

The Application of Delta Gamma Normal Value at Risk to Measure the Risk in the Call Option of Stock

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ABSTRACT

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Call options of stock have a nonlinear dependence on market risk factors, thus encouraging the development of a method capable of measuring the risk of call option of stock, namely the Delta Gamma Normal Value at Risk (DGN VaR) method. The DGN VaR method can provide a more accurate VaR estimate than Delta Normal VaR (DN VaR) because of the Delta and Gamma sensitivity measures in the formula. The DGN VaR method uses the second-order Taylor Polynomial approach to approximate the return of stock price underlying the call option. This research applies the DGN VaR method to analyze the risk of call options of Atlassian Corporation (TEAM) and MicroStrategy Incorporated (MSTR). Both companies operate in the technology sector and are among the top 100 largest software companies based on market capitalization for the analysis period September 21, 2022 to September 21, 2023. The analyzed options in this research consist of in-the-money and out-of-the-money options with several strike prices (K). For in-the-money options, the strike prices are \$105, \$110, and \$115 for TEAM, and \$150, \$160, and \$170 for MSTR, while for out-of-the-money options, the strike prices are \$190, \$195, and \$200 for TEAM, and \$330, \$340, and \$350 for MSTR with varying confidence levels of 80%, 90%, 95%, and 99%. Based on the results of the analysis, the DGN VaR for the analyzed in-the-money option has a greater value than the DGN VaR for the analyzed out-of-the-money option.



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A. INTRODUCTION

Investing in the capital market is an alternative for developing owned assets to obtain profits in the future. However, apart from making a profit, investing in the capital market is also not free from risks (losses). In an effort to overcome this, various strategies have been developed that are able to minimize risk (limit the investor losses) (Miftahurrohman et al., 2021). One of the strategies is called hedging, which utilizes derivative instruments in its implementation (Hull & White, 1987). Derivative instruments are financial instruments that derive their value from the price of underlying assets, such as equity instruments, fixed-income instruments, foreign currencies, and commodities (Chan et al., 2019). Derivative instruments include options, futures, forwards, swaps, and warrants obtained through underlying assets such as stocks, bonds, commodities, and others (Wróblewski et al., 2023). Among derivative instruments, options with stocks as the underlying asset will be the focus of this research.

According to Gumanti (2017), stock options can be used by investors as a hedging tool, which will provide new opportunities for investors to gain profits amidst the volatility of stock prices. An option is a legal agreement between two parties, the option writer and the option

holder, whereby the option holder (option buy) grants the option writer the right to purchase or sell specific assets at a predetermined price (strike price) and within a predetermined window of time (expiration date) (Chance & Brooks, 2015). Put options are stock options that allow the option contract holder the right to sell stocks at a specific price and time, while call options are on the other hand, give the right to purchase stocks at a specific price and time (Higham, 2004). Just like other investment tools, options still have a risk of loss.

Recently, many advancements have been made in calculating the risk value of investing because forecasting risk levels is an important task in financial risk management (Hoga & Demetrescu, 2023). Knowledge about risk is important for the investor (Sarpong et al., 2018). There have been many advancements in calculating the risk of investing recently. The risk measurement that has been widely applied is Value at Risk (VaR) (Khindanova & Rachev, 2019). The relevance of VaR to insurance and financial institutions has attracted much attention in the financial econometric literature (Nieto & Ruiz, 2016). Many studies in developing Latin American countries (Ozun & Cifter, 2007), Southeast Asian countries (Cheong et al., 2011), European Union nations (Iglesias, 2015), Nordic markets (Jobayed, 2017), South African market (Mabitsela et al., 2015; Naradh et al., 2021), and other regions have studied VaR.

The VaR concept has been widely used in financial market risk management since its introduction in the 1990s by JP Morgan (Lu, 2022). VaR is defined as an estimate of the greatest loss in an asset or portfolio's value under normal market conditions over a given amount of time and confidence level (Dimitrova et al., 2021). According to Sultra et al. (2021), VaR can be used to quantify risk and estimate the maximum potential loss that may occur in the future. Measurements using VaR are generally carried out by investors to estimate potential losses due to market risk because the VaR method can be easily used by various financial institutions (Amin et al., 2018). Unlike stocks, which depend linearly on market risk factors, options have a nonlinear dependence on these factors. Delta Normal VaR (DN VaR) and Delta Gamma Normal VaR (DGN VaR) have been developed using the Taylor Polynomial concept to approximate the return value of the stocks underlying the call option (Sulistianingsih et al., 2019). DN VaR uses a first-order Taylor Polynomial, while DGN VaR uses a second-order Taylor Polynomial (Date & Bustreo, 2016). As the name suggests, risk measurement of options using DN VaR only incorporates the Delta Greeks in its formula, whereas DGN VaR incorporates both Delta and Gamma Greeks. Delta is the sensitivity of an option's price to changes in the price of the underlying asset. Gamma is the rate of change of Delta with respect to changes in the price of an underlying asset (in this study is stocks) (Chen & Yu, 2013).

This research used the DGN VaR method utilized in several previous studies. Mina and Ulmer (1999) applied the Delta Gamma approximation to calculate VaR for four test portfolios. The research concluded that the Delta Gamma approximation closely matched the results from full Monte Carlo simulation, even for extreme portfolios. Then, Britten-Jones and Schaefer's research (1999) focused on estimating risk in nonlinear assets and found that the DGN VaR method produced portfolio value estimates close to the actual value. Then, Duffie and Pan (2001) also conducted research using the DGN VaR method along with Fourier transformation techniques to calculate risk for large portfolios with market and credit risks. They concluded that the analytical VaR approach was more computationally efficient compared to the Monte Carlo simulation for a certain level of accuracy. The study by Cui et al. (2013) utilized a

nonlinear portfolio consisting of 10 call and put options each on ten different companies to estimate risk using parametric VaR estimations. They found that the DGN VaR method outperformed the DN VaR method in terms of accuracy and computational efficiency. Sulistianingsih et al. (2019) used a portfolio consisting of one stock (Exxon Mobile Corporation (XOM)) and two options from JD.com, Inc (JD) and Eni. S.p.A. (E). They concluded that both the DN VaR and DGN VaR methods can estimate the maximum loss effectively. Furthermore, the DGN VaR method is considered superior to the DN VaR method. This research applies the DGN VaR method that utilizes the option Greeks (Delta and Gamma) on stock call options of Atlassian Corporation (TEAM) and MicroStrategy Incorporated (MSTR) for the period from September 21, 2022, to September 21, 2023.

B. METHODS

This research applies the DGN VaR method, which utilizes the option Greeks (Delta and Gamma), and focuses on call options of stocks. Several conditions in this study include the use of European call options on stocks that do not pay dividend and the assumption that stock log returns are normally distributed.

1. Delta Gamma Normal VaR (DGN VaR)

Call option risk is measured using the DGN VaR method with a second-order Taylor polynomial approach to approximate the return of the underlying stock. This method was developed because options have a nonlinear dependence on market risk factors. Measuring option risk using DGN VaR has several assumptions. The first assumption is that changes in option pricing and stock returns have a nonlinear relationship. The second assumption is that the stock returns that underlying the options are assumed to be normally distributed with mean zero ($E[\Delta S] = 0$) and variance σ^2 ($Var[\Delta S] = \sigma^2$) (Sulistianingsih et al., 2019).

In the DGN VaR method, the option value (C_t) is only affected by the stock price (S_t), while the strike price (K), maturity time (φ), stock volatility value (σ), and risk-free interest rate (r) is constant then the option price can be formulated as follows:

$$C_t \approx f(S_t) \quad (1)$$

Based on Equation (1), the return of an option in period t up to T can be expressed in Equation (2) as follows (Lehar, 2000):

$$\Delta C = C_T - C_t \approx f(S_t + \Delta S) - f(S_t) \quad (2)$$

Option prices can be derived using the second-order Taylor Polynomial approach which is formulated as follows:

$$\begin{aligned} \Delta C &\approx f(S_t + \Delta S) - f(S_t) \\ \Delta C &\approx \left(\frac{\partial C_t}{\partial S_t} \right) \Delta S + \frac{1}{2} \left(\frac{\partial^2 C_t}{\partial S_t^2} \right) (\Delta S)^2 \end{aligned} \quad (3)$$

where $\frac{\partial C_t}{\partial S_t}$ is denoted as Delta (δ) and $\frac{\partial^2 C_t}{\partial S_t^2}$ is denoted as Gamma (γ) of an option, so that Equation (3) can be written as:

$$\Delta C \approx \delta \Delta S + \frac{\gamma}{2} (\Delta S)^2 \quad (4)$$

Based on Equation (4), the mean and variance obtained for a holding period (hp) in Equation (5) and Equation (6) are as follows (Dowd, 2007; Sulistianingsih et al., 2019):

$$\begin{aligned} E[\Delta C] &= hp \cdot E \left[\delta (\Delta S) + \frac{\gamma}{2} (\Delta S)^2 \right] \\ &= hp \cdot \delta E(\Delta S) + \frac{\gamma}{2} E(\Delta S)^2 \\ &= 0 \end{aligned} \quad (5)$$

and

$$\begin{aligned} Var[\Delta C] &= hp \cdot Var \left[\delta (\Delta S) + \frac{\gamma}{2} (\Delta S)^2 \right] \\ &= hp \left(\delta^2 \sigma^2 + \frac{\gamma^2}{4} \sigma^4 \right) \end{aligned} \quad (6)$$

Then, substitute the mean and variance of the option in the general estimate of VaR to obtain the DGN VaR for stock options as follows (Sulistianingsih et al., 2019).

$$\begin{aligned} VaR &= Z_\alpha \sigma - \mu \\ VaR_{DGN} &= Z_\alpha \sqrt{hp \left(\delta^2 \sigma^2 + \frac{\gamma^2}{4} \sigma^4 \right)} - 0 \\ VaR_{DGN} &= \sqrt{hp} \left(Z_\alpha \sigma \sqrt{\left(\delta^2 + \frac{\gamma^2}{4} \sigma^2 \right)} \right) \end{aligned} \quad (7)$$

2. Kupiec Backtesting

When making investment decisions, the accuracy of the VaR model is critical. Backtesting is a tool for determining the accuracy of the VaR model's forecast (Patra & Padhi, 2015). Backtesting is at the core of financial supervision activities because the accuracy of risk measurement has implications for solvency capital, which must be taken into account by financial institutions (Evers & Rohde, 2014). In this research, backtesting will be measured based on the frequency of losses that occur in the tail of the distribution. This method is called Kupiec Backtesting. Assume that n represents the total number of observations, x is the frequency of losses beyond the Value at Risk (VaR) threshold, which is the number of observations greater than the VaR, and p is the tolerance limit for the VaR deviation. The value

of p is determined as one minus the VaR confidence level ((Dowd, 2007); (Jorion, 2007); (Rosadi, 2009)). Then, the variable x follows a binomial distribution with:

$$P(x|n, p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n \quad (8)$$

The hypothesis test used in the Kupiec Test, as described by Rosadi (2009) is as follows:

$$H_0 : P(x) \leq p \text{ and } H_1 : P(x) > p$$

where $P(x) \leq p$ indicates that the VaR Model is suitable for use. The statistic for the Kupiec Test is then defined as:

$$\Phi = P(X > x | p^* = p) = 1 - P(X \leq x | p^* = p), \quad (9)$$

where $P(X \leq x | p^* = p) = \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x}$ is the cumulative distribution of the binomial distribution. Therefore, Equation (9) can be expressed as follows:

$$\Phi = 1 - \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x} \quad (10)$$

H_0 is rejected if Φ is less than the significance level. If H_0 is not rejected, it can be concluded that the VaR model is suitable for use. The basic concept in Kupiec Backtesting is to verify whether or not the amount of losses estimated by the DGN VaR is greater than Φ . The Kupiec Backtesting in this research is conducted by developing an R program.

C. RESULT AND DISCUSSION

1. Research Data

In this research, investment risk will be estimated for Atlassian Corporation (TEAM) and MicroStrategy Incorporated (MSTR) stock options using the DGN VaR method. The data analyzed consisted of 252 observations from September 21, 2022, to September 21, 2023. Descriptive statistics of closing stock price and stock price returns can be seen in Table 1.

Table 1. Descriptive Statistics of Stock

Characteristics	Closing Prices		Returns	
	TEAM	MSTR	TEAM	MSTR
Observations	252	252	251	251
Mean	168.125	278.617	-0.001	0.002
Minimum	116.340	136.630	-0.342	-0.230
Maximum	242.460	461.830	0.159	0.150
Standard Deviation	27.127	74.548	0.044	0.052
Variance	735.856	5557.335	0.002	0.003

Based on the results of the descriptive statistics shown in Table 1, the closing price data for TEAM and MSTR stocks amounts to 252 observations, while the return data for TEAM and

MSTR stock prices amounts to 251 observations. The lowest closing price of stocks owned by TEAM occurred on November 22, 2022, at \$116.34, while the highest closing price of stocks owned by TEAM occurred on October 5, 2022, at \$242.46. Furthermore, the lowest closing price of stocks owned by MSTR occurred on December 29, 2022, at \$136.63, while the highest closing price of stocks owned by MSTR occurred on July 13, 2023, at \$461.83. The lowest stock price return was -0.342 for TEAM and -0.230 for MSTR, while the highest stock price return was 0.159 for TEAM and 0.150 for MSTR. The movement of closing prices and stock returns from TEAM and MSTR for the period 21 September 2022 to 21 September 2023 can be seen in Figure 1.

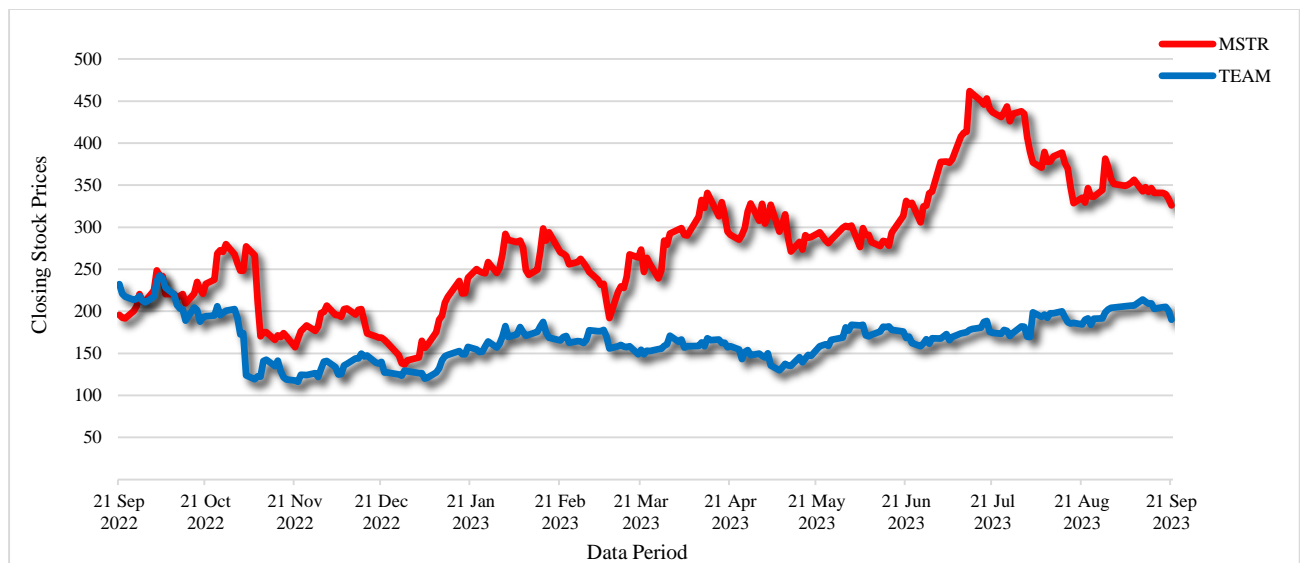


Figure 1. Graphs of Closing Stock Prices Underlying The Options

Based on Figure 1, the stock price movement of TEAM experienced a drastic decline during the period from September to November, but in the subsequent period, the stock price returned to a stable movement, while the stock price movement of MSTR tended to be fluctuating and experienced a drastic increase during the period from May to July. These stock price movements affect the risk, return, and attractiveness to investors. In investing, an investor expects to obtain high returns with minimal losses, but stocks with high returns tend to have high risks as well. The return conditions of TEAM and MSTR stocks are shown in the following Figure 2.

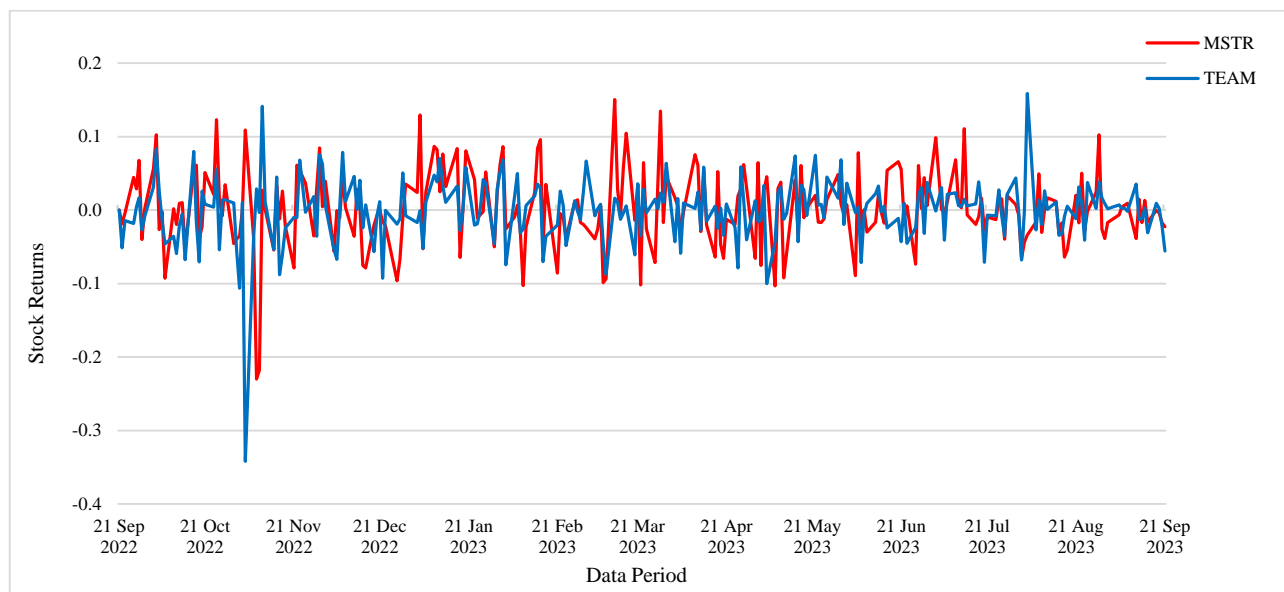


Figure 2. Graphs of Stock Returns Underlying The Options

Based on Figure 2, it can be seen that the stock return movements of TEAM and MSTR are quite fluctuating, where the stock return movement of TEAM stock ranges from -0.4 to 0.2, while the return movement of MSTR ranges from -0.3 to 0.2. Both of these stock returns indicate that the return yields are highly variable, ranging from very high to very low, even negative.

2. Return Normality Test

A normality test is a test regarding the normality of data distribution. The normality test carried out on stock return data is a requirement that must be met when estimating risk using DGN VaR. The results of the normality test for TEAM and MSTR stock return data using the Kolmogorov-Smirnov test are presented in Table 2.

Table 2. TEAM and MSTR Stock Return Data Normality Test

Characteristics	TEAM	MSTR
Sample Observation	251	251
Kolmogorov-Smirnov Z	0.080	0.065
Asymp.Sig. (2-tailed)	0.080	0.240

Table 2 indicates that the analyzed return data is normally distributed because the p-value is higher than the significance level (0.05).

3. Delta Gamma Normal VaR

In this research, the price of the stock underlying the analyzed option (S_t) is determined based on the closing prices of TEAM and MSTR companies on September 21, 2023, namely \$189.95 and \$326.06. The risk-free interest rate (r) utilized in this research is 5.5%. Then, the volatility value obtained from TEAM stock return data is 0.6993 and MSTR is 0.8206. Additionally, there are two categories of options based on the strike price (K) namely, in-the-money, where the strike price is lower than the stock price at the transaction time, and out-of-the-money, where the strike price is higher than the stock price at the transaction time. The in-the-money options used in the research were \$105, \$110, and \$115 for TEAM and \$150, \$160,

and \$170 for MSTR, while the out-the-money option used in the research were \$190, \$195, and \$200 for TEAM and \$330, \$340, and \$350 for MSTR, with the time to maturity (φ) for each strike price being 0.1151 years. Before estimating the VaR of stock options, determining several components is needed in the VaR analysis of stock options. The values of d_1 , d_2 , $N(d_1)$, $N(d_2)$, Delta, and Gamma for TEAM and MSTR stock with different execution prices in detail can be seen in Table 3.

Table 3. Values of d_1 , d_2 , $N(d_1)$, $N(d_2)$, Delta and Gamma

Stock	K	d_1	d_2	$N(d_1)$	$N(d_2)$	Delta	Gamma
TEAM	\$105	2.6393	2.4016	0.9958	0.9918	0.9958	0.0003
	\$110	2.4436	2.2059	0.9927	0.9863	0.9927	0.0004
	\$115	2.2566	2.0189	0.9880	0.9783	0.9880	0.0007
	\$190	0.1444	-0.0933	0.5574	0.4628	0.5574	0.0087
	\$195	0.0351	-0.2026	0.5140	0.4197	0.5140	0.0088
	\$200	-0.0714	-0.3091	0.4715	0.3786	0.4715	0.0088
MSTR	\$150	2.9460	2.6670	0.9984	0.9962	0.9984	0.0001
	\$160	2.7146	2.4357	0.9967	0.9926	0.9967	0.0001
	\$170	2.4972	2.2183	0.9937	0.9867	0.9937	0.0002
	\$330	0.1191	-0.1598	0.5474	0.4365	0.5474	0.0044
	\$340	0.0121	-0.2669	0.5048	0.3948	0.5048	0.0044
	\$350	-0.0919	-0.3708	0.4634	0.3554	0.4634	0.0044

Table 3 indicates that the values of d_1 , d_2 , $N(d_1)$, and $N(d_2)$ for TEAM and MSTR tend to get smaller when the value of the strike price (K) gets bigger. In addition, the Delta value also tends to get smaller when the strike price (K) gets bigger. This means that the sensitivity of the option price to changes in the stock price is getting smaller, so small changes in the underlying stock price have an increasingly minimal impact on the option price. Then, the Gamma value tends to get bigger when the strike price (K) gets bigger. This condition shows that for every unit change in the underlying stock price, the Delta value will experience a bigger change. After all components are obtained, VaR will be calculated using the DGN VaR method. This research uses a holding period (hp) of 1 day with confidence levels of 80%, 90%, 95%, and 99%. The DGN VaR for TEAM and MSTR stock options is presented in Table 4.

Table 4. DGN VaR on TEAM and MSTR Stock Options

Stock	Strike Price (K)	DGN VaR				
		80%	90%	95%	99%	
TEAM	In The Money	\$105	0.0370	0.0563	0.0723	0.1023
		\$110	0.0369	0.0562	0.0721	0.1019
		\$115	0.0367	0.0559	0.0717	0.1015
	Out Of The Money	\$190	0.0207	0.0315	0.0405	0.0572
		\$195	0.0191	0.0291	0.0373	0.0528
		\$200	0.0175	0.0267	0.0342	0.0484
MSTR	In The Money	\$150	0.0435	0.0663	0.0851	0.1203
		\$160	0.0434	0.0662	0.0849	0.1201
		\$170	0.0433	0.0660	0.0847	0.1197
	Out Of The Money	\$330	0.0239	0.0363	0.0466	0.0660
		\$340	0.0220	0.0335	0.0430	0.0608
		\$350	0.0203	0.0308	0.0395	0.0558

Table 4 shows the call option VaR of TEAM at a strike price of \$105, with the lowest confidence level of 80% being 0.0370, while the VaR for the same strike price with the highest confidence level of 99% is 0.1023. Furthermore, the VaR for call option of MSTR at a strike price of \$150 with the lowest confidence level of 80% is 0.0435, while the VaR for the same strike price with the highest confidence level of 99% is 0.1203. This indicates alignment with the research conducted by Sulistianingsih et al. (2024), which suggests that the higher the level of confidence used to calculate VaR, the larger the VaR estimate obtained.

The highest estimated investment loss for call option of TEAM occurred at a strike price of \$105 and a 99% confidence level of 0.1023 or 10.23% of the total investment value. Thus, if the investor invests \$1,000,000, the maximum risk of loss in the investment is \$102,300. Furthermore, the highest estimated investment loss for call option of MSTR occurred at a strike price of \$150 and a 99% confidence level of 0.1203 or 12.03% of the total investment value. Thus, if the investor invests \$1,000,000, then the maximum risk of loss that the investor has to be encountered is \$120,300.

The estimated VaR for in-the-money option of TEAM for strike price of \$105 with a confidence level of 80% is 0.0370, while the VaR of the out-of-the-money option for strike price of \$200 with the same confidence level is 0.0175. Furthermore, the estimated VaR for in-the-money option of MSTR for a strike price of \$150 with a confidence level of 80% is 0.0435, while the VaR of the out-of-the-money option for a strike price of \$350 with the same confidence level is 0.0203. This shows that the higher the strike price of the call option, the smaller the VaR estimate that will be obtained and the VaR for the analyzed in-the-money option has a greater value than the DGN VaR for the analyzed out-of-the-money option.

4. Kupiec Backtesting

After obtaining the DGN VaR, the validity of the VaR model was then tested using the Kupiec Backtesting test. The results of the Kupiec Backtesting test are presented in Table 5.

Table 5. Kupiec Backtesting of DGN VaR

Stock	Strike Price (K)		80%		90%		95%		99%	
			PTL	PV	PTL	PV	PTL	PV	PTL	PV
TEAM	In The Money	\$105	0.159	0.940	0.080	0.833	0.032	0.884	0.008	0.459
		\$110	0.159	0.940	0.080	0.833	0.032	0.884	0.008	0.459
		\$115	0.159	0.940	0.080	0.833	0.032	0.884	0.008	0.459
	Out Of The Money	\$190	0.255	0.014	0.195	0.000	0.155	0.000	0.076	0.000
		\$195	0.271	0.003	0.203	0.000	0.155	0.000	0.100	0.000
		\$200	0.283	0.001	0.219	0.000	0.183	0.000	0.112	0.000
MSTR	In The Money	\$150	0.143	0.987	0.076	0.883	0.048	0.488	0.008	0.459
		\$160	0.143	0.987	0.076	0.883	0.048	0.488	0.008	0.459
		\$170	0.143	0.987	0.076	0.833	0.048	0.488	0.008	0.459
	Out Of The Money	\$330	0.259	0.009	0.179	0.000	0.135	0.000	0.076	0.000
		\$340	0.271	0.003	0.195	0.000	0.143	0.000	0.100	0.000
		\$350	0.287	0.000	0.203	0.000	0.151	0.000	0.108	0.000

Table 5 indicates the Kupiec backtesting results show that the PTL (percentage of tail losses) value on stock options for both TEAM and MSTR with in-the-money strike prices do not exceed the tail losses limit, and the PV (p-value) is greater than the significant level. As a result, H_0 is rejected and it can be concluded that VaR using the DGN VaR method is valid and suitable for use. However, this method is unable to predict the risk value of stock options with out-of-the-money strike prices.

D. CONCLUSION AND SUGGESTIONS

Based on the result, it can be concluded that the highest estimated investment loss using DGN VaR at a 99% confidence level on the TEAM call option occurs at a strike price of \$105, which is 10.23% of the total investment value. Then, the highest estimated investment loss on the MSTR call option at a 99% confidence level occurs at the strike price of \$150, which is 12.03% of the total investment value. Second, the DGN VaR for the analyzed in-the-money option has a greater value than the DGN VaR for the analyzed out-of-the-money option. For the next research, it can be applied to employ more Greeks, such as vega and rho, in option risk measurement.

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