

Power Domination Number On Shackle Operation with Points as Linkage

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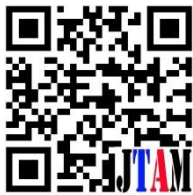
ABSTRACT

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The Power dominating set is a minimum point of determination in a graph that can dominate the connected dots around it, with a minimum domination point. The smallest cardinality of a power dominating set is called a power domination number with the notation $\gamma_p(G)$. The purpose of this study is to determine the Shackle operations graph value from several special graphs with a point as a link. The result operation graphs are: Shackle operation graph from Path graph Shack (P_m, v, n) , Shackle operation graph from Sikel graph Shack (C_m, v, n) , Shackle operation graph from Star graph Shack (S_m, v, n) . The method used in this paper is axiomatic deductive method in solving problems. Understanding the axiomatic method itself is a method of deductive proof principles that applies in mathematical logic by using theorems that already exist in solving a problem. In this paper begins by determining the paper object that is the Shackle point operations result graph. Next, determine the cardinality of these graphs. After that, determine the point that has the maximum degree on the graph as the dominator point of power domination. Then, check whether the nearest neighbor has two or more degrees and analyze its optimization by using a ceiling function comparison between zero forcing with the greatest degree of graph. Thus it can be determined γ_p minimal and dominated. The results of the power domination number study on Shackle operation graph result with points as connectors are $\gamma_p(\text{Shack}(P_m, v, n)) = n - 1$, for $m \geq 2$ and $n \geq 1$; $\gamma_p(\text{Shack}(C_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$; $\gamma_p(\text{Shack}(S_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$.



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A. INTRODUCTION

Mathematics is very fundamental in the development of science and technology. Almost in all branches of science use mathematics in problem solving. For example, in the fields of science and social sciences. In this case, a topic that is often discussed in the branch of mathematics and at the same time became the forerunner to technological development, namely about Graph Theory.

Graph Theory is one of the subject matter of Discrete Mathematics which has long been known and widely applied in various fields (Munir & Rinaldi, 2012). In representing the visual of a graph that is by expressing objects with nodes, dots, circles, points, or vertices, while the relationship between objects is expressed by lines or edges (Imelda Roza, Narwen, 2014).

One of the interesting topics in Graph Theory is dominating set. The definition of dominating set is a concept of determining the minimum possible points on a graph by

determining points as dominating points so that they can reach the points around them (Slamin, Dafik, & Wasposito, 2018). Dominating set has experienced many developments, such as: locating dominating set, independent dominating set, power dominating set, and others (Zou et al., 2011).

The application of power domination number in daily life, one of them is the placement of LBS (Load Break Switch) on the electrical system that is useful as an electrical circuit breaker. The goal is to make the number of LBS focal points minimal and efficient in their use (Bawono, 2017).

This paper will discuss the power dominating set. Power dominating set is a way to determine the minimum possible points in a graph that can dominate the connected points around it, with the minimum number of dominating points. The smallest cardinality of a power dominating set is called a power domination number with the notation $\gamma_p(G)$ (Benson et al., 2018). To find out the minimum limit of the number of power domination numbers in a graph that is by using the ceiling function comparison between zero forcing with $Z(G)$ notation with the largest degree of a graph (Benson et al., 2015).

Previous studies have investigated Power dominating sets by (Benson et al., 2018), (Chang, Dorbec, Montassier, & Raspaud, 2012), (Department, 2012), (Dorbec & Klavžar, 2014), (Ferrero, Hogben, Kenter, & Young, 2017a), (Stephen, Rajan, Ryan, Grigorious, & William, 2015), (Seema & A.Vijayakumar, 2011), (Ferrero, Varghese, & Vijayakumar, 2011), (Wang, Chen, & Lu, 2016), (Ferrero, Hogben, Kenter, & Young, 2017), (Bozeman et al., 2019), and (Dorbec, Henning, Löwenstein, Montassier, & Raspaud, 2013). The theorem as a reference in this research is **Theorem 1**: for any graph $\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$ (Benson et al., 2015). Meanwhile, the graph that will be examined in this research is the Shackle operations result from several special graphs with a point as the link. The operation result graphs are: Shackle operation graph with points as linkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$. Thus the title can be obtained namely "Power Domination Number on Shackle Operation Graph with Points a Linkage".

B. METHODS

The method used in this paper is axiomatic deductive method in solving problems. Understanding the axiomatic method itself is a paper method using the principles of deductive validity proof in mathematical logic by using existing theorems in solving a problem (Lestari, 2015).

In this paper begins by determining the object of the paper that the Shackle point operation result graphs. Next determine the cardinality of these graphs. After that, determine the point that has the maximum degree on the graph as the dominator point of Power Domination. Then, check whether the nearest neighbor has two or more degrees and analyze its optimization by using the comparison of the ceiling function between zero forcing with the largest degree of graph (Yang & Marzetta, 2013). Thus, it can be determined γ_p minimal and dominated.

The graph that will be examined in this research is the Shackle operation result graphs from several special graphs with a point as the link. The operation result graphs are: Shackle operation graph with points as linkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$.

C. RESULTS AND DISCUSSION

In the results and discussion will be explained about the power dominating number on the Shackle operation result graphs. The results of this paper are several new theorems about power dominating numbers on the Shackle operation result graphs. The initial step in this paper is to determine the cardinality of the Shackle operation result graphs. Then, the second step is to determine the power dominating number on the Shackle operation result graphs. The operation result graphs are: Shackle operation graph with points as linkage from Path graph $Shack(P_m, v, n)$, Shackle operation graph from Cycle graph $Shack(C_m, v, n)$, Shackle operation graph from Star graph $Shack(S_m, v, n)$. The following are the results of the paper.

1. Shackle Operation Result Graph From Path Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as linkage from the Path graph $Shack(P, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the truth of the theorem.

◇ **Theorem C.1.** For $m \geq 2$ and $n \geq 1$, the power domination number of the graph $Shack(P_m, v, n)$ is $\gamma_p(Shack(P_m, v, n)) = n - 1$.

Proof.

Shackle operational result graph from Path graph $Shack(P_m, v, n)$ is a graph with set point $V(Shack(P_m, v, n)) = \{A_i; 1 \leq i \leq n\} \cup \{x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$ and the set of sides $E(Shack(P_m, v, n)) = \{A_i x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\} \cup \{A_{i+1} x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$. So $p = |V(Shack(P_m, v, n))| = mn + m + n, q = |E(Shack(P_m, v, n))| = 3mn + m - n - 1, |Z(Shack(P_m, v, n))| = mn + 1$ and $|\Delta(Shack(P_m, v, n))| = 2m$.

For $m \geq 2$ and $n \geq 1$ it will be shown that $\gamma_p(Shack(P_m, v, n)) \geq n - 1$ by selecting the set of dominator points association that is $S = \{A_i; 1 \leq i \leq n\}$. Suppose $\gamma_p(Shack(P_m, v, n)) < n$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $Shack(P_m, v, n)$ with $\gamma_p(Shack(P_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(Shack(P_m, v, n)) \geq n - 1$.

Next it will be shown that $\gamma_p(Shack(P_m, v, n)) \leq n - 1$ by selecting the set of dominator points association from $Shack(P_m, v, n)$ which is $S = \{A_i; 1 \leq i \leq n\}$ which observes other points, then every node in $(Shack(P_m, v, n))$ is observed by $S = \{A_i; 1 \leq i \leq n\}$, so $\gamma_p(Shack(P_m, v, n)) \leq |S| = n - 1$. Therefore $\gamma_p(Shack(P_m, v, n)) \geq n - 1$ and $\gamma_p(Shack(P_m, v, n)) \leq n - 1$, then $\gamma_p(Shack(P_m, v, n)) = n - 1$. The set S contains n elements, namely $S = \{A_i; 1 \leq i \leq n\}$. Thus the value $\gamma_p(Shack(P_m, v, n)) = n - 1$ is obtained. The following will show an example of graph $Shack(P_6, v, 4)$ can be seen in Figure 1.

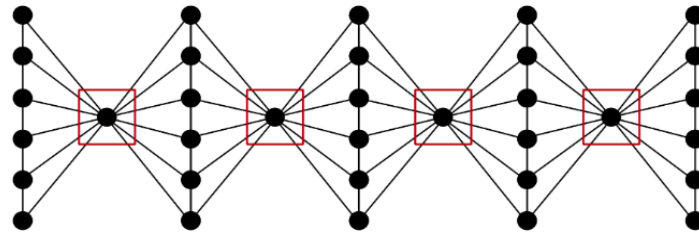


Figure 1. Power Dominating Set Graph ($Shack(P_6, v, 4)$)

To strengthen the evidence above, based on Theorem 1, namely $\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. It will be shown that $n - 1 \geq \left\lceil \frac{mn+1}{2m} \right\rceil$.

$$\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$$

$$n - 1 \geq \left\lceil \frac{mn + 1}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \geq \left\lceil \frac{mn}{2m} + \frac{1}{2m} \right\rceil$$

$$\leftrightarrow n - 1 \geq \left\lceil \frac{m}{2} + \frac{1}{2m} \right\rceil$$

$$\text{Thus } n - 1 \geq \left\lceil \frac{mn+1}{2m} \right\rceil. \quad \blacksquare$$

2. Shackle Operation Result Graph from Cycle Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as linkage from Cycle graph $Shack(C_m, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the theorem truth.

◇ **Theorem C.2.** For $m \geq 3$ and $n \geq 1$, the power domination number of the graph $Shack(C_m, v, n)$ is $\gamma_p(Shack(C_m, v, n)) = n - 1$.

Proof.

Shackle operational result graph from Cycle Shack $Shack(C_m, v, n)$ graph is a set of vertices $V(Shack(C_m, v, n)) = \{A_i; 1 \leq i \leq n\} \cup \{x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$ and the set of sides $E(Shack(C_m, v, n)) = \{A_i x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\} \cup \{x_{ij} x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$. So

$$p = |V(Shack(C_m, v, n))| = mn + m + n, \quad q = |E(Shack(C_m, v, n))| = 3mn + m, \quad |Z(Shack(C_m, v, n))| = mn - m + 2 \text{ and } |\Delta(Shack(C_m, v, n))| = 2m.$$

For $m \geq 3$ and $m \geq 1$, it will be shown that $\gamma_p(Shack(C_m, v, n)) \geq n - 1$ by selecting the set of dominator points that is $S = \{A_i; 1 \leq i \leq n\}$. Suppose $\gamma_p(Shack(C_m, v, n)) < n - 1$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $Shack(C_m, v, n)$ with $\gamma_p(Shack(C_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(Shack(C_m, v, n)) \geq n - 1$.

Next it will be shown that $\gamma_p(Shack(C_m, v, n)) \leq n - 1$ by selecting the set of dominator points from $Shack(C_m, v, n)$ which is $S = \{A_i; 1 \leq i \leq n\}$ which observes other points, then every node in $(Shack(C_m, v, n))$ is observed $S = \{A_i; 1 \leq i \leq n\}$, so $\gamma_p(Shack(C_m, v, n)) \leq |S| = n - 1$. Therefore $\gamma_p(Shack(C_m, v, n)) \geq n - 1$ and $\gamma_p(Shack(C_m, v, n)) \leq n - 1$, then

$\gamma_p(\text{Shack}(C_m, v, n)) = n - 1$. The set S contains $n - 1$ elements namely $S = \{A_i; 1 \leq i \leq n\}$. Thus the value $\gamma_p(\text{Shack}(C_m, v, n)) = n - 1$ is obtained.

To strengthen the evidence above, based on Theorem 1, namely $\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. It will be shown that $n - 1 \geq \left\lceil \frac{mn - m + 2}{2m} \right\rceil$.

$$\begin{aligned} \gamma_p(G) &\geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil \\ n - 1 &\geq \left\lceil \frac{mn - m + 2}{2m} \right\rceil \\ &\Leftrightarrow n - 1 \geq \left\lceil \frac{mn}{2m} - \frac{m}{2m} + \frac{2}{2m} \right\rceil \\ &\Leftrightarrow n - 1 \geq \left\lceil \frac{n}{2} - \frac{1}{2} + \frac{2}{2m} \right\rceil \\ \text{Thus } n - 1 &\geq \left\lceil \frac{mn - m + 2}{2m} \right\rceil. \quad \blacksquare \end{aligned}$$

The following will show an example of graph $\text{Shack}(C_4, v, 2)$ can be seen in Figure 2.

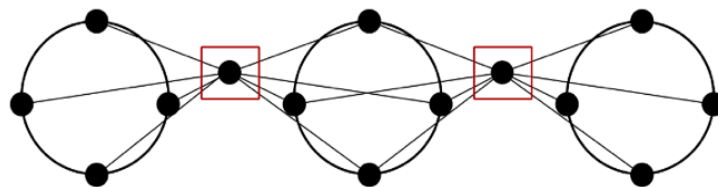


Figure 2. Power Dominating Set Graph ($\text{Shack}(C_4, v, 2)$)

3. The Operation Graph from Star Graph

In this section, the power dominations number theorem on the Shackle operation graph with points as linkage from the Star graph $\text{Shack}(S_m, v, n)$ will be presented, followed by theorem evidence. Then an example is given as a visualization of the truth of the theorem.

◇ **Theorem C.3.** For $m \geq 3$ and $n \geq 1$, the power domination number of the graph $\text{Shack}(S_m, v, n)$ is $\gamma_p(\text{Shack}(S_m, v, n)) = n - 1$.

Proof.

Shackle operation graph from Star graph $\text{Shack}(S_m, v, n)$ is a graph with set point $V(\text{Shack}(S_m, v, n)) = \{A_i; 1 \leq i \leq n\} \cup \{x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$ and the set of sides $E(\text{Shack}(S_m, v, n)) = \{A_i x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\} \cup \{x_{ij} x_{ij}; 1 \leq i \leq n - 1; 1 \leq j \leq m\}$. so $p = |V(\text{Shack}(S_m, v, n))| = mn + m + 2n$, $q = |E(\text{Shack}(S_m, v, n))| = 3mn + m + 2n$, $|Z(\text{Shack}(S_m, v, n))| = mn - m + 2m - 7$ and $|\Delta(\text{Shack}(S_m, v, n))| = 2m + 2$.

For $m \geq 3$ and $m \geq 1$, it will be shown that $\gamma_p(\text{Shack}(S_m, v, n)) \geq n - 1$ by selecting the set of dominator points namely $S = \{A_i; 1 \leq i \leq n\}$. Suppose $\gamma_p(\text{Shack}(S_m, v, n)) < n - 1$ by taking $n - 2$, then a node will be observed. As an illustration, consider the graph $\text{Shack}(S_m, v, n)$ with $\gamma_p(\text{Shack}(S_m, v, n)) = n - 2$. It appears that there are nodes that have more than one neighbor so that the other points are not observed. So the supposition above is wrong then $\gamma_p(\text{Shack}(S_m, v, n)) \geq n - 1$.

Next, it will be shown that $\gamma_p(\text{Shack}(S_m, v, n)) \leq n - 1$ by selecting the set of dominator points from $\text{Shack}(S_m, v, n)$ yaitu $S = \{A_i; 1 \leq i \leq n\}$ which observes other points, then every

node in $(Shack(S_m, v, n))$ is observed by $S = \{A_i; 1 \leq i \leq n\}$, so $\gamma_p(Shack(S_m, v, n)) \leq |S| = n - 1$. Therefore $\gamma_p(Shack(S_m, v, n)) \geq n - 1$ and $\gamma_p(Shack(S_m, v, n)) \leq n - 1$, then $\gamma_p(Shack(S_m, v, n)) = n - 1$. The set S contains $n - 1$ elements namely $S = \{A_i; 1 \leq i \leq n\}$. Thus the value $\gamma_p(Shack(S_m, v, n)) = n - 1$ is obtained.

To strengthen the evidence above, based on Theorem 1, namely $\gamma_p(G) \geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil$. will be shown that $n - 1 \geq \left\lceil \frac{mn+n+2m-7}{2m+2} \right\rceil$.

$$\begin{aligned} \gamma_p(G) &\geq \left\lceil \frac{Z(G)}{\Delta(G)} \right\rceil \\ n - 1 &\geq \left\lceil \frac{mn + n + 2m - 7}{2m + 2} \right\rceil \\ \Leftrightarrow n - 1 &\geq \left\lceil \frac{n(m + 1) + 2m - 7}{2(m + 1)} \right\rceil \\ \Leftrightarrow n - 1 &\geq \left\lceil \frac{n(m + 1)}{2(m + 1)} + \frac{2m - 7}{2m + 2} \right\rceil \\ \Leftrightarrow n - 1 &\geq \left\lceil \frac{n}{2} + \frac{2m - 7}{2m + 2} \right\rceil \\ \text{Thus } n - 1 &\geq \left\lceil \frac{mn+n+2m-7}{2m+2} \right\rceil. \quad \blacksquare \end{aligned}$$

The following will be shown an example of graph $Shack(S_4, v, 2)$ can be seen in Figure 3.

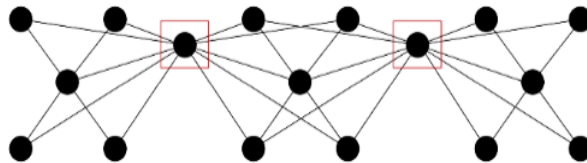


Figure 3. Power Dominating Set $(Shack(S_4, v, 2))$

D. CONCLUSIONS AND SUGGESTIONS

The following are conclusions obtained from the results and previous discussions.

1. Cardinalities of points (orders) and sides (sizes) on Shackle operational result graphs with points as linkage in this paper are as follows:
 - a. $|V(Shack(P_m, v, n))| = mn + m + n$ and $|E(Shack(P_m, v, n))| = 3mn + m - n - 1$;
 - b. $|V(Shack(C_m, v, n))| = mn + m + n$ and $|E(Shack(C_m, v, n))| = 3mn + m$;
 - c. $|V(Shack(S_m, v, n))| = mn + m + 2n + 1$ and $|E(Shack(S_m, v, n))| = 3mn + m + 2n$.
2. The power domination number on the Shackle operation result with points as linkage graphs in this paper are as follows:
 - a. $\gamma_p(Shack(P_m, v, n)) = n - 1$, for $m \geq 2$ and $n \geq 1$;
 - b. $\gamma_p(Shack(C_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$;
 - c. $\gamma_p(Shack(S_m, v, n)) = n - 1$, for $m \geq 3$ and $n \geq 1$;

From the results of the Power Dominating Number research, the researchers provide advice to other researchers to be able to examine the topic with special graphs and other operating results graphs. And can determine applications related to Power Dominating Set to complete solutions in everyday life.

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