steps based on the applied research design, and these steps consisted of three stages. The first stage was preparing the experiment (design preparation). This stage consisted of two actions: reviewing literature and interviewing teachers as well as designing the hypothetical learning trajectory. The literature review was conducted to investigate theoretical perspectives. Class teachers were interviewed to reveal in-depth characteristics of students and identify the teachers' factual teaching practices. Moreover, students' possible difficulties in learning were explored to reveal their conjecture thinking. The teacher's understanding of the student's learning process could underlay the development of pedagogical problem-solving (Schneider & Gowan, 2013) Steffe, 2004). The combination of theoretical and factual perspectives formed the basis for designing the hypothetical learning trajectory. The hypothetical learning trajectory consisted of three elements: learning objectives, learning activities, and conjectures of students' learning process.

The second stage was designing the experiment. In this stage, the designed hypothetical learning trajectory was implemented to prepare the experiment. Students' thinking was observed from their work and interviewed several students. The interview explored students' learning opportunities and barriers when learning. The final stage of the design research was conducting a retrospective analysis. At this stage, all data obtained at the design experiment stage were analyzed by comparing the hypothetical learning trajectory and the students' actual learning processes. In this stage, the possible causes were analyzed, and any possibilities were synthesized to improve the hypothetical learning trajectory. The results of the analysis were then employed to improve the hypothetical learning trajectory in the next cycles.

This research was conducted through lesson study activities in Public Junior High School by applying lesson study for eight cycles (Figure 1). The subjects were 32 students from class

VIII; they consisted of 14 male students and 18 female students. The qualitative data were collected through interviews, observations, and documents. the research instrument was validated by experts consisting of observation and interview instruments. The teachers were interviewed to explore their teaching practices and identify the students' difficulties in learning and conjecture learning. The semi-structured interview covered three aspects: teaching practices, students' learning difficulties, and students' thought assumptions. These data were employed in the first research stage and the final stage, namely the preparation of a hypothetical learning trajectory.

Furthermore, the students were interviewed to explore their learning potential and learning obstacles during learning activities. This study observed the students' actual learning process in the classroom and compared this learning process with the conjectures of a hypothetical learning trajectory. Finally, documentation was conducted by collecting the results of students' work. The learning process was also recorded using a recorder and handycam, and the result of this recording served as an additional data source if some data had not been observed. These data were obtained in the second phase of this study and analyzed using the r-retrospective analysis to produce ALT, as shown in Figure 3.

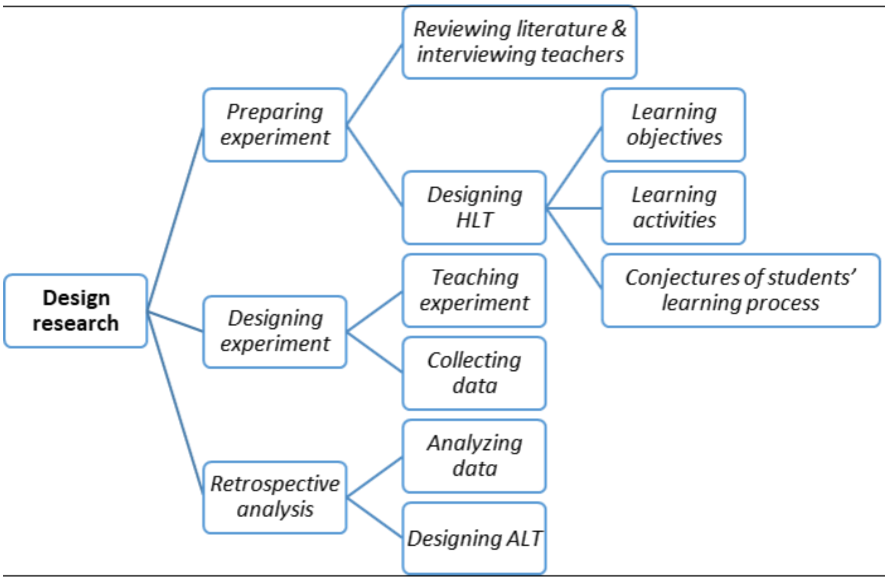

Figure 3. Design Research

Based on the design research stages, the data were analyzed retrospectively using a hypothetical learning trajectory as a guideline for the analysis. The first step of the analysis focused on implementing learning activities to check whether the implemented activities followed the design. Therefore, the students' strategies to construct their knowledge during learning could be revealed. In the next step, the students' actual learning processes were obtained from empirical observations and comparison analysis with the learning objectives. This step explored to what extent the learning objectives were achieved and how the students' understand the concepts learned. Furthermore, the students' actual learning processes were compared with the conjecture or prediction of their learning processes in the hypothetical learning trajectory. The interaction between the hypothetical learning trajectory and empirical observations was analyzed to develop instructional theory.

C. RESULT AND DISCUSSION

This section will describe the design research stages of this research to describe the learning trajectory of problem-based learning with literacy and numeracy reinforcement. However, this section only describes one of the eight cycles that have been done. Lesson study activities combined with this research stage. In the first stage, research preparation was carried out collaboratively in accordance with the first and second lesson study stages, namely studying curriculum and formulating goals, and plans. The second stage in this research, the experimental design is in accordance with the third stage of lesson study, namely conducting. While the third stage in the research, retrospective analysis is in accordance with the fourth lesson study stage, namely reflecting.

1. Stage 1. Preparing the Experiment

This stage consists of two activities: (1) reviewing literature and interviewing teachers and (2) designing the hypothetical learning trajectory. In the stage of literature review and interviews with teachers, several agreements were made. First, this study was conducted in class VIII D. Second, the employed material was Cartesian coordinate material. Third, the learning model employed a problem-based learning model. Fourth, the learning was conducted by strengthening literacy and numeracy. Fifth, the students were not used to learning to find a concept. Sixth, the students had insufficient cooperation and communication skills.

The results of the literature review and interviews with teachers are used to prepare the hypothetical learning trajectory. The designed hypothetical learning trajectory consists of three main components: determination of learning objectives, learning activities, and conjectures of the learning process. Learning objectives can be done in two ways. First, problem-based learning enables learners to find the original point position to another. Second, problem-based learning enables learners to find an original point position toward a particular point.

Meanwhile, the developed learning activities include three learning activities: literacy, numeracy task-sharing, and numeracy task-jumping. The first activities consist of students' conjecture responses (hypothetical learning process), including (1) reading literacy; (2) understanding meanings in literacy texts; (3) discussing the mutual group to reveal characteristics of ants; (4) discussing how ants can walk in rows, defend their territory, exchange information, find food sources, and build nests; and (5) finding strategies for ants that get lost, cannot return to their nest, and find the colony.

The second learning activities consist of students' conjecture responses (hypothetical learning process: (1) reading words of mathematical problems; (2) representing ant trajectory as a rectangle; (3) describing an ant's trajectory at Cartesian coordinates; (4) representing a hole when the ant comes out of the center point O (0,0); (5) determining the ant's coordinates of trajectory point; and (6) drawing the ant's possible trajectories and determining its coordinate points.

The third learning activity consists of students' conjecture responses: (1) reading contextual problems; (2) understanding contextual problems very close to the students' environment; (3) interpreting the information in pictures, and (4) determining the position of a place on the position of the school as the center point. The completed and developed hypothetical learning trajectory is presented in Figure 4.

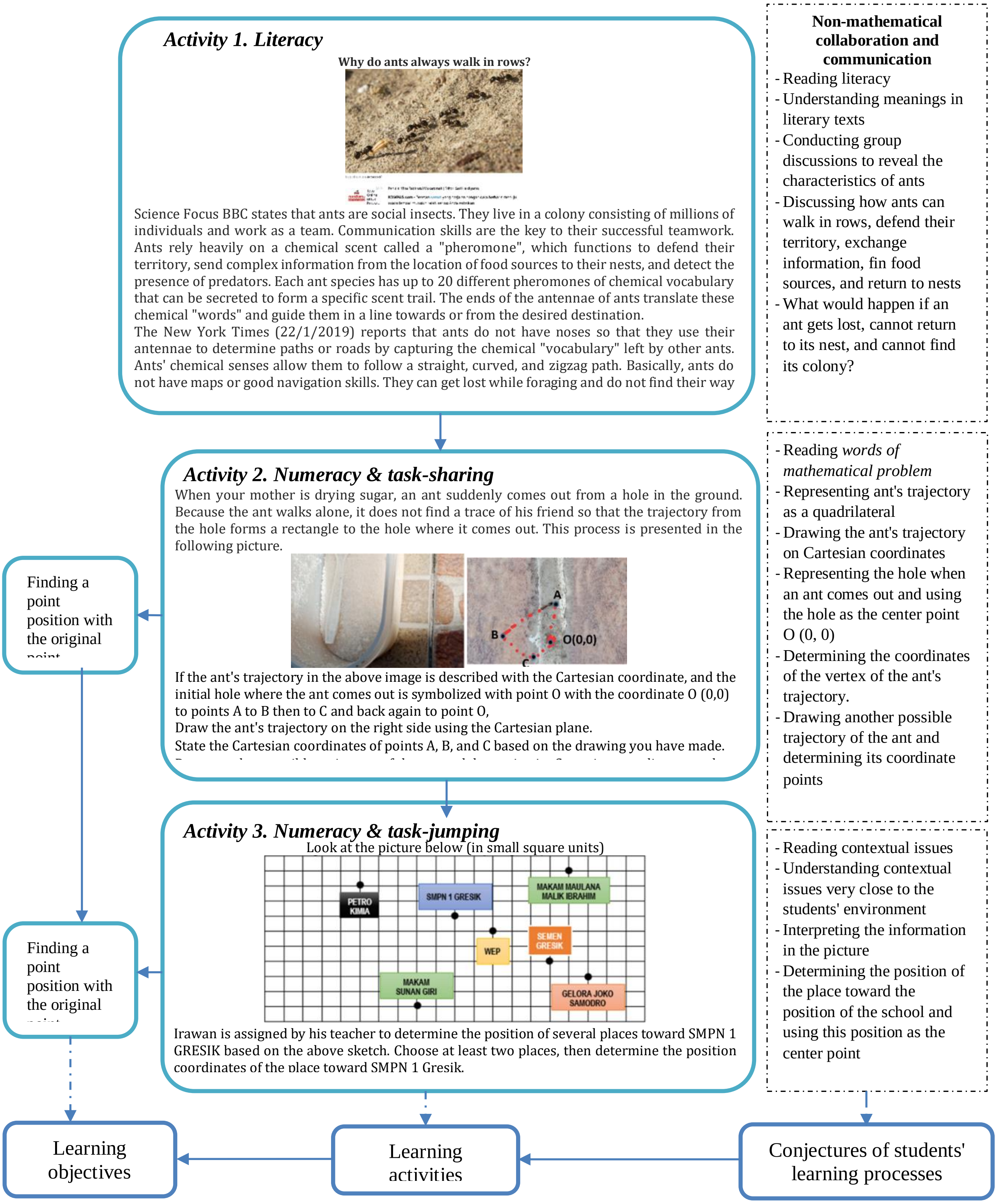

Figure 4. Hypothetical Learning Trajectory

2. Stage 2. Designing the Experiment

This stage consists of two activities: teaching experiment executed by one model teacher and collecting data by another teacher. The teaching experiment begins with preliminary activities that lead to the developed hypothetical learning trajectory. The preliminary activity begins with delivering learning objectives, apperceptions, and motivations. In the apperception activity, students are reminded of the material learned, namely the point position on Cartesian coordinates. The teachers and students do question and answer sessions to determine the position of some points in Cartesian coordinates, as shown in Figure 5.

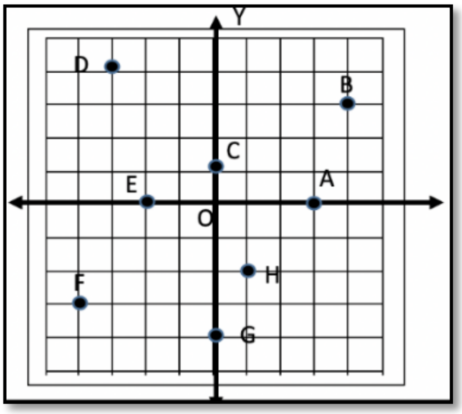

Figure 5. Cartesian Coordinates

Determine the coordinate positions of points A, B, C, D, E, F, G, and H in the graph below. The students respond by answering the teachers' questions about the position of points A, B, C, D, E, F, G, and H on Cartesian coordinates, alternately. Some students totally cannot answer the questions, and others have some wrong answers. Such a condition agrees with previous conjectures so that teachers should provide assistance in the form of directions to the students to calculate the distance of the point to the X-axis and Y-axis. However, some students have difficulty distinguishing between the X-axis and the Y-axis so that the teacher should provide assistance again.

In the motivation section, the teacher conveys the benefit of studying the material by showing a map to find the location of a place based on the maps and GPS. The students respond to this explanation by listening. These main activities consist of three learning activities. Each activity will describe the development of students' understanding of Cartesian coordinates, especially the topic of point position to the center point and other points obtained through problem-based learning with literacy and numeracy empowerment. The results of this study are in accordance with research conducted by Jale Balaban-Sali and Pounaki who examined student literacy and communication (Balaban-Sali, 2020) (Pounaki et al., 2017).

3. Activity 1. Literacy

At the initial stage, the teacher divides the students into small groups. The teacher gives the student worksheets and asks them to understand the reading of non-mathematical literacy in the early part of the student worksheets (Activity 1. Literacy in Figure 4). Literacy texts trigger a student to discuss with other students so that they receive a significant impact at the beginning of learning. The process of understanding non-mathematical literacy can encourage the students to talk to other students. However, non-mathematical literacy has not led to mathematical concepts. This study employed the literary entitled "Why do Ants always Walk in Rows?" When understanding literacy, three questions following conjectures on the hypothetical learning trajectory emerge. First, what are the characteristics of ants? Second, how can ants walk in rows, defend the territory, exchange information, find food sources, and return to their nests? Third, what would happen if ants get lost, cannot return to their nests, and cannot find their colony? These questions emerge because the teacher continuously encourages the students so that they can ask questions about the content and meaning of literacy to their classmates. Non-mathematical literacy given at the beginning of the learning can successfully serve as a stimulus for students to collaborate and communicate. Moreover, non-mathematical literacy will encourage the students to think about the next flow of thoughts related to mathematical concepts, as shown in Figure 6.

Figure 6. Students' Discussion about Literacy

4. Activity 2. Numeracy and Task-Sharing

From Activity 1 about non-mathematical literacy, the students have understood that an ant's walking in trajectory lines can be revealed. Moreover, the students can represent the ant's row trajectory by drawing the line on Cartesian coordinates. Non-mathematical literacy in Activity 1 is brought into mathematical thinking in the form of numeracy problems; meanwhile, that in Activity 2 is in the form of factual problems (See Figure 4).

Activity 2 is in the form of numeracy problems prepared for task-sharing. The numeracy question that contains contextual problems is extended and related to reading literacy. The numeracy question narrates the case of sugar approached by one ant. The ant comes out from a hole in the tiles where the sugar is placed. The ant comes out from the hole, approaches the sugar, and enters the hole again forming a quadrilateral trajectory. The students draw the ant's trajectory within Cartesian coordinates on checkered papers. The hole where the ant first exits and reenters becomes the center point of the O (0,0) coordinate. The other three points are named points A, B, and C. Furthermore, during the discussion, the students could find the coordinates of the four points that form the ant's quadrilateral trajectory. Each group of students differently draws the ant's trajectory, as shown in Figure 7.

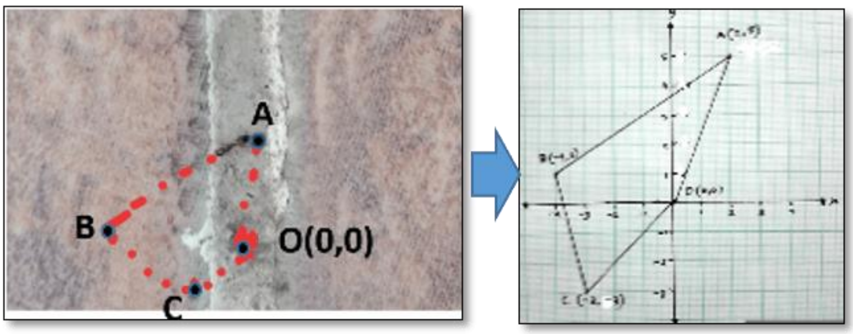

Figure 7. Ant's Trajectory (Left) Drawn on Cartesian Coordinates (Right)

To reinforce their understanding, the students draw other possible forms of ant's trajectories and determine the coordinates of each point again. Then, they discuss the determined coordinates. Some students draw a straight line until it touches the X-axis line to determine the X value. Then, they draw a straight line touching the Y-axis to determine the Y value. Until the end of Activity 2, the students have understood the concept of the point position toward the center point coordinates.

5. Activity 3. Numeracy and Task-Jumping

In this activity, the students expand their understanding scheme. The numeracy questions in the form of task-jumping have enabled the students to determine the position of the school toward several surrounding places (see questions in Activity 3 in Figure 4). After understanding the concept of the point position toward the coordinate center, they apply it in this case. The location point of the school plan is represented as the center point of coordinates, as presented in Figure 8.

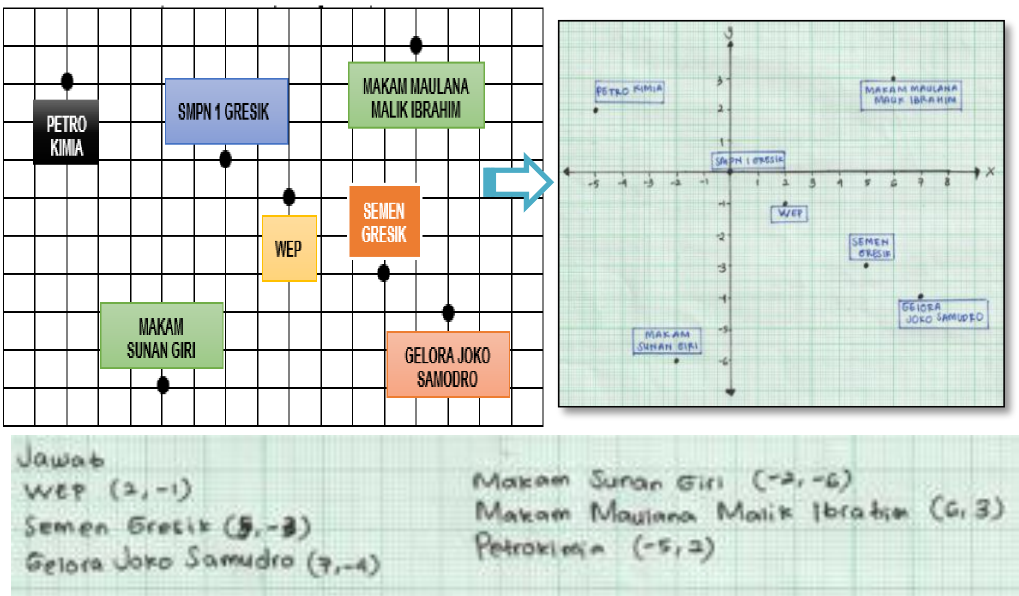

Figure 8. School Plan and Its Point Coordinates

At the end of this activity, the students expand their understanding by analogizing the approach to determine one point position to another. As a result, they can formally understand the concept of one position to another. The results of this study are in line with research conducted by James M. Moser (Moser, 2021).

6. Stage 3. Retrospective Analysis

The result of this study is in the form of students' actual learning trajectory in problem-based learning with the literacy and numeracy reinforcement. The data analysis has revealed that the actual learning trajectory focuses on four levels of modelling: situational, referential, general, and formal levels. At the situational level, the students receive the literacy readings that become their reference to collaborate and communicate. At this level, the students capture the initial idea of the trajectory of ants walking in rows. They can represent the ant's trajectory in the form of images. Then, on the same level, the students are given the problem situations that lead to mathematical concepts. In this situation, an ant comes out from the hole and approaches the sugar in the jar. Since the ant is alone, it cannot survive for long. Therefore, it only approaches the sugar, crosses at three points, and enters the same hole.

The next thinking stage is the referential level. At this level, the students get an idea to describe an ant's trajectory on Cartesian coordinates. The three points visited by ants are named points A, B, and C. Meanwhile, the holes where the ants exit and enter are represented as a center point or coordinate axis O (0, 0). Furthermore, the students think to determine the coordinates. At this stage, the connection between mathematical concepts is necessary. The concept tells how the student can think to determine the coordinates and discover the concept of distance from the point to the coordinate axis. This referential stage leads to achieve the mathematical concept, namely understanding and determining the point position toward the center point or coordinate axis.

The next level after the referential level is the general level, where the students understand not only the point position to the center point or coordinate axis but also one point position to another. At this level, the students are challenged with another question about open numeracy problems in the form of task-jumping. They are asked to determine the position of the school toward certain surrounding places following the prepared plan.

The trajectory of thinking experienced by the students in the next activity is analogizing the original point of the school on a plan as the center point of coordinates. Therefore, at this level, the students can understand how to determine the position of one point to another in Cartesian coordinates. This thinking trajectory has led the students to the level of formal thinking. The ability to think at the level of formal mathematics is very important for students to have (Cruz, 2018). The actual learning trajectory of this learning is presented in Figure 9.

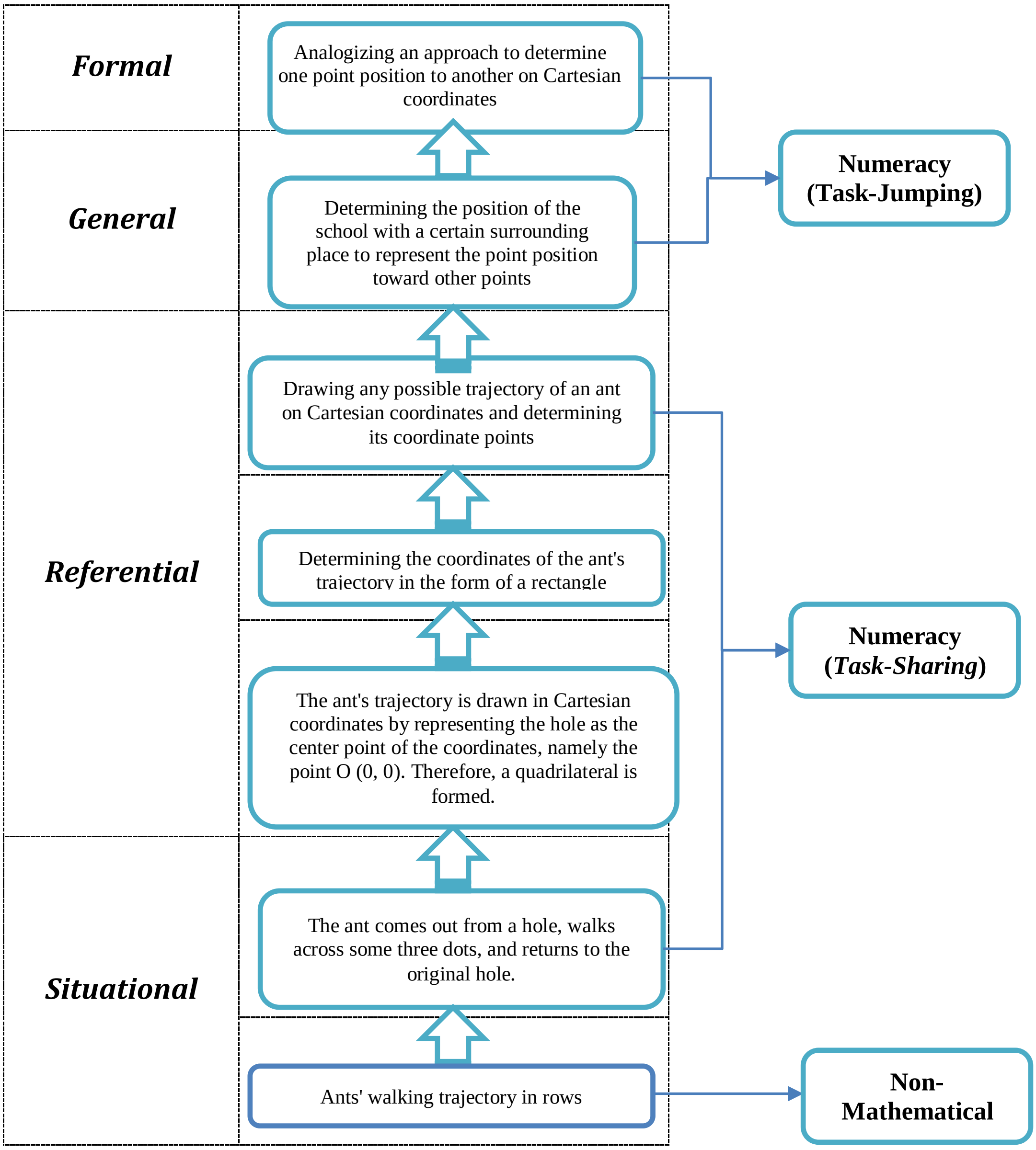

Figure 9. The Problem-Based Learning Trajectory with the Literacy and Numeracy Reinforcement on the Concept of Cartesian Coordinates

D. CONCLUSION AND SUGGESTIONS

This study concludes that problem-based learning with literacy and numeracy reinforcement can help students understand Cartesian coordinates. In this study, the students' learning trajectory in understanding the concept of one point position to another point in Cartesian coordinates constitutes the flow of student learning. This flow is obtained through a series of activities at different levels or modelling levels. The students' learning flow begins at the situational level by understanding non-mathematical literacy which becomes a reference for students to collaborate and communicate. Therefore, a mathematical understanding could emerge. At the referential level, the students extend their understanding from non-mathematical contextual problems to interrelated mathematical contextual problems. This referential stage leads to achieve the mathematical concept, namely understanding and determining the point position toward the center point or coordinate axis. The referential level is derived from the general level in which the students understand not only the point position to the center point or coordinate axis but also the point position to another point. The trajectory of thinking experienced by the students in the next activity is the analogizing process to understand the concept of one point position to another point in Cartesian coordinates. This thinking trajectory has led the students to the level of formal thinking.

This research suggests several points. First, the research on problem-based learning trajectory with literacy and numeracy empowerment can help students to achieve a level of formal thinking. Therefore, similar research should help students understand other mathematical concepts. Second, this study highly recommends that teachers investigate learning planning that considers students' conjecture responses in the form of hypothetical learning trajectories. As a result, the expected learning objectives can be achieved more optimally.

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