Robust Optimization of Vaccine Distribution Problem with Demand Uncertainty

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ABSTRACT

This study proposes a multi objective optimization model for vaccine distribution problems using the Maximum Covering Location Problem (MCLP) model. The objective function of the MCLP model in this study is to maximize the fulfillment of vaccine demand for each priority group at each demand point. In practice, the MCLP model requires data on the amount of demand at each demand point, which in reality can be influenced by many factors so that the value is uncertain. This problem makes the optimization model to be uncertain linear problem (ULP). The robust optimization approach converts ULP into a single deterministic problem called Robust Counterpart (RC) by assuming the demand quantity parameter in the constraint function is in the set of uncertainty boxes, so that a robust counterpart to the model is obtained. Numerical simulations are carried out using available data. It is found that the optimal value in the robust counterpart model is not better than the deterministic model but is more resistant to changes in parameter values. This causes the robust counterpart model to be more reliable in overcoming uncertain vaccine distribution problems in real life. This research is limited to solving the problem of vaccine distribution at a certain time and only assumes that the uncertainty of the number of requests is within a specified range so that it can be developed by assuming that the number of demand is dynamic.

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A. INTRODUCTION

Vaccine distribution modeling has been widely used to carry out optimal distribution planning. Vaccine distribution is often faced with several problems which can be grouped based on the characteristics of the problems faced (De Boeck et al., 2020). Outlines the general characteristics and challenges inherent in the distribution chain, both from location selection problems and storage problems. Constraints in selecting vaccine distribution locations are supported by several literature studies by modeling the problem of selecting vaccine distribution locations that can cover maximum demand points using the Maximum Coverage Location Problem (MCLP) model (Alzahrani & Hanbali, 2021; Batanović et al., 2009; Jayalakshmi & Singh, 2017; Lim et al., 2016; Lusiantoro et al., 2022; Máximo et al., 2017) where based on several studies it can be obtained that the MCLP model can provide a solution for determining efficient vaccine distribution locations based on the distance between the point of demand and distribution center. Then the problem of limited distribution capacity was studied by Chen et al. (2022); Hovav & Tsadikovich (2015) which provides an illustration that in

distribution problems, capacity constraints can influence the optimal location of distribution centers.

On the other hand, vaccine distribution based on priority has a big impact when faced with the problem of limited supply in pandemic conditions. Gamchi et al. (2021) modeled the vaccine distribution problem by dividing potential vaccine recipients based on priority groups using multi-objective optimization. Based on the previous explanation, the Maximum Covering Location Problem (MCLP) Model is a problem model that aims to maximize the number of vaccine recipients based on the service coverage of health facilities. To optimize vaccine distribution, a comparison between equitable distribution approaches and priority-based distribution is an important concern. Equitable distribution of vaccines can provide fairer vaccination opportunities for the entire community. The MCLP model is generally a single objective model that models vaccine distribution evenly. In a crisis such as a pandemic, the distribution of vaccines based on priority can reduce the negative impact of the spread of disease. Vaccine distribution based on priority groups can be completed using a multi-objective optimization model with the objective function being that vaccine requests are sorted based on priority groups.

Furthermore, it is known that the MCLP Model uses population data in an area obtained from data provided by the government, in reality, the number of individuals in a population can be random so it can cause the number of vaccine demand in a population to become uncertain. Uncertainty in the number of requests can affect the optimal value of the MCLP model, this is because the optimization model solution has high sensitivity to changes in parameters (Bertsimas et al., 2011). Robust optimization is an optimization model that considers the uncertainty of parameter values. Robust optimization focuses on worst-case optimization, where the worst-case constraints are calculated based on a set of convex parameter uncertainties (Gabrel et al., 2014). The resulting solution must be feasible for every uncertain parameter in the specified uncertainty set.

To provide a better picture, a robust optimization model for uncertainty in the number of requests for vaccine distribution problems has been developed. In Ziaei & Pishvaee (2019) vaccine supply chain network problems with vaccine demand uncertainty and cost issues are modeled. Yang & Rajgopal (2019) have modeled the vehicle routing problem with uncertainty in vaccine demand and vehicle travel time. In Wang et al. (2023) have modeled the problem of vaccine distribution with uncertainty in the amount of supply and demand. Robust optimization can provide better worst-case performance compared to non-robust optimization (Gülpinar et al., 2013).

Based on several literature studies above, this research aims to (1) develop a vaccine distribution problem model using the MCLP model based on priority groups with multi-objective problems; (2) then compare the classic MCLP model which distributes vaccines evenly using objective optimization -single with vaccine distribution based on priority; (3) developing uncertainty assumptions about the number of vaccine demand parameters in the MCLP model and solving this problem with a robust optimization approach; and (4) comparing models with a definite number of requests which are then called deterministic models and models with the number of requests uncertain which is then called the robust counterpart model. It is hoped that this research can provide an overview of how the priority-based vaccine

distribution model works and how to overcome uncertainty in the number of requests in the model.

B. METHODS
1. Maximum Covering Location Problem

The classic Maximum Covering Location Problem model aims to determine the location of potential distribution centers that can cover demand points maximally. Refers to (Church & Revelle, 1974), MCLP has the following form:

Objective function:

$$max \sum_{i \in J} a_i y_i$$

constraint function:

$$\sum_{j \in N_i} x_j \geq y_i, \forall i \in I. \tag{1}$$

$$\sum_{j \in J} x_j = P.$$

$$x_j, y_i \in \{0,1\}$$

Where is

I : denotes the set of demand nodes

J : denotes the set of facility sites

N_i : $\{j \in J | d_{ij} \leq S\}$

a_i : population to be served at the demand node i

d_{ij} : the shortest distance from node i to node j;

S : the distance beyond which a demand point is considered "uncovered"

P : the number of facilities to be located

x_j : $x_j = 1$ if a facility is allocated to the site j, $x_j = 0$ otherwise

y_i : $y_i = 1$ if demand point i covered, $y_i = 0$ otherwise

In this research, the author attempts to develop the basic concept of binary demand fulfillment decisions (y_i) to be more specific, adding several other assumptions from several related papers such as capacity constraints, and adding the assumption that a_i is uncertain and can be solved by robust optimization.

2. Robust Optimization

The robust optimization approach in this study refers to (Ben-Tal et al., 2009; Ben-Tal & Nemirovski, 2002; Bertsimas et al., 2011; Den Hertog, 2013; Gorissen et al., 2015; Sozuer & Thiele, 2016). For example, suppose there is an uncertain linear problem with the following form:

Objective function:

$$\min c^T x.$$

constraint function: (2)

$$Ax \leq b.$$
$$(A, b, c) \in \mathcal{U}.$$

where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^m \times \mathbb{R}^n$, dan $b \in \mathbb{R}^n$, is an uncertain decision variable and $\mathcal{U}$ is the notation of the uncertainty set. The robust optimization approach transforms uncertain linear problems (ULP) into a single deterministic problem called Robust Counterpart (RC). Problem (RC) can be equivalently written as a problem with a linear objective function and uncertainty in the constraint function only. Assume that $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ are certain, the robust counterpart formulation of equation (2) is defined in equation (3)

Objective function:

$$\min c^T x.$$

constraint function (3)

$$a_i^T(\zeta)x \leq b_i, i = 0, 1, \ldots, m.$$
$$\zeta \in \mathcal{Z}.$$

where $\mathcal{Z} \in \mathbb{R}^L$ is the set of uncertainties and ζ is a variable that controls the *range* of uncertainties. The solution $x \in \mathbb{R}^n$ is called robust feasible if it satisfies all uncertainty constraints for all $\zeta \in \mathcal{Z}$. Suppose a and b is a generalized representation of a_i and b_i, then the constraints in equation (3) can be written into equation (4) as follows:

$$a^T(\zeta)x \leq b, \forall \zeta \in \mathcal{Z}. \tag{4}$$

Define the uncertain parameters as in equation (4).

$$a(\zeta) = \bar{a} + P\zeta. \tag{5}$$

where $\bar{a} \in \mathbb{R}^n$ is the nominal value vector and $P \in \mathbb{R}^{n \times L}$ is a matrix of confounders that cause uncertainty. The set $\mathcal{U}$ is defined as in equation (6).

$$\mathcal{U} = \{a | a = \bar{a} + P\zeta, \zeta \in \mathcal{Z}\}. \tag{6}$$

Substitute the uncertain parameters in equation (5) to the uncertain constraints in equation (4), and we get the result as in equation (7).

$$(\bar{a} + P\zeta)^T x \leq b, \forall \zeta \in \mathcal{Z}. \tag{7}$$

The simplest uncertainty set is *Box Uncertainty*. The *box* uncertainty set can be expressed in equation (8).

$$\mathcal{Z} = \{\zeta : \|\zeta\|_\infty \le \rho\}. \tag{8}$$

with ρ is an adjustable parameter to measure uncertainty. The robust counterpart formulation assuming the uncertainty is in the uncertainty box can be constructed by substituting equation (8) into inequality (7) to obtain inequality (9).

$$(\bar{a} + P\zeta)^T x \le b, \forall \, \zeta : \|\zeta\|_\infty \le \rho. \tag{9}$$

The solution obtained from the robust optimization problem is "*the best worst-case*" solution, that is, the solution obtained is the best solution for all the worst conditions that may occur. In the vaccine distribution problem, the worst condition that occurs is that there is an unfulfilled demand, so the worst-case formulation is made in constraint (9) as follows:

$$\max_{\zeta : \|\zeta\|_\infty \le \rho} (\bar{a} + P\zeta)^T x \le b. \tag{10}$$

Based on the norm definition, the robust counterpart formulation is obtained as follows:

Objective function:
$$\min c^T x.$$
constraint function $\tag{11}$
$$\bar{a_i}^T x + \rho \|P^T x\|_1 \le b$$
$$\zeta \in \mathcal{Z}.$$

The solution for solving the MCLP model with an uncertain amount of vaccine demand is carried out by assuming that the demand quantity parameter in the constraint function is in the set of uncertainty boxes, so that a robust counterpart to the MCLP model is obtained.

3. Research Data

To implement the model created, numerical simulations were carried out using available data. The MCLP model aims to determine the location of vaccine distribution centers that can cover demand points optimally. Therefore this research requires these data: (a) potential vaccine distribution center candidates; (b) service capacity of potential vaccine distribution center candidates; (c) potential vaccine demand points; (d) number of vaccine requests at each request point; and (e) the distance between potential vaccine distribution centers and vaccine demand points. Based on availability, the data used as a parameter for the number of vaccine requests is population data based on age and sub-district, sub-district data, and DKI Jakarta Health Center data as follows:

Table 1. Total Population of DKI Jakarta by Age and Village 2021

No.	Village	Age (Years)							
		00-09	10-19	20-29	30-39	40-49	50-59	60-69	70+
1	Cengkareng Barat	13.033	13.821	12.630	14.061	13.731	9.042	4.609	2.309
2	Cengkareng Timur	17.492	17.137	15.713	18.542	17.365	10.608	5.068	1.835
3	Duri Kosambi	17.077	16.596	15.989	17.581	15.357	10.263	5.263	1.892
4	Kapuk	30.016	29.825	27.013	31.093	28.030	17.260	8.023	2.275
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
267	Panggang Island	1.414	1.201	1.320	1.171	987	651	308	124

Table 2. Village

No.	Village Name
1	Cengkareng Barat
2	Cengkareng Timur
3	Duri Kosambi
4	Kapuk
⋮	⋮
267	Panggang Island

Table 3. Health Center

No.	Name of Health Center	Service Capacity
1	Kecamatan Cengkareng	2.2230
2	Cengkareng Brt. I	8.474
3	Cengkareng Brt. II	21.691
4	Kelurahan Cengkareng Timur	17.085
⋮	⋮	⋮
315	Kelurahan Pulau Panggang	16.203

After determining the distribution centers, demand points, and priority groups used, this research calculates the distance between each distribution center and each demand point using geographical location and distance formula using Google API *syntax* for Spreadsheet as follows:

a. Latitude of the health center:

```
function getlatloc(address) {
var location = Maps.newGeocoder().geocode(address);
var lat = location["results"][0]["geometry"]["location"]["lat"]; return lat;}
```

b. Longitude of the health center:

```
function getlngloc(address) {
var location = Maps.newGeocoder().geocode(address);
var lng = location["results"][0]["geometry"]["location"]["lng"]; return lng;}
```

c. The latitude of the neighborhood:

```
function getlat(address) {
var location = Maps.newGeocoder().geocode(address);
var lat =(location["results"][0]["geometry"]["bounds"]["northeast"]["lat"]
+lokasi["results"][0][geometry"] ["bounds"]["southwest"]["lat"])/2; return lat;}
```

d.   Longitude of the neighborhood:

```
function getlng(address) {
var location = Maps.newGeocoder().geocode(address);
var lng = (location["results"][0]["geometry"]["bounds"]["northeast"]["lng"]
+lokasi["results"][0]["geometry"] ["bounds"]["southwest"]["lng"])/2; return lng;}
```

After getting the geographical location, the formula for the distance between squared points is $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, where: d is distance; x_1 is the latitude of the 1st point; x_2 is the latitude of the 2nd point; y_1 is the longitude of the 1st point; y_2 is the longitude of the 2nd point. Conversion of distance in the corner range of Jakarta: $1°$ Latitude = 110,570 meters; $1°$ Longitude = 110,900 meters.

C.   RESULT AND DISCUSSION
1.   Vaccine Distribution Problem Model Formulation

This study formulates a vaccine distribution problem model using the maximum covering location problem model that refers to the research of (Church & Revelle, 1974; Lim et al., 2016; Lusiantoro et al., 2022). The MCLP model formulation of the vaccine distribution problem begins by defining the problem index and the decision variables used. This study assumes that vaccines are distributed through health facilities to demand points which are divided into several groups based on priority, therefore defining i as the demand point index, j as the priority group index, and k as the candidate health facility index with $i \in \mathcal{I}, j \in \mathcal{J}$, and $k \in \mathcal{K}$. Then this study uses the assumption of relaxation of decision variables that refer to research (Lim et al., 2016) which allows partial fulfillment of demand by more than one health facility, therefore defined the variable $x_k \in \{0,1\}$ as a health facility selection decision variable k and $y_{ijk} \in [0,1]$ as the decision variable for the proportion of vaccine demand fulfillment.

This research develops the basic assumptions of the proportion of services in the research of Lim et al. (2016) and Lusiantoro et al. (2022), which assumes that the sum of the number of requests is not equal to the number of vaccine fulfillments. Suppose a_{ij} is the number of priority group vaccine requests j at the demand point i, the number of vaccine requests obtained by each priority group at each demand point is defined as the number of vaccine requests multiplied by the proportion of vaccine fulfillment or can be written as $a_{ij}y_{ijk}$. Meanwhile, the amount of vaccine demand fulfillment by health facilities is defined as the service capacity of health facilities multiplied by the proportion of services provided. Suppose w_{ijk} is the proportion of healthy facility services and C_k is the service capacity of the health facility k then the amount of vaccine demand fulfillment can be written as $w_{ijk}C_kx_k$. The constraints ensure that the number of vaccine requests obtained by each priority group at each demand point is no more than the sum of the fulfillment of vaccine requests by health facilities. Therefore, the constraint can be written in the following equation:

$$a_{ij}y_{ijk} \le w_{ijk}C_kx_k, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}.$$

Furthermore, this study generalizes the assumptions of the division of service categories in research (Lim et al., 2016) by dividing health facility services into two categories, namely the

optimal service category and partial service. Suppose d_{ijk} is the distance parameter between health facilities k with the priority group j at the demand point i, d_o is the optimal service distance, and d_f is the maximum service distance, then the value of w_{ijk} is determined by conditions (i), (ii), and (iii) and is illustrated in Figure 1.

   i. If $d_{ijk} \leq d_o$ then $w_{ijk} = 1$.

   ii. If $d_o < d_{ijk} \leq d_f$ then $w_{ijk} = \frac{d_f - d_{ijk}}{d_f - d_o}$.
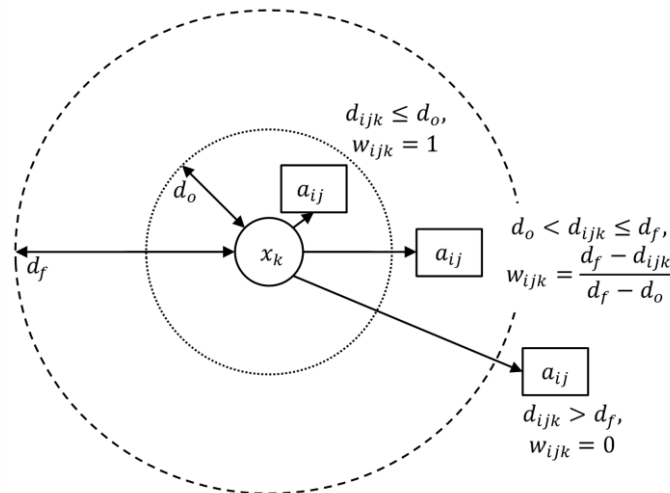
   iii. If $d_{ijk} > d_f$ then $w_{ijk} = 0$.

Figure 1. Illustration of health facility service levels

Figure 1 shows that if the demand point i, j has a distance from the distribution center k less than optimal services distance (d_o), then the demand point gets 100 percent service from the distribution center, or can be written as $a_{ij} y_{ijk} \leq C_k x_k$, and if it located between optimal services distance and maximum service distance, let's say it's gets 60 percent service from the distribution center can be written as $a_{ij} y_{ijk} \leq 0,6 \times C_k x_k$. The basic assumptions in the MCLP problem model used include: (a) the proportion of vaccine demand fulfillment is no more than 100% (Lusiantoro et al., 2022); (b) facilities used are limited (Church & Revelle, 1974); (c) the number of vaccines distributed is limited (Lusiantoro et al., 2022); and (d) the capacity of each health facility is limited (Lim et al., 2016). The MCLP model in this study is formulated as follows:

    The set

$\mathcal{I}$	The set of demand points (index:i)
$\mathcal{J}$	The set of priority groups (index:j)
$\mathcal{K}$	The set of candidate health facilities (index:k)

    Parameters

a_{ij}	Prioritized vaccine demand j at the point of demand i vulnerable
d_{ijk}	Distance from point of demand i to the facility k
d_o	Optimal service distance covered by a facility
d_f	The farthest service distance covered by a facility
P	Number of health facilities to be used
C_k	Health facility vaccine storage capacity k

TC Total vaccines distributed

Variables

w_{ijk} Health facility service level k to the point of demand i

x_k $x_k = 1$ if candidate facilities k is used, $x_k = 0$ otherwise

y_{ijk} Vaccine demand fulfillment rate at the point i with priority j of health facilities k

The objective function of the MCLP model in this study is to maximize the fulfillment of vaccine demand for each priority group at each demand point. On the other hand, to perform vaccine distribution based on priority, the objective function is modified by sorting the objective function into a multi-objective problem as follows:

Objective function:

$$\max \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} a_{ij} y_{ijk}, \quad j = 1,2,\dots,m. \tag{12}$$

constraint function:

$$\sum_{k \in \mathcal{K}} y_{ijk} \leq 1, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{13}$$

$$\sum_{k \in \mathcal{K}} x_k \leq P. \tag{14}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} a_{ij} y_{ijk} \leq TC. \tag{15}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} a_{ij} y_{ijk} \leq C_k x_k, \forall k \in \mathcal{K}. \tag{16}$$

$$a_{ij} y_{ijk} \leq w_{ijk} C_k x_k, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{17}$$

$$x_k \in \{0,1\}, \forall k \in \mathcal{K}. \tag{18}$$

$$y_{ijk} \in [0,1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{19}$$

where the objective function (12) is to maximize the fulfillment of vaccine demand based on priority, this objective function can be solved using the lexicographic method. Constraint (13) ensures that the proportion of vaccine demand fulfillment is not more than 100%. Constraint (14) limits the number of facilities to be used. Constraint (15) limits the amount of vaccine distributed. Constraint (16) limits the fulfillment of vaccine demand based on the service capacity of health facilities. Constraint (17) limits the amount of vaccine demand fulfillment based on the coverage of health facility services. Constraints (18) and (19) are the limits of the decision variables.

2. Robust Optimization of Vaccine Distribution Problem

This study assumes that all parameters are known and have an exact value except for the number of vaccine requests or the population of vulnerable individuals. The uncertainty of the population of vulnerable individuals is overcome by robust optimization by assuming that the uncertainty is in a set of uncertainties. Based on the model created, uncertainty exists in the

objective function (12), constraint functions (15), (16), and (17) so that the model has additional constraints, namely $a_{ij} \in \mathcal{U}$. Uncertainty in this problem is assumed to only exist in the constraint function. Vaccine demand uncertainty a_{ij} is assumed to be in the uncertainty *box*. The affine form of the vaccine demand parameter can be expressed as follows:

$$a_{ij} = \bar{a}_{ij} + P_{ij}\zeta, \forall \, \zeta : \|\zeta\|_\infty \leq \rho. \tag{20}$$

where

    a. a_{ij} *affine* of the parameter of the number of vaccine requests for each priority group j at the demand point i with $a_{ji} \in \mathbb{R}$.

    b. $\bar{a}_{ij}$ is the number of vaccine requests for each priority group j at the demand point i which is obtained from the nominal value with $\bar{a}_{ij} \in \mathbb{R}$.

    c. P_{ij} is the confounding constant for the uncertainty in the constraint function with $P_{ij} \in \mathbb{R}$.

    d. ζ is the uncertainty control variable for the constraint function, and ρ is an *adjustable* parameter that determines the uncertainty value.

Robust optimization assumes that the uncertainty is *constraint-wise*, so the development of the *robust counterpart* model can be focused on constraints that have vulnerable population uncertainty. Note that constraints (15), (16), and (17) have vulnerable population uncertainty, i.e. $(\bar{a}_{ij} + P_{ij}\zeta)y_{ijk}$, suppose used $\rho = 1$ then $\forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, t = t_d$ can be *treated* as follows:

$$(\bar{a}_{ij} + P_{ij}\zeta)y_{ijk}, \forall \, \zeta : \|\zeta\|_\infty \leq 1 \equiv \bar{a}_{ij}y_{ijk} + P_{ij}\zeta y_{ijk}, \forall \, \zeta : \|\zeta\|_\infty \leq 1.$$
$$\equiv \max_{\zeta : \|\zeta\|_\infty \leq 1} \bar{a}_{ij}y_{ijk} + P_{ij}\zeta y_{ijk}.$$
$$\equiv \bar{a}_{ij}y_{ijk} + \max_{\zeta : \|\zeta\|_\infty \leq 1} P_{ij}\zeta y_{ijk}.$$
$$\equiv \bar{a}_{ij}y_{ijk} + \|P_{ij}y_{ijk}\|_1.$$
$$\equiv \bar{a}_{ij}y_{ijk} + |P_{ij}y_{ijk}|.$$

Because $y_{ijk} \in [0,1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$, then it is obtained:

$$\bar{a}_{ij}y_{ijk} + |P_{ij}y_{ijk}| \equiv \bar{a}_{ij}y_{ijk} + |P_{ij}|y_{ijk}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{21}$$

Based on the above analysis, the vaccine distribution problem model with uncertainty in the number of requests is equivalent to the robust counterpart model as follows:

Objective function:

$$\max \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \bar{a}_{ij}y_{ijk}, \quad j = 1,2,\dots,m. \tag{22}$$

constraint function:

$$\sum_{k \in \mathcal{K}} y_{ijk} \leq 1, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{23}$$

$$\sum_{k \in \mathcal{K}} x_k \leq P. \tag{24}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \bar{a}_{ij} y_{ijk} + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} |P_{ij}| y_{ijk} \leq TC. \tag{25}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \bar{a}_{ij} y_{ijk} + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} |P_{ij}| y_{ijk} \leq C_k x_k, \forall k \in \mathcal{K}. \tag{26}$$

$$\bar{a}_{ij} y_{ijk} + |P_{ij}| y_{ijk} \leq w_{ik} C_k x_k, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{27}$$

$$x_k \in \{0,1\}, \forall k \in \mathcal{K}. \tag{28}$$

$$y_{ijk} \in [0,1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{29}$$

3. Numerical Simulation

The case study used in this research is the distribution of the COVID-19 vaccine in DKI Jakarta. Suppose there are 7.5 million doses of vaccine to be distributed where vaccine distribution is carried out through health centers to urban villages and the population in each urban village is divided by age. Based on the data owned, the number of vaccine requests (a_{ij}) shown in Table 1 and the capacity of each health center (C_k) shown in Table 3, then it can be determined that $|\mathcal{I}| = 267, |\mathcal{J}| = 8$, and $|\mathcal{K}| = 315$. The vaccine recipient groups are sorted into elderly (over 60 years old), children (less than 10 years old), adolescents (10 to 30 years old), and adults (30 to 60 years old). The complete order of priority groups in this simulation is as follows:

a. Population aged 70 years and over
b. Population aged 60 years to 69 years
c. Population aged 0 years to 9 years
d. Population aged 10 years to 19 years
e. Population aged 20 years to 29 years
f. Population aged 50 years to 59 years
g. Population aged 40 years to 49 years
h. Population aged 30 years to 39 years

Furthermore, it is assumed that each health center can optimally serve the surrounding demand points with a distance of 1 km and it is possible to serve demand points up to 5 km. based on these assumptions, the values of $d_o = 1$ and the value of $d_f = 5$. Since each health center is assumed to have an optimal service distance of 1 km, the service area of each health center is ± 3 km², on the other hand, DKI Jakarta has an area of ± 660 km², so it can be assumed that the number of health centers that will be used to carry out vaccine distribution is 220 units.

Numerical simulation is first performed by assuming that the number of requests is an exact value as in Table 3 with the model in equations (12)-(19). To shorten the simulation results, this study accumulates the number of fulfilled requests based on priority. For comparison, this study also conducted simulations for the equal distribution of vaccines where the objective function is solved directly without sorting by priority. The objective function for this case study is shown in the following equation:

$$\max \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \bar{a}_{ij} y_{ijk}. \tag{30}$$

The number of vaccine requests and simulation results are shown in Table 4.

Table 4. Number of Vaccine Requests and Fulfillment by Priority

No.	Demand Amount	Vaccine Fulfillment with Equitable Distribution		Vaccine Fulfillment with Priority Based Distribution	
1	302.867	95.986	(32%)	302.756	(100%)
2	668.899	330.383	(49%)	668.689	(100%)
3	1.788.144	1.375.542	(77%)	1.787.135	(100%)
4	1.825.288	1.328.904	(73%)	1.824.380	(100%)
5	1.718.630	1.136.297	(66%)	1.427.036	(83%)
6	1.262.456	855.743	(68%)	166.135	(13%)
7	1.825.432	1.180.760	(65%)	133.031	(7%)
8	1.869.879	1.196.377	(64%)	49.397	(3%)
Total	11.261.595	7.499.992	(67%)	6.358.559	(56%)

Table 4 shows that equal distribution of vaccines can provide greater fulfillment of vaccine demand at 67 percent compared to distribution based on priorities at 56 percent, but distribution of vaccines based on priorities can provide fulfillment of vaccine demand for early priorities first, as in priorities 1 to 4 which are fulfilled by 100 percent first. Furthermore, simulations were carried out assuming that the amount of vaccine demand is uncertain with uncertainty in the range of -30 to 30 from the nominal value in Table 3. The amount of vaccine demand and simulation results are shown in Table 5.

Table 5. Number of Vaccine Requests and Fulfillment by Priority with Uncertainty Number of Vaccine Requests

No.	Demand Amount	Vaccine Fulfillment with Equitable Distribution		Vaccine Fulfillment with Priority Based Distribution	
1	302.867	0	(0%)	302.756	(100%)
2	668.899	35.735	(5%)	668.689	(100%)
3	1.788.144	1.357.755	(76%)	1.787.135	(100%)
4	1.825.288	1.404.380	(77%)	1.824.326	(100%)
5	1.718.630	1.268.102	(74%)	1.284.568	(75%)
6	1.262.456	517.270	(41%)	333.578	(26%)
7	1.825.432	1.418.682	(78%)	89.484	(5%)
8	1.869.879	1.450.952	(78%)	28.320	(2%)
Total	11.261.595	7.452.876	(66%)	6.318.856	(56%)

Table 5 shows similar results to Table 4 where the distribution of vaccines evenly can provide greater demand fulfillment than the distribution of vaccines based on priorities, but with the change or uncertainty in the amount of demand causes differences in the amount of demand in each priority group and it can be obtained that the fulfillment of vaccine demand in the robust counterpart model is smaller than the deterministic model. The robust counterpart model is claimed to provide an optimal value that is more resistant to changes in parameter values, therefore, in this study a sensitivity analysis will be carried out by changing vaccine demand parameters in the range of $[-30,30]$. The simulation results are shown in Table 6.

Table 6. Sensitivity Analysis Simulation

Range of Change	Deterministic Model	Robust Counterpart Model
$[-30,0]$	Feasible	Feasible
$[0,30]$	Infeasible	Feasible
$[-30,30]$	Infeasible	Feasible

Based on the simulation results, it is found that changes in parameter values greater than the nominal value cause the constraints in the deterministic model to become infeasible so that the model does not have an optimal value and in the robust counterpart model, this problem does not occur because the robust counterpart model considers the worst case that can occur in changing the value of the vaccine demand parameter.

D. CONCLUSION

Based on the analysis carried out in section C.1, it was found that the priority-based distribution problem can be performed by modeling the MCLP model's single-objective objective function into a multi-objective one. Thus, from the numeric simulation shown in Table 4, the equal vaccine distribution problem provides a better optimal value than vaccine distribution based on priority, while vaccine distribution based on priority ensures the fulfillment of demand at a higher priority. This problem is influenced by the number of facilities used and the distance of service owned by health facilities, so further analysis is needed.

Lastly, from section C.2, found that the robust optimization approach to vaccine demand uncertainty can be modeled using robust optimization by assuming the demand uncertainty is in the uncertainty box set and from simulation shows in table 5 and 6, it found that the optimal value in the robust counterpart model is not better than the deterministic model but is more resistant to changes in parameter values. This illustrates that if the amount of vaccine demand that occurs in society is uncertain, the robust counterpart model will still be feasible and still can be used compared to the deterministic model.

This research is limited to solving the problem of vaccine distribution at a certain time and only assumes that the uncertainty of the number of requests is within a specified range so that it can be developed by assuming that the number of requests is dynamic, for example, to know the number of requests on any day if the infection rate is known. This problem can be solved using dynamic system models such as the SIR model or using statistical models such as linear regression models. In addition, to solve dynamic uncertain optimization problems, you can use an adjustable robust counterpart optimization model.

REFERENCES

Alzahrani, A. S., & Hanbali, A. Al. (2021). Maximum Coverage Location Model for Fire Stations with Top Corporate Risk Locations. *International Journal of Industrial Engineering and Operations Management*, *03*(02), 58–74. https://doi.org/10.46254/j.ieom.20210201

Batanović, V., Petrović, D., & Petrović, R. (2009). Fuzzy logic based algorithms for maximum covering location problems. *Information Sciences*, *179*(1–2), 120–129. https://doi.org/10.1016/j.ins.2008.08.019

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust Optimization*. Princeton University Press .

Ben-Tal, A., & Nemirovski, A. (2002). Robust optimization - Methodology and applications. *Mathematical Programming, Series B*, *92*(3), 453–480. https://doi.org/10.1007/s101070100286

Bertsimas, D., Brown, D. B., & Caramanis, C. (2011). Theory and applications of robust optimization. In *SIAM Review* (Vol. 53, Issue 3, pp. 464–501). https://doi.org/10.1137/080734510

Chen, Y., Tao, R., & Downs, J. (2022). Location Optimization of COVID-19 Vaccination Sites: Case in Hillsborough County, Florida. *International Journal of Environmental Research and Public Health*, *19*(19). Article e12443 https://doi.org/10.3390/ijerph191912443

Church, R., & Revelle, C. (1974). *The Maximal Covering Location Problem [Tesis]*. The Johns Hopkins University. https://doi.org/10.1007/BF01942293

De Boeck, K., Decouttere, C., & Vandaele, N. (2020). Vaccine distribution chains in low- and middle-income countries: A literature review. *Omega (United Kingdom)*, *97(8).* Article e102097 https://doi.org/10.1016/j.omega.2019.08.004

Den Hertog, D. (2013). *Practical Robust Optimization-an introduction*. https://www.lnmb.nl/conferences/2013/programlnmbconference/denhertog.pdf

Gabrel, V., Murat, C., & Thiele, A. (2014). Recent advances in robust optimization: An overview. *European Journal of Operational Research*, *235*(3), 471–483. https://doi.org/10.1016/j.ejor.2013.09.036

Gorissen, B. L., Yanikoğlu, I., & den Hertog, D. (2015). A practical guide to robust optimization. *Omega (United Kingdom)*, *53*, 124–137. https://doi.org/10.1016/j.omega.2014.12.006

Gülpinar, N., Pachamanova, D., & Çanakoglu, E. (2013). Robust strategies for facility location under uncertainty. *European Journal of Operational Research*, *225*(1), 21–35. https://doi.org/10.1016/j.ejor.2012.08.004

Hovav, S., & Tsadikovich, D. (2015). A network flow model for inventory management and distribution of influenza vaccines through a healthcare supply chain. *Operations Research for Health Care*, *5*(2), 49–62. https://doi.org/10.1016/j.orhc.2015.05.003

Jayalakshmi, B., & Singh, A. (2017). A hybrid artificial bee colony algorithm for the cooperative maximum covering location problem. *International Journal of Machine Learning and Cybernetics*, *8*(2), 691–697. https://doi.org/10.1007/s13042-015-0466-y

Lim, J., Claypool, E., Norman, B. A., & Rajgopal, J. (2016). Coverage models to determine outreach vaccination center locations in low and middle income countries. *Operations Research for Health Care*, *9(2)*, 40–48. https://doi.org/10.1016/j.orhc.2016.02.003

Lusiantoro, L., Tri, S., Mara, W., & Rifai, A. P. (2022). A Locational Analysis Model of the COVID-19 Vaccine Distribution. *Operations and Supply Chain Management*, *15*(2), 240–250. http://doi.org/10.31387/oscm0490344

Máximo, V. R., Nascimento, M. C. V., & Carvalho, A. C. P. L. F. (2017). Intelligent-guided adaptive search for the maximum covering location problem. *Computers and Operations Research*, *78(2)*, 129–137. https://doi.org/10.1016/j.cor.2016.08.018

Shamsi Gamchi, N., Torabi, S. A., & Jolai, F. (2021). A novel vehicle routing problem for vaccine distribution using SIR epidemic model. *OR Spectrum*, *43*(1), 155–188. https://doi.org/10.1007/s00291-020-00609-6

Sozuer, S., & Thiele, A. C. (2016). *Robustness Analysis in Decision Aiding, Optimization, and Analytics* (M. Doumpos, C. Zopounidis, & E. Grigoroudis, Eds.; Vol. 241). Springer International Publishing. https://doi.org/10.1007/978-3-319-33121-8

Veenstra, M., Roodbergen, K. J., Coelho, L. C., & Zhu, S. X. (2018). A simultaneous facility location and vehicle routing problem arising in health care logistics in the Netherlands. *European Journal of Operational Research*, *268*(2), 703–715. https://doi.org/10.1016/j.ejor.2018.01.043

Wang, X., Jiang, R., & Qi, M. (2023). A robust optimization problem for drone-based equitable pandemic vaccine distribution with uncertain supply. *Omega (United Kingdom)*, *119*. https://doi.org/10.1016/j.omega.2023.102872

Yang, Y., & Rajgopal, J. (2019). Outreach Strategies for Vaccine Distribution: A Multi-Period Stochastic Modeling Approach. *Operations Research Forum*, *2*(2). https://doi.org/10.1007/s43069-021-00064-1

Ziaei, Z., & Pishvaee, M. S. (2019). The design of the vaccine supply network under uncertain condition: A robust mathematical programming approach. *Journal of Modelling in Management*, *14*(4), 841–871. https://doi.org/10.1108/JM2-07-2018-0093