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The Impact of Peer Pressure Mathematical Models Armed Criminal Groups with Criminal Mapping Area

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ABSTRACT

Model Armed Criminal Groups is mathematically realistic to be considered in the study of mathematical science. The aim of this research is to form a mathematical model of social cases of criminal acts. The given model is a criminal form that adopts the conformity of the conditions in the susceptible, exposed, infected, and recovered (SEIR) disease distribution model. The research method used is literature study and analysis. The research results show that there are 2 non-negative equilibrium, and one of them is stability analysis. Stability analysis is only carried out at equilibrium that does not contain a zero value with the Routh-Hurwitz criteria. In the results of other research the trajectories show that population growth tends not to experience fluctuations, this indicates that the population is growing towards stability rapidly. In case studies in the field, this marks a cycle of crime that quickly subsides or only occurs in a short period of time and does not occur in a sustainable manner. Overall the susceptible population, the exposed population, the infected population, and the recovered population experience the same conditions.

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A. INTRODUCTION

The development and progress of the modern world today cannot be separated from mathematics. All human activities are related to mathematical knowledge. The role of mathematics in everyday life problems as well as in other disciplines is usually presented in the form of mathematical modeling (Sooknanan et al., 2013). Mathematical modeling can describe complex situations in everyday life in a simpler way (Pratama et al., 2023). A mathematical model is a set of equations that express real behavior and is made based on assumptions. Assumptions that are closer to the actual conditions will produce better mathematical models (Monuteaux et al., 2015). Mathematical models are used in various fields, both in the natural sciences, medical engineering, and social sciences such as economics and social engineering.

The development of the world of research in mathematics does not only focus on exact knowledge but mathematics is also integrated with the social field. Symptoms and social behavior of a society that continues to experience dynamics and changes are interesting to be formulated into mathematical modeling (Pratama et al., 2022). People's behavior such as addiction, crime, pornography, and terrorism, can be modeled mathematically more simply.

The ultimate goal of this form of modeling is to be taken into account by policymakers to control the variables that factor in the high or low social behavior of the community (Zhilla & Lamallari, 2015) (Vaughn & DeLisi, 2018). The integration of applied mathematics with other sciences will answer problems with a scientific and scientific approach. Mathematics knowledge integrated with social sciences, for example, will become a new role model for the development of other applied sciences in the future.

The focus of the development of mathematical modeling carried out in this research is to determine the behavior of armed criminal groups through social contacts in society. Conducted a study by modeling the growth of criminal groups which are considered infectious diseases in the form of susceptible, infected, and recovered (SIR) models (Short et al., 2008) (Ferreira et al., 2020). Both studies focus on developing and exploring SIR models. Meanwhile, research has been conducted that focuses on several forms of criminal behavior in groups. Criminal acts committed in groups will further affect the growth rate of other criminal groups (Moore & Bergner, 2016) (Pratama, 2022). Both studies focus on developing and exploring SIR models. Meanwhile, research has been conducted that focuses on several forms of criminal behavior in groups.The example of brawling behavior between student groups always gives birth to rival brawl groups that become the strongest rivals of the brawling group. Likewise, in other forms of criminal cases, it always affects the growth of new criminal groups.

Many criminal group cases, it is assumed that criminal group members only commit criminal crimes, but do not recruit new members. Mathematically the criminal group model is developed into susceptible, exposed, infected, and recovered (SEIR) forms (Zaman et al., 2017)(Puspitasari et al., 2021). The results of both studies show that the SEIR model is rational to consider for criminal forms of group behaviour. The development of the SEIR model is realistic to consider because before a person commits a crime, interaction is needed between groups who will commit a crime and have committed a crime (Nur, 2020) (Garriga & Phillips, 2022) (Feltran et al., 2022). Research conducted by Nur (2020) Garriga & Phillips (2022) Feltran et al. (2022) also confirms that the closer the interaction between criminal group members, the greater the potential for criminal groups to recruit their members. The tendency of interaction between groups is mostly done by group members who have relationships. Relationships in criminal groups are mostly done because friends, lovers, family, or strong relationships influence others.

The focus of the research study is on the types of actions of armed criminal groups. Sharp weapons used to carry out criminal piercings include; machetes, knives, bows, swords, slingshots, and so on. Armed criminal groups are groups that actively recruit with various methods (D'Orsogna & Perc, 2015)(Pratama et al., 2023). Based on observations and interviews conducted at the Merauke resort police, common goals, patriotic spirit, ideology, family ties, and existence are some examples of recruitment factors that often become the basis for the group to recruit. In this study, the focus of the modeling is carried out on four compartments, namely the population that is susceptible to becoming members of a criminal group susceptible, the population of criminal groups that cannot recruit new members exposed, the population of criminal groups that can recruit new members infected and the population that has been ceased to commit a crime recovered. From these four compartments, a mathematical model of social cases, namely criminal acts, will be formed. A mathematical model formed with peer pressure and in stable conditions is the goal of the research. The final stage of the research is also discussed in the simulation section without recruitment behavior and through recruitment by criminal groups.

B. METHODS

Literature study is the basis for this type of research to model the actions of armed criminal groups. The research stages are arranged based on the following research steps; assumption testing, unit analysis, model formulation, stability analysis, eigenvalues, and numerical simulation. The formulation of the criminal group behavior model adopts the susceptible, infected, and recovered (SIR) epidemiological model (J. Kuhn, 2022) (Peter et al., 2018). The stages of this research process with the SIR model are as shown in Figure 1.

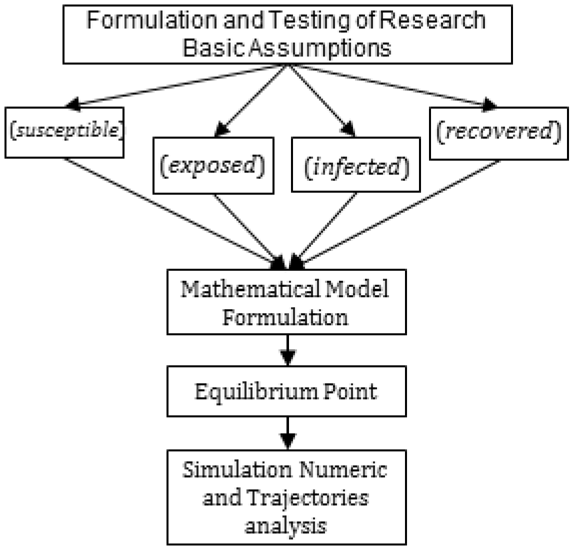

Figure 1. Flow diagram of the research

The SIR model can be used to analyze the behavior of armed criminals who are considered to be infectious diseases. The community population in the mapping of criminal areas is divided into four community groups (Schmidt et al., 2021) (Feltran et al., 2022). The population that becomes a vulnerable group to become a new member of a criminal group is N. Populations N are vulnerable to becoming members of criminal groups because of social interactions. All individuals who are not criminal members become part of the population N.

The population of groups that commit criminal acts is divided into two, criminal groups with the ability to recruit new members P_1 and criminal groups that are passive or do not recruit members P_2. Meanwhile, those representing criminal groups who repent or do not commit crimes of armed crimes are symbolized by R. Assuming positive population growth due to births and individual movements Perko (2001), the population growth rate is α, will automatically increase the population N and each population N, P_1, P_2 and R can decrease due to the natural death rate of each individual of δ_1, δ_2, δ_3 and δ_4. Populations P_1 that are actively recruiting vulnerable populations N have an effect of τ, which results in a population shifting N into the population P_1. Individuals from the group N can move to the group population P_1, due to the influence of themselves with a transfer rate of β, then the transfer rate is equal to $\beta\tau P_1$.

While in the population P_1 changes can occur due to the displacement of individual populations to population groups P_2 with a rate of change that is ω. On the basic assumptions of the study, two recovery measures were given. The first recovery is intervention from the government or the authorities so that the recovery rate is given at a rate of ϕ_1. The second recovery is intervention from the surrounding environment, namely local cultural laws or customary norms, at a rate of ϕ_2. All forms of recovery are mathematically given the form $(\phi_1 + \phi_2)$. The phenomenon that is also considered is the ineffectiveness of the form of recovery that is carried out, causing the population to P_1 increase from the population R. The ineffectiveness of this form is given for certain cases where the displacement rate is $(\lambda - \lambda f_1)$, where λ is the recovery rate constant.

The population P_2 can be reduced because of the recovery intervention carried out, an analogy as in the population P_1 that recovery is carried out in two forms, namely the rate ϕ_2 at the government rehabilitation stage and the rate at ϕ_3 the social environment stage or customary law/norms. Population movement P_2 -to- R is equal to $(\phi_2 + \phi_3)$. In the population P_2, it is also assumed that there is a cycle of saturation in the behavior of armed criminal acts so that without going through the recovery process, the individual returns to being a population that is not a sharp-armed criminals. The ineffectiveness of the proposed form of recovery is $(\lambda - \lambda f_2)$. Meanwhile, in the successful condition of the population during recovery, it will increase the population N at a rate of displacement $(\lambda f_1 + \lambda f_2)$. In the flow chart the assumptions that have been described are as shown in Figure 2.

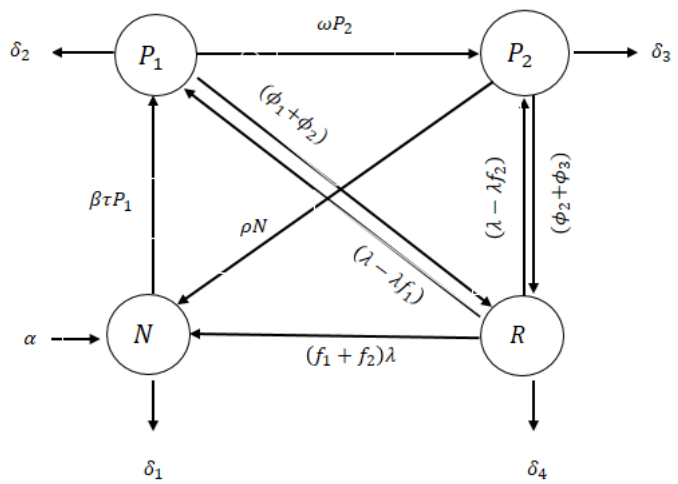

Figure 2. Compartment Diagram of the Criminal Group Population Model

From the schematic diagram of the mathematical modeling of the behavior of armed criminals, the following non-linear mathematical differential equations are obtained;

$$\frac{dN}{dt}=\alpha+(\lambda f_1+\lambda f_2)R+\rho P_2N-\beta\tau P_1N-\delta_1N,$$
$$\frac{dP_1}{dt}=\beta\tau P_1N-\omega P_1P_2-(\phi_1+\phi_2)P_1-\delta_2P_1,$$
$$\frac{dP_2}{dt}=\omega P_1P_2-(\phi_2+\phi_3)P_2+(\lambda-\lambda f_1)R-\rho P_2N-\delta_3P_2,\qquad(1)$$
$$\frac{dR}{dt}=(\phi_1+\phi_2)P_1+(\phi_2+\phi_3)P_2-\delta_4R.$$

where $N\geq 0$, $P_1\geq 0$, $P_2\geq 0$ and $R\geq 0$.

Dimensional analysis in a model (1) will be given in accordance with the assumptions that have been given previously. The dimensions of the model variable (1) are considered because taking the parameter values it is more realistic in accordance with the actual conditions. The descriptions and dimensions are presented in the followinTable 1.

Table 1. Model Description and Dimensions

Parameter	Unit	Parameter	Unit
$N\geq 0$	$[M][T]^{-1}$	ω	$[T]^{-1}$
$P_1\geq 0$	$[M][T]^{-1}$	ϕ_1	$[T]^{-1}$
$P_2\geq 0$	$[M][T]^{-1}$	ϕ_2	$[T]^{-1}$
$R\geq 0$	$[M][T]^{-1}$	ϕ_3	$[T]^{-1}$
α	$[M][T]^{-1}$	λ	$[T]^{-1}$
δ_1	$[T]^{-1}$	τ	-
δ_2	$[T]^{-1}$	f_1	-
δ_3	$[T]^{-1}$	f_2	-
δ_4	$[T]^{-1}$	$(1-f_1)$	-
β	$[T]^{-1}$	$(1-f_2)$	-

C. RESULT AND DISCUSSION

1. Equilibrium without Armed Criminal Groups

This section, model (1) is given the form without the existence of criminal groups. This is very possible because the actions or methods taken are effective in creating a conducive society. The differential equation operation used in the model (1) is with conditions $\frac{dN}{dt}=0$, $\frac{dP_1}{dt}=0$, $\frac{dP_2}{dt}=0$ and $\frac{dR}{dt}=0$. Mathematically, the equilibrium model (1) is obtained as follows, $E_0=(N^*,0,P_2^*,R^*)$. These conditions make it possible that schemes for population growth in the presence of armed criminal groups do not exist. In addition, it can be said that the population in an area does not experience recruitment of criminal behavior. Population growth without criminal gangs is the quotient between the constant rate of natural population growth and natural deaths in individual populations. The smaller the death rate of the population, the greater the impact of population growth. On the other hand, the lower the birth rate, the lower the death rate.

2. Equilibrium with Armed Criminal Groups

Equilibrium with armed criminal groups is a mathematical model involving the variables of armed criminal groups in the population. In model (1) and by solving the differential equation, the equilibrium point is obtained $E_1 = \left(N^*, P_1^*, P_2^*, R^*\right)$. The equilibrium point obtained is the solution of the quadratic equation in model (1), namely;

$$ax^2 + bx + c = 0, \tag{2}$$

where $a \neq 0$, with

$$x_{1,2}^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{3}$$

There are several possible roots that arise from (2), because it is determined by the value of the determinant symbolized by D (Verhulst, 2017). There are two real roots $D \geq 0$, $x_1 + x_2 > 0$, and $x_1 \cdot x_2 > 0$. There are two negative real roots if $D \geq 0$, $x_1 + x_2 < 0$, and $x_1 \cdot x_2 > 0$. Two real roots with different signs if $D \geq 0$ and $x_1 \cdot x_2 < 0$. The two real roots are equal if $D \geq 0$ dan $x_1 \cdot x_2 > 0$. The two real roots are inverse if $D > 0$ and $x_1 + x_2 = 0$. The two real roots are opposite if $D > 0$ and $x_1 \cdot x_2 < 0$. All imaginary roots if $D < 0$. The possible values to be analyzed in function (2) are positive real ones. Taking the positive real values that emerge from equation (2) is $N^* = x^*$, so that the equilibrium $E_1 = \left(N^*, P_1^*, P_2^*, R^*\right)$ becomes,

$$P_1^* = \frac{w_1 x^* + w_2}{\omega \beta \tau \delta_4 x^* + w_3},$$

$$P_2^* = \frac{x^* \beta \tau - \delta_2 \phi_2 \phi_1}{\omega},$$

$$R = \frac{x^* w_4 + w_5}{w_6 \omega},$$

where x^* are the positive real roots that allow the Routh-Hurwitz criteria to be tested to fulfill the stability requirements of the model (1).

3. Equilibrium Analysis

a. Equilibrium without Armed Criminal Groups

The analysis in this section is given for model (1) without the behavior of armed criminal groups. The analysis given is an analysis of the stability of the equilibrium point without criminal groups. Equilibrium $E_0 = \left(N^*, 0, 0, 0\right)$ is the basic basis for this analysis to be carried out. The next step is to form a Jacobian matrix, for the model equation (1), as follows,

$$J_{jacob}\left(E_0\right) = \begin{bmatrix} j_{aa} & j_{ab} & j_{ac} & 0 \\ j_{ba} & j_{bb} & j_{bc} & 0 \\ 0 & j_{cb} & j_{cc} & 0 \\ 0 & j_{db} & j_{dc} & j_{dd} \end{bmatrix}, \tag{4}$$

where
$j_{aa}=(f_1+f_2)\lambda-\beta\tau P_1^*-\delta_1$, $j_{ab}=-\beta\tau N^*$, $j_{ac}=\rho$, $j_{ad}=(f_1+f_2)\lambda$, $j_{cc}=-\rho-\delta_3-\phi_2-\phi_3$, $j_{dd}=\dfrac{\beta\tau\alpha}{\delta_1}-\delta_2-\phi_1-\phi_2$, $j_{cd}=-\lambda f_1+\lambda$, $j_{db}=\phi_1+\phi_2$, $j_{dc}=\phi_2+\phi_3$, $j_{dd}=-\delta_4$.

The characteristic equations that emerge from the Jacobian matrix are,

$$\lambda^3+A_2\lambda^2+A_1\lambda+A_0=0, \tag{5}$$

the roots of the negative real part that arise from equation (5) as a solution to the equation and meet the Routh-Hurwitz criteria. The Hurwitz matrix that appears is,

$M_1=[A_2]$,

$M_2=\begin{bmatrix}A_2 & 1\\ 0 & A_1\end{bmatrix}$,

$M_3=\begin{bmatrix}A_2 & 1 & 0\\ A_0 & A_1 & A_2\\ 0 & 0 & A_0\end{bmatrix}$.

Conditions that meet the Routh-Hurwitz criteria as a locally asymptotically stable point are, $A_2>0$, $A_2\cdot A_1>0$ and $A_2\cdot A_1\cdot A_0-A_0^2>0$ or conditions that are the same for $A_1>0$ and $A_2A_1>A_0$.

b. Equilibrium with Armed Criminal Groups

This section will show an equilibrium analysis involving the behavior of armed criminal groups. The population that engages in criminal group behavior is assigned to the entire model (1). The equilibrium point analyzed is $E_1=(N^*,P_1^*,P_2^*,R^*)$. Equilibrium point analysis with criminal group behavior shows the form of the Jacobian matrix,

$$J_{jacob}(E_1)=\begin{bmatrix}j_{aa} & j_{ab} & j_{ac} & j_{ad}\\ j_{ba} & j_{bb} & j_{bc} & 0\\ 0 & j_{cb} & j_{cc} & j_{cd}\\ 0 & j_{db} & j_{dc} & j_{dd}\end{bmatrix}, \tag{6}$$

where
$j_{aa}=-\beta\tau P_1-\delta_1$, $j_{ab}=-\beta\tau N$, $j_{ac}=\rho$, $j_{ad}=(f_1+f_2)\lambda$, $j_{ba}=\beta\tau P_1$, $j_{bb}=N\beta\tau-\omega P_2-\delta_2-\phi_1-\phi_2$, $j_{bc}=-\omega P_1$, $j_{cb}=\omega P_2$, $j_{cc}=\omega P_1-\rho-\delta_3-\phi_2-\phi_1$, $j_{cd}=-\lambda f_1+\lambda$, $j_{db}=\phi_1+\phi_2$, $j_{dc}=\phi_2+\phi_3$ and $j_{dd}=-\delta_4$,

The characteristic equations associated with the Jacobian matrix are,

$$\lambda^4+B_3\lambda^3+B_2\lambda^2+B_1\lambda+B_0=0, \tag{7}$$

the roots of the negative real part that arise from equation (7) as a solution to the equation and meet the Routh-Hurwitz criteria. The Hurwitz matrix that appears is,

$N_1=[B_3]$,

$$N_2 = \begin{bmatrix} B_3 & 1 \\ 0 & B_3 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} B_3 & B_1 & 0 \\ 1 & B_2 & B_0 \\ 0 & B_3 & B_1 \end{bmatrix},$$

$$N_4 = \begin{bmatrix} B_3 & B_1 & 0 & 0 \\ 1 & B_2 & B_0 & 0 \\ 0 & B_3 & B_1 & 0 \\ 0 & 1 & B_2 & B_0 \end{bmatrix}.$$

Conditions that meet the Routh-Hurwitz criteria as a locally asymptotically stable point are, $B_3 > 0$, $B_2 \cdot B_3 - B_1 > 0$, $B_3 \cdot B_2 \cdot B_1 - \left(B_1^2 + B_3^2 B_0\right) > 0$ and $B_3 \cdot B_2 \cdot B_1 \cdot B_0 > 0$.

4. Reproduction Number

The variable of criminal groups that interacts directly with the assumption of the behavior of armed criminal groups is the differential equation of criminal group behavior. There are two forms of function involving the behavior of criminal groups from equation (1), namely

$$\begin{aligned} \frac{dP_1}{dt} &= \beta\tau P_1 N - \omega P_1 P_2 - (\phi_1 + \phi_2)P_1 \\ &- \delta_2 P_1, \\ \frac{dP_2}{dt} &= \omega P_1 P_2 - (\phi_2 + \phi_3)P_2 + (\lambda - \lambda f_1)R \\ &- \rho P_2 N - \delta_3 P_2 \end{aligned} \tag{8}$$

The reproduction number associated with the model form (8) is as follows,

$$R_0 = \frac{(\omega\tau\beta P_1 - w)N}{P_2 m - P_1 n + h}. \tag{9}$$

where,

$$w = \tau\beta\rho - \tau\beta\delta_3 - \tau\beta\phi_2 - \tau\beta\phi_3,$$

$$m = \omega\rho + \omega\delta_3 + \omega\phi_2 + \omega\phi_3,$$

$$n = \omega\delta_2 + \omega\phi_1 + \omega\phi_2,$$

$$\begin{aligned} h &= \rho\delta_2 + \rho\phi_1 + \rho\phi_2 + \delta_2\delta_3 + \delta_2\phi_2 + \delta_2\phi_3 \\ &+ \delta_3\phi_1 + \delta_3\phi_2 + \phi_1\phi_2 + \phi_2\phi_3 + \phi_1\phi_3 + \phi_2^2. \end{aligned}$$

There are several possible values R_0 that represent its representation, the first condition being the condition for $R_0 < 1$, it is clear that gradually the population with heavily armed criminal groups will disappear. Each individual from the population who is recruited into a criminal group on average produces less than one individual recruitment into a new criminal group. In field conditions, such conditions are usually due to the failure of recruitment in the criminal group itself. The second condition for $R_0 = 1$, this means that criminal groups both individuals and groups will remain in the population. The existence of this group will continue to exist in the population, but it does not have a significant impact on the recruitment carried out and does not get a good response from the general public. In the third condition, if $R_0 > 1$ the armed criminal groups will spread into the population. This signifies the successful recruitment and raising of criminal gangs. In these conditions, the tendency for social conflicts to occur is very large, due to a security and order crisis.

5. Numerical Simulation

Numerical simulation is given to analyze the trajectories of the movement of the overall population distribution. The system solution model (1) will also be displayed numerically as well as the reproduction number R_0 on the model. Parameter values are taken based on relevant assumptions and references in this study. Each of these parameters $\alpha = 19$, $\mu = 0.5$, $\lambda = 0.052$, $f_1 = 0.44$, $f_2 = 0.2$, $\omega = 0.21$, $\phi_1 = 0.115$, $\phi_2 = 0.1$, $\phi_3 = 0.05$, $\tau = 0.0501$, $\mu = 0.5$, $\beta = 0.9$, $\delta_1 = 0.5$, $\delta_2 = 0.6$, $\delta_3 = 0.8$, $\delta_4 = 0.6$, and $\rho = 0.1$. The results of numerical simulation in model (1) show two non-negative equilibrium that have characteristics, namely $E_0 = (40.952983, 0, 1.1357627, 0.28394068)$ and $E_1 = (26.82173, 4.837348, 1.878057, 2.20289)$. At equilibrium E_0, it shows a condition without a population of armed criminal groups. Under these conditions, recruitment does not go well or there is no recruitment rate. Conditions like this are certainly unstable in people's lives, so the equilibrium is E_0 ignored in the follow-up analysis. This condition E_1, there is an equilibrium that will be tested for the stability of the model (1). The growth of criminal groups can be seen clearly and will be shown by trajectories of individual growth with criminal groups. Equilibrium E_1, resulting in eigen values are $\lambda_1 = -0.1165059\,24914891$, $\lambda_2 = -0.11650592\,4914891$, $\lambda_3 = -0.5230014\,3317347$ dan $\lambda_4 = -0.5962595\,44396749$. All eigenvalues meet the Routh-Hurwitz criteria, so the point E_1 is a local asymtotic equilibrium point.

The reproduction number model (1), a shape that comes from a stable asymtotic local equilibrium point is simulated, namely equilibrium $E_1 = (26.82173, 4.837348, 1.878057, 2.20289)$. The R_0 representative results with the model (1) are $R_0 = 0.09346982\,322$. It is clear that the number of values $R_0 > 1$, causes the behavior of criminal groups to be very massively incorporated into people's lives. In other words, the behavior of criminal groups spreads in the population growth model (1). The incident also illustrates that, on average, each individual who is a member of a criminal group successfully recruits more than one new member. So that the criminal behavior committed is transmitted to the new members who join. Such a process continues to run linearly over time.

6. Trajectories Model

Based on the parameter values, trajectories analysis was performed from each population. Trajectories analysis is given to look at the movement of population growth and the distribution of individuals who adopt the behavior of armed criminals. Assuming the initial conditions equation $N(0) = 45$, $P_1(0) = 15$, $P_2(0) = 12$ dan $R(0) = 9$. All initial values taken are in the time dimension t, so the growth trajectories are as shown in Figure 3, Figure 4, Figure 5, and Figure 6.

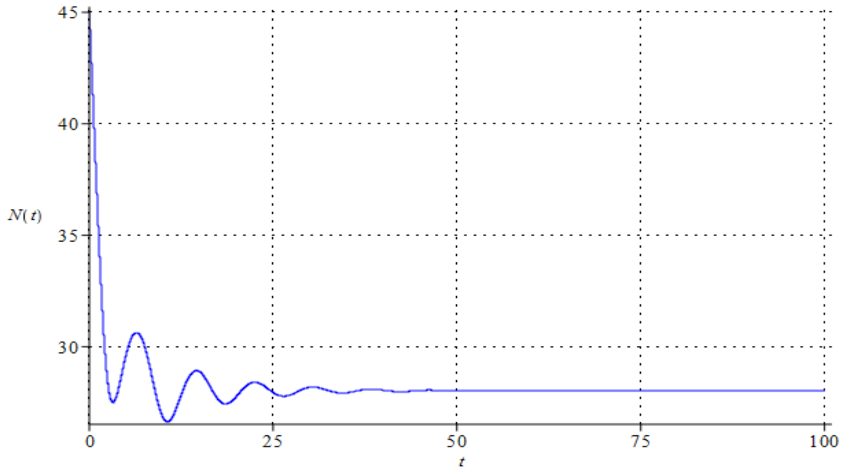

Figure 3. Trajectories Population Susceptible N

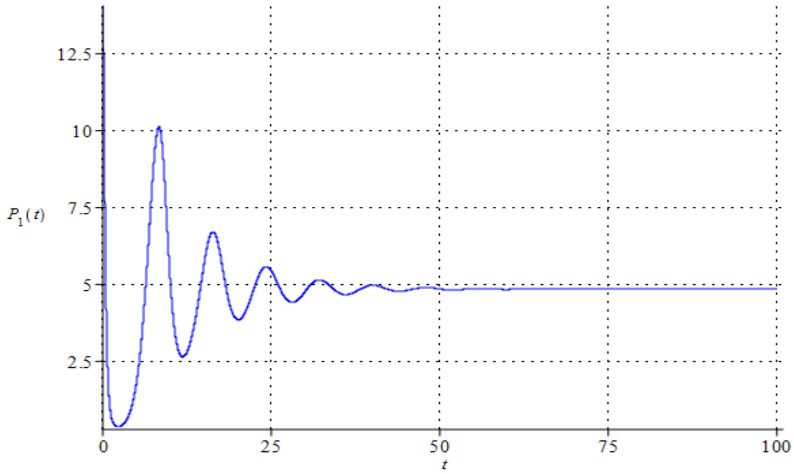

Figure 4. Trajectories Population Exposed P_1

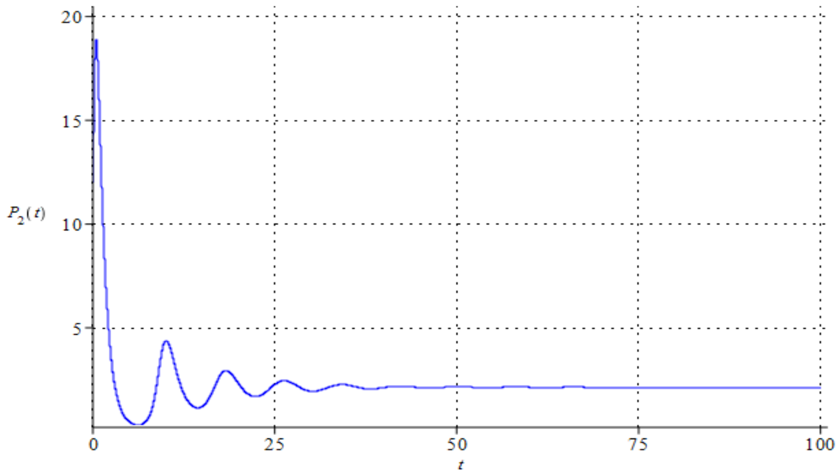

Figure 5. Trajectories Population Infected P_2

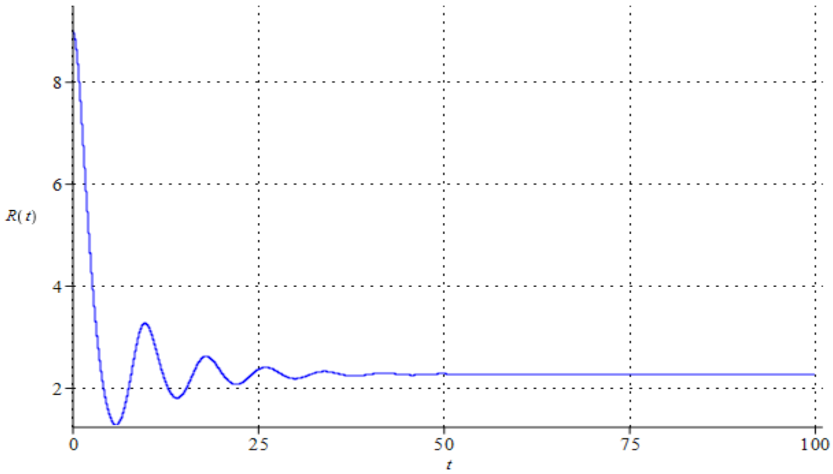

Figure 6. Trajectories Population Recovered R

Trajectories correspond to all figures in the numerical simulation model (1). Population growth tends not to experience sharp fluctuations, this indicates the population is growing towards stability rapidly. In case studies in the field, this marks a cycle of crime that quickly subsides or only occurs in a short period of time and does not occur continuously. The results of this research are in line with previous research which states that criminal behavior will decrease when it reaches the peak of behavioral saturation. Other research also states that criminal behavior is a cyclical behavior that is repeated and continuous. This is in line with social research studies which say that crimes committed by humans will never disappear. Overall the population is susceptible (N), the population is exposed (P_1), the population is infected (P_2), and the population is recovered (R). Population growth fluctuates at the beginning of the interaction, but as time goes by the interactions are getting closer to the equilibrium point. In the population of criminal groups that are actively recruiting P_1 population growth is very volatile at the initial interaction compared to the other three populations. The results of this research are in line with research which states that criminal groups that recruit always approach potential members. This is in line with humanitarian theory which continues to take a social approach to following or looking for followers in criminal behavior. Recruitment interaction is a strong reason for the fluctuating trajectories process that occurs in model 1. The description, an analysis of the model (1) is given with the condition that it only involves criminal groups that do not recruit members or only involve criminal groups that are actively recruiting new members. This analysis is given to see events that may occur in the community's ecosystem. Each trajectory is given as shown in Figure 7 and Figure 8.

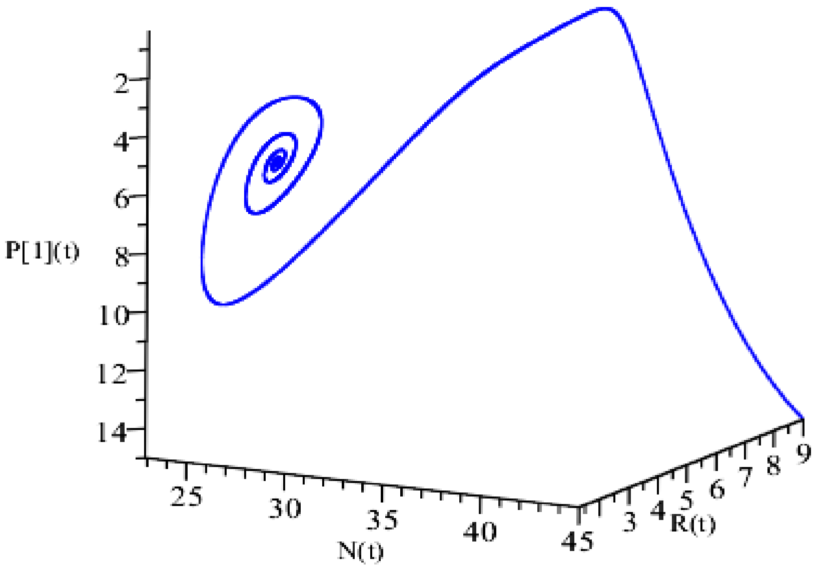

Figure 7. Trajectories Population $N(t), P_1(t)$ and $R(t)$

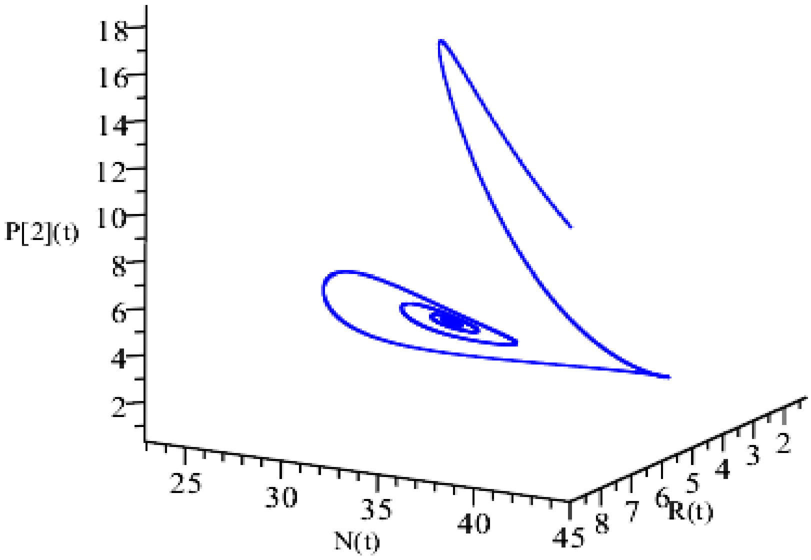

Figure 8. Trajectories Population $N(t), P_2(t)$ and $R(t)$

Figure 7 and figure 8 it is clear that there is a significant difference in population growth with the group variables recruiting new members and those not recruiting members. In figure 8, the criminal growth is clearly visible towards great stability. Meanwhile, the condition is inversely proportional to the population with criminal groups that do not recruit. The growth in figure 7 is not significant in the criminal group, but is more consistent and does not fluctuate. The condition that may occur is that criminal behavior lasts longer in criminal groups that do not recruit new members, while in criminal groups that recruit new members it ends faster or is handled by the authorities. Writing the results and discussion can be separated into different sub or can also be combined into one sub. The summary of results can be presented in the form of graphs and figures. The results and discussion sections must be free from multiple interpretations (Vaughn & DeLisi, 2018). The discussion must answer research problems, support and defend answers with results, compare with relevant research results, state the limitations of the study carried out and find novelty.

D. CONCLUSION AND SUGGESTIONS

Model (1) is based on assumptions that are close to real conditions in the community. The assumptions built are formed from observations, interviews, and relevant theories in the study. After the assumptions are built, the mathematical model that is formed is arranged in a differential equation. The population in model (1) consists of a population that is susceptible to being a member of a susceptible criminal group (N), a population of a criminal group that cannot recruit new members exposed (P_1), a population of a criminal group that can recruit new members is infected (P_2) and a population that has stopped committing crimes recovered (R). The model formed is model (1), namely:

$$\frac{dN}{dt} = \alpha + (\lambda f_1 + \lambda f_2)R + \rho P_2 N - \beta\tau P_1 N - \delta_1 N,$$

$$\frac{dP_1}{dt} = \beta\tau P_1 N - \omega P_1 P_2 - (\phi_1 + \phi_2)P_1 - \delta_2 P_1,$$

$$\frac{dP_2}{dt} = \omega P_1 P_2 - (\phi_2 + \phi_3)P_2 + (\lambda - \lambda f_1)R - \rho P_2 N - \delta_3 P_2,$$

$$\frac{dR}{dt} = (\phi_1 + \phi_2)P_1 + (\phi_2 + \phi_3)P_2 - \delta_4 R.$$

The model (1) above was analyzed by mathematical linearization under conditions $\frac{dN}{dt} = 0$, $\frac{dP_1}{dt} = 0$, $\frac{dP_2}{dt} = 0$ and $\frac{dR}{dt} = 0$. The results show that model (1) gives 2 non-negative equilibrium, each of which is $E_0 = \left(N^*, 0, P_2^*, R^*\right)$ and $E_1 = \left(N^*, P_1^*, P_2^*, R^*\right)$. Stability analysis is carried out only at equilibrium that does not contain zero. Consideration to see a positive criminal population growth rate at equilibrium E_1. Equilibrium stability was tested with the Routh-Hurwitz criteria which resulted in the corresponding eigenvalues in the characteristic equation that appeared. Each of these eigenvalues are $\lambda_1 = \text{-}0.116505924914891$, $\lambda_2 = \text{-}0.116505924914891$, $\lambda_3 = \text{-}0.52300143317347$ and $\lambda_4 = \text{-}0.596259544396749$. Trajectories are also given in the analysis of research results to observe fluctuating population growth trajectories. The results as a whole show the condition of the population growing only at the beginning of growth and soon ending towards stability. This condition will accelerate criminal behavior in the community to reduce and not experience ongoing cases. Conditions like this can occur with appropriate intervention from the authorities. Social engineering by the government can also reduce the crime rate which continues to grow under certain conditions. In the next research, we recommend a criminal group model that is formed from the alle effect or fear effect on criminal groups. The nature of fear in humans always exists, so the intervention to reduce criminal behaviour by intervening in fear. So that this fear intervention will reduce criminal behaviour in the mathematical model.

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