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Confidence Interval for Variance Function of a Compound Periodic Poisson Process with a Power Function Trend

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ABSTRACT

This research is a follow-up research of Utama (2022) on asymptotic distribution of an estimator for variance function of a compound periodic Poisson with the power function trend. The objectives of this research are (i) to formulate a confidence interval for the variance function of a compound periodic Poisson process with a power function trend and (ii) to prove the convergence to $1-\alpha$ probability of the parameter included in the confidence interval. This research process begins with a review of the existing formulation of the variance function estimator and its asymptotic distribution. Next, the confidence interval for the variance function of the compound periodic Poisson process with a power function trend is formulated and the convergence to $1-\alpha$ is determined. After obtaining the confidence interval, the research continued by conducting computer simulations to confirmed the results obtained analytically. The results obtained show that the confidence interval for the variance function of a compound periodic Poisson process with a power function trend converges to $1-\alpha$ both analytically and numerically for different finite time intervals.

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A. INTRODUCTION

Uncertain phenomena or events can be modeled with a stochastic process. Such phenomena or events include determining the estimated total future claims Sakthivel & Rajitha (2017), the occurrence of natural disasters Tse (2014), risk theory Zhang et al. (2014) and queuing systems (Li et al., 2014)(Nosova, 2019). Therefore, stochastic processes have an important role in solving problems in various fields in real life (Crescenzo et al., 2015). Stochastic process is a process to quantify the relationship of a set of random events at a given time interval Ross (2014), to describe the occurrence of uncertain phenomena or events in the future (Acuña-ureta et al., 2021). Stochastic process is categorized into two, namely discrete time stochastic processes and continuous time stochastic processes (Mangku, 2021). However, this research only focuses on one form of continuous-time stochastic processes, namely the Poisson process.

A Poisson process $\{N(t); t \geq 0\}$ has a certain intensity function where the number of phenomena or occurrences at each time interval is Poisson distributed (Last & Penrose, 2018). Poisson process are categorizeded into homogeneous and non-homogeneous Poisson process. a process is called periodic when it has a periodic intensity function (Mangku, 2001). Periodic Poisson processes have been widely used to model phenomena or events including events in the field of communication Belitser et al. (2012), finance Engle (2000), seasonal extreme rainfall events Ngailo et al. (2016), flood events Barbier et al. (2013), and the occurrence of earthquakes (Shao & Lii, 2011). A compound Poisson process is a process that has a sequence independent of the Poisson process ($N(t)$) and has i.i.d. random variables (Mangku, 2021).

The study of compound homogeneous Poisson processes has been widely developed. However, this process cannot be used if there are events that have a higher probability to occuring at a certain time intervals compared to the other time intervals. With cases like this, it is necessary to consider that time is influential so that the process used is non-homogeneous Poisson process. a specific cases of compound non-homogeneous Poisson process is compound periodic Poisson process.

The study of compound periodic Poisson processes has been developed by many researchers. The first research was conducted by Ruhiyat et al. (2013), and Mangku et al. (2013), namely the estimation of the mean functions of compound periodic Poisson process. Furthermore, in Makhmudah et al. (2016) researched the estimation of the variance function in the compound periodic Poisson processuses the power function tren to estimate the mean function of the compound Poisson process. Fajri et al. (2022) that uses the trend of the power function to formulate the $\hat{V}_{n,b}(t)$ in the periodic Poisson process. Then the modification of the estimator by Utama et al. (2022) was carried out to determine the asymptotic distribution of $\hat{V}_{n,b}(t)$. This research is focused on formulating confidence intervals on $\hat{V}_{n,b}(t)$ which has been discussed in (Utama et al., 2022). a related work on confidence interval can also be found in (Muhamad et al., 2022).

B. METHODS

This research focuses on theory development to formulate confidence intervals with the following research flow:

1. Preliminary study
   a. Studying theories relevant to the estimator of the variance function to be analyzed.
   b. Exploring the mathematical foundation to find interval estimators for existing models.
   c. Studying the formulations of estimators and the asymptotic distribution of the existing estimator.

2. Formulate confidence interval for the variance function and prove that probability of the parameter being in the confidence interval convergence to $(1-\alpha)$, analytically and numerically.
3. Conduct computer simulations using R and Scilab software with the following parameters:
   a. The selected α values were 1%, 5%, dan 10%.
   b. The selected τ values were 1 and 5, where $\tau = 1$ representing small period, and $\tau = 5$ representing large period.
   c. The length of the observation interval used was $n = 20\tau$, 50τ, dan 100τ which respectively represent small, medium and large observation intervals, .
   d. The variance function estimators used were repeated 1000 times.

Assume $\{N(t), t \geq 0\}$ is non-homogeneous Poisson random variable whose rate (λ) has two functions, namely a periodic function λ_c having a positive period τ, and a power function trend. Thus, its intensity function at each $s > 0$ is formulated as

$$\lambda(s) = \lambda_c(s) + as^b, \tag{1}$$

$\lambda_c(s)$ denotes a function of periodicity with period τ, as^b denotes the power function trend, and a denotes the slope of the trend. The power b is a real number in the $0 < b < \frac{1}{2}$, and λ_c isn't assumed to have parameters of any kind, except that it's a periodic function, which can be written as

$$\lambda_c(s) = \lambda_c(s + k\tau) \text{ for each } s \geq 0 \text{ and } k \in \mathbb{N}. \tag{2}$$

Assume $\{Y(t), t \geq 0\}$ is a process that states

$$Y(t) = \sum_{i=1}^{N(t)} X_i, \tag{3}$$

where $N(t)$ denotes a Poisson process for which of intensity function is λ and $\{X_i, i = 1,2, \dots\}$ are i.i.d random variable with $\mu < \infty$ and $\sigma^2 > 0$ (Kruczek et al., 2017). Suppose the variance functions of the process $Y(t)$ is denoted by $V(t)$ which is formulated as

$$V(t) = E[N(t)]E[N(X^2)] = \Lambda(t)\,\mu_2 \tag{4}$$

where

$$\Lambda(t) = \int_0^t \lambda_c(s)\,ds. \tag{5}$$

Let $t_r = t - \left\lfloor \frac{t}{\tau} \right\rfloor \tau$, $\lfloor x \rfloor$ denotes the greatest integer that less than or equal to x, for every $x \in \mathbb{R}$. Let $k_{t,\tau} = \left\lfloor \frac{t}{\tau} \right\rfloor$ for each $t \geq 0$ where t is real number, then it is obtained

$$t = k_{t,\tau}\,\tau + t_r; 0 \leq t_r < \tau. \tag{6}$$

Let the periodic part of the $\{N(t), t \geq 0\}$ has a global intensity, denoted by $\theta = \frac{1}{\tau}\int_0^\tau \lambda_c(s)ds$, and it is assumed that $\theta > 0$. Hence, it is obtained that

$$\Lambda(t) = k_{t,\tau}\,\tau\theta + \Lambda_c(t_r) + \frac{a}{b+1} t^{b+1}. \tag{7}$$

By using the substitution method of substituting equation (7) into (4), the process $Y(t)$ is obtained as follows

$$V(t) = \left(k_{t,\tau}\,\tau\theta + \Lambda_c(t_r) + \frac{a}{b+1} t^{b+1}\right) \mu_2. \tag{8}$$

In (Utama et al., 2022), the result of modification of the $\hat{V}_{n,b}(t)$ estimator are obtained as in the following Equation

$$\hat{V}_{n,b}(t) = \left(\left(1 + k_{t,\tau}\right) \hat{\Lambda}_{c,n,b}(t_r) + k_{t,r}\hat{\Lambda}^{c}_{c,n,b}(t_r) + \frac{a}{b+1} t^{b+1}\right) \hat{\mu}_{2,n} \tag{9}$$

where,

$\hat{\Lambda}_{c,n,b} = \left(\left(\frac{\tau}{n}\right)^{1-b} (1-b) \sum_{k=1}^{n,\tau} \frac{1}{k^b} N([k\tau, k\tau + t_r]) - \hat{a}_{m,b} t_r n^b (1-b)\right)$,

$\hat{\Lambda}^{c}_{c,n,b}(t) = \left(\left(\frac{\tau}{n}\right)^{1-b} (1-b) \sum_{k=1}^{n,\tau} \frac{1}{k^b} N([k\tau + tr, k\tau + \tau]) - \hat{a}_{m,b}(\tau - t_r) n^b (1-b)\right)$,

where $\hat{a}_{m,b} = \left(\frac{N([0,m])(b+1)}{m^{b+1}} - \frac{\tilde{\theta}_n(b+1)}{m^b}\right)$ for $n \to \infty$,

$\tilde{\theta}_n = \left(\frac{1-b}{b^2 n^{1-b} \tau^b} \sum_{k=1}^{k_{n,\tau}} \frac{1}{k^b} N([k\tau, k\tau + \tau]) - \frac{(1-b)(1+b) n^b N([0,n])}{b^2 n^{1+b}}\right)$ (Utama et al., 2022),

$\hat{\mu}_{2,n} = \frac{1}{N([0,n])} \sum_{i=1}^{N([0,n])} X_i^2$ where $\delta > 0$, $\hat{\mu}_{2,n} = 0$ when $N([0,n]) = 0$ (Makhmudah et al., 2016).

Theorem 1 (Asymptotic Distribution of Variance Function Estimators)
Let $\lambda(s)$ satisfy equation (1) and is locally integrable. If the process $Y(t)$ satisfies equation (3), then

$$\sqrt{n^{1-b}} \left(\hat{V}_{n,b}(t) - V(t)\right) \xrightarrow{d} Normal\left(0, \left(\left(1 + k_{t,\tau}\right)^2 t_r a\tau(1-b) + k_{t,\tau}^2 (\tau - t_r) a\tau(1-b)\right) \mu_2^2\right)$$

as $n \to \infty$ (Utama et al., 2022).

C. RESULT AND DISCUSSION

1. Confidence Interval of the Variance Function

A confidence interval is an interval estimate with a confidence coefficient (Hogg et al., 2019). Variance is used to measure the average amount of fluctuation of a random variable from an expected value (Ghahramani, 2016). Based on Theorem 1, we obtain $\mu = 0$ and $\sigma^2 = \left(1 + k_{t,\tau}\right)^2 a\tau t_r (1-b)\mu_2^2 + k_{t,\tau}^2 a\tau(1-b)(\tau - t_r)\mu_2^2$. From the expected value and variance above, the confidence interval of the variance function with a confidence level of $1 - \alpha$ for $0 < \alpha < 1$ can be written as follows

$$I_{V,n} = \left[\hat{V}_{n,b}(t) + \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\sqrt{V_n}\, , \hat{V}_{n,b}(t) - \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\sqrt{V_n}\right], \tag{10}$$

with Φ denotes the standardized normal distribution function and

$$V_n = \frac{\left(\left(1 + k_{t,\tau}\right)^2 t_r \hat{a}_{m,b} \tau(1-b) + k_{t,\tau}^2 (\tau - t_r) \hat{a}_{m,b} \tau(1-b)\right) \hat{\mu}_{2,n}^2}{n^{1-b}} \tag{11}$$

2. Convergence of Confidence Interval for Variance Functions

Theorem 2 (The Convergence of $\boldsymbol{V(t)} \in \boldsymbol{I_{V,n}}$)

For the confidence interval $I_{V,n}$ for the variance function $V(t)$, we have that

$$P\big(V(t) \in I_{V,n}\big) \to 1 - \alpha,$$

as $n \to \infty$.

Proof: Based on $I_{V,n}$, $P\big(V(t) \in I_{V,n}\big)$ can be described as

$$P\left(\hat{V}_{n,b}(t) - \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right) \leq V(t) \leq \hat{V}_{n,b}(t) + \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)\right)$$

Let $\hat{V}_{n,b}(t) = q$ and $V(t) = l$, hence

$$= P\left(q - \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right) \leq l \leq q + \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)\right)$$

$$= P\left(-\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right) \leq -q + l \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)\right)$$

$$= P\left(\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right) \geq q - l \geq -\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)\right)$$

$$= P\left(-\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right) \leq q - l \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)\right)$$

$$= P\left(\frac{-\Phi^{-1}\left(1-\frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)}{\left(\sqrt{V_n}\right)} \leq \frac{q-l}{\left(\sqrt{V_n}\right)} \leq \frac{\Phi^{-1}\left(1-\frac{\alpha}{2}\right)\left(\sqrt{V_n}\right)}{\left(\sqrt{V_n}\right)}\right)$$

$$= P\left(-\Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \leq \frac{q-l}{\left(\sqrt{V_n}\right)} \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right).$$

According Studentize version of Theorem 1, it is obtained that

$$\left(\frac{q - l}{\left(\sqrt{V_n}\right)}\right) = \left(\frac{\hat{V}_{n,b}(t) - V(t)}{\left(\sqrt{V_n}\right)}\right) \xrightarrow{d} Normal\ (0,1),$$

as $n \to \infty$. Thus, $P\big(V(t) \in I_{V,n}\big)$ converges to

$$P\left(-\Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \leq Z \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right)$$

for $n \to \infty$, and Z is a standardized random variable that has an expected value of 0 and variance of 1. The results of the description from probability function above has been simplified by (Muhamad et al., 2022) as follows

$$P\left(-\Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \leq Z \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right) \Leftrightarrow P\left(Z \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right) - \left(1 - P\left(Z \leq \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right)\right)$$

Hence,

$$= \Phi\left(\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right) - \left(1 - \Phi\left(\Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\right)\right)$$

$$= \left(1 - \frac{\alpha}{2}\right) - \left(1 - \left(1 - \frac{\alpha}{2}\right)\right) = 1 - \frac{\alpha}{2} - 1 + 1 - \frac{\alpha}{2}$$

$$= 1 - \alpha.$$

3. Simulation of Confidence Intervals for Variance Functions

In this research, the simulation process aims to strengthen the results of analytical confidence interval formulation. The simulation process was carried out using R and Scilab software. R software's used to realize the data of $I_{V,n}$ for variance function, while Scilab software's used to show the illustration image of $I_{V,n}$ for variance function with 1000 times iterations. The results for confidence interval simulation are shown in Table 1.

Table 1. Simulation results of confidence interval ($I_{V,n}$) for $t = 0.05, \tau = 1$ with 1000 repetitions

α	n	The number of $I_{V,n}$ that contain parameters	The number $I_{V,n}$ that dont contain parameters	Percentage $I_{V,n}$ that contain parameters	Percentage $I_{V,n}$ that dont contain parameters	Absolute error
1%	20	986	14	98.6%	1.4%	0.4%
	50	987	13	98.7%	1.3%	0.3%
	100	991	9	99.1%	0.9%	0.1%
5%	20	944	56	94.4%	5.6%	0.6%
	50	949	51	94.9%	5.1%	0.1%
	100	950	50	95.0%	5.0%	0.0%
10%	20	895	105	89.5%	10.5%	0.5%
	50	897	103	89.7%	10.3%	0.3%
	100	901	99	90.1%	9.9%	0.1%

Based on Table 1, the simulation results show that the percentage of $I_{V,n}$ that does not contain parameters for $t = 0.05$ and $\tau = 1$ with 1000 repetitions converges to α. For $\alpha = 1\%$ the percentage range is from $0.9\% - 1.4\%$, for $\alpha = 5\%$ the percentage range is from $5.0\% - 5.6\%$, while for $\alpha = 10\%$ the percentage range is from $9.9\% - 10.5\%$. Therefore, the absolute errors tend to be relatively small, in the range of $0.0\% - 0.6\%$. Likewise, the percentage of $I_{V,n}$ that contains parameters will corverge to $1 - \alpha$, with $\alpha = 1\%$ in the range $98.6\% - 99.1\%$, $\alpha = 5\%$ in the range $94.4\% - 95.0\%$, and $\alpha = 10\%$ in the range $89.5\% - 90.1\%$, The results from Table 1 also show that if α is larger, then the number of $I_{V,n}$ parameters is smaller, i.e. for $\alpha = 1\%$ ranges from $986 - 991$, $\alpha = 5\%$ ranges from $944 - 950$, and $\alpha = 10\%$ ranges from $895 - 901$. The illustrative results of the confidence interval ($I_{V,n}$) using only 100 repetitions for $\tau = 1$ can be viewed in Figure 1.

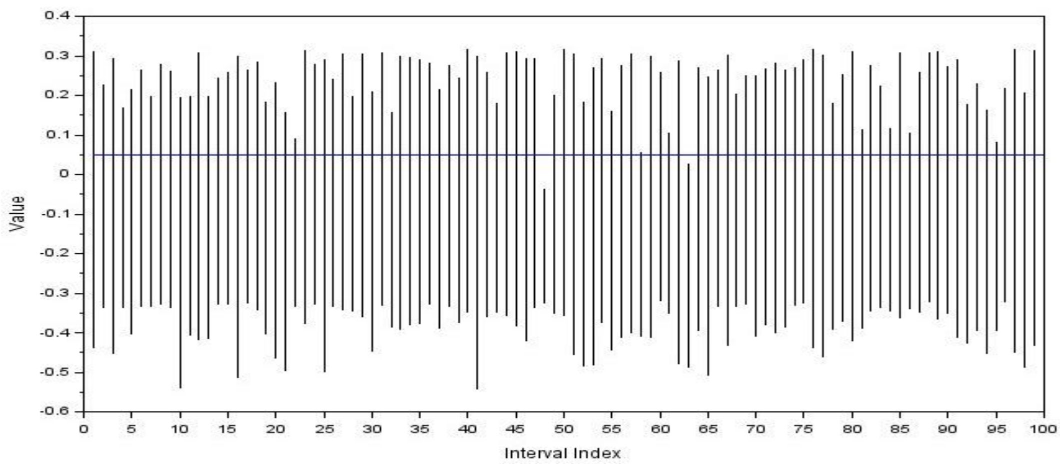

Figure 1. Confidence interval of the variance function with $t = 0.05$, α =1%, n =100, and $\tau = 1$

Figure 1 is an illustration result using Scilab software by inputting 100 repetitions of the $I_{V,n}$ with $t = 0.05$ and $\tau = 1$. In Figure 1 there is a horizontal line and vertical lines. The vertical line is the confidence intervals of the variance function, while horizontal line represents the variance function value. The simulation results of the $I_{V,n}$ above show 2 vertical lines that dont intersect with the horizontal line. This shows that a variance function value isn't in the 2 $I_{V,n}$. Table 1 shows that for $\alpha = 1\%$ and $n = 100$ there are 9 $I_{V,n}$ that dont contain parameters. Thus, in the 101st to 1000th repetitions there are 7 $I_{V,n}$ that dont contain a variance function value. The simulation results using $t = 0.05$ and $\tau = 5$ for 1000 repetitions are shown in Table 2.

Table 2. Simulation results of confidence interval ($I_{V,n}$) for $t = 0.05, \tau = 1$ with 1000 repetitions

$\boldsymbol{\alpha}$	$\boldsymbol{n}$	**The number of $I_{V,n}$ that contain parameters**	**The number $I_{V,n}$ that dont contain parameters**	**Percentage $I_{V,n}$ that contain parameters**	**Percentage $I_{V,n}$ that dont contain parameters**	**Absolute error**
1%	100	984	16	98.4%	1.6%	0.6%
	250	988	12	98.8%	1.2%	0.2%
	500	991	9	99.1%	0.9%	0.1%
5%	100	948	52	94.8%	5.2%	0.2%
	250	949	51	94.9%	5.1%	0.1%
	500	951	49	95.1%	4.9%	0.1%
10%	100	898	102	89.8%	10.2%	0.2%
	250	898	102	89.8%	10.2%	0.2%
	500	900	100	90.0%	10.0%	0.0%

Based on Table 2, the simulation results show that the percentage of $I_{V,n}$ that does not contain parameters for $t = 0.05$ and $\tau = 1$ with 1000 repetitions converges to α. For $\alpha = 1\%$ the percentage range is from $0.9\% - 1.6\%$, for $\alpha = 5\%$ the percentage range is from $4.9\% - 5.2\%$, while for $\alpha = 10\%$ the percentage range is from $10.0\% - 10.2\%$. Therefore, the absolute errors tend to be relatively small, within the range of $0.0\% - 0.6\%$. Likewise, the percentage of $I_{V,n}$ that contains parameters will corverge to $1 - \alpha$, with $\alpha = 1\%$ in the range $98.4\% - 99.1\%$, $\alpha = 5\%$ in the range $94.8\% - 95.1\%$, and $\alpha = 10\%$ in the range $89.8\% - 90.0\%$, The results from Table 2 also show that if α is larger, then the number of $I_{V,n}$ parameters is smaller, i.e. for $\alpha = 1\%$ ranges from $984 - 991$, $\alpha = 5\%$ ranges from $948 - 951$, and $\alpha = 10\%$ ranges from $898 - 900$. The illustrative results of the confidence interval ($I_{V,n}$) using only 100 repetitions with $\tau = 5$ can be viewed in Figure 2.

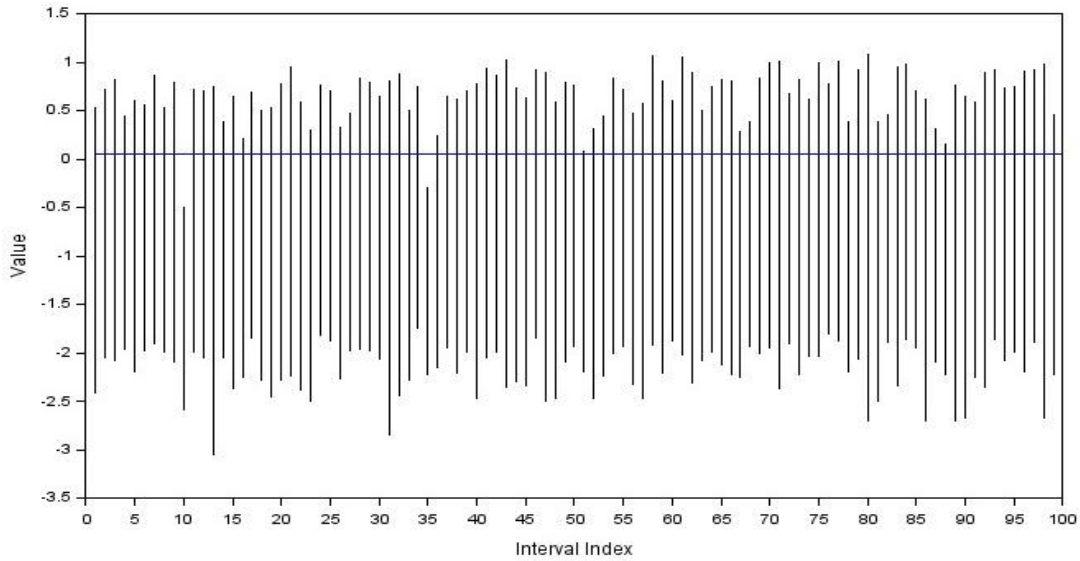

Figure 2. Confidence interval of the variance function with $t = 0.05$, α =1%, n =100, and $\tau = 5$

Figure 2 is an illustration result using Scilab software by inputting 100 repetitions of the $I_{V,n}$ with $t = 0.05$ and $\tau = 5$. In Fiture 2 there is a horizontal line and vertical lines. The vertical line is the $I_{V,n}$ of the variance function, while horizontal line represents the variance function value. The simulation results of the $I_{V,n}$ above show 2 vertical lines that dont intersect with the horizontal line. This shows that the value of the variance function isn't in the 2 $I_{V,n}$. Table 2 shows that for $\alpha = 1\%$ and $n = 100$ there are 16 $I_{V,n}$ that dont contain parameters. Thus, for the 101st to 1000th repetitions there are 14 $I_{V,n}$ that dont contain a variance function value.

Based on the simulation results of $I_{V,n}$ given in Table 1 and 2 show that the probability of the variance function $V(t)$ covered by the $I_{V,n}$ is consistant with the analytical result, that is converges to $1 - \alpha$, as $n \to \infty$. The result of this research are in accordance with research conducted by (Muhamad et al., 2022) which obtained that the probability of parameters being covered in the confidence interval is getting nearer to $1 - b$. If the real level used is 1%, 5%, and 10%, then the confidence interval obtained will approach $1 - a$ namely 99% for $\alpha = 1\%$, 95% for $\alpha = 5\%$, and 90% for $\alpha = 10\%$ at a finite observation time $[0, n]$.

D. CONCLUSION AND SUGGESTIONS

The results formulation show the confidence interval for the $V(t)$ of the periodic Poisson process by adding a power function trend is obtained

$$I_{V,n} = \left[\hat{V}_{n,b}(t) + \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\sqrt{V_n}\ , \hat{V}_{n,b}(t) - \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)\sqrt{V_n}\right] \quad ,$$

with Φ denotes the standardized normal distribution function and

$$V_n = \frac{\left(\left(1 + k_{t,\tau}\right)^2 t_r \hat{a}_{m,b}\tau(1 - b) + k_{t,\tau}^2(\tau - t_r)\hat{a}_{m,b}\tau(1 - b)\right)\hat{\mu}_{2,n}^2}{n^{1-b}}.$$

The convergence of the probability that the $V(t)$ of a compound periodic Poisson process included in the confidence interval is

$$P(V(t) \in I_{V,n}) \rightarrow 1 - \alpha; n \rightarrow \infty.$$

The simulation results of the $I_{V,n}$ show that the probability of the variance function $V(t)$ covered by the $I_{V,n}$is consistant with the analytical result, that is converges to $1 - \alpha$, as $n \rightarrow \infty$. The future research can use other intensity functions and intervals of observation to show more varied simulation outputs.

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