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Numerical Simulation of Fluid Flow in the Narrow Strait with Density Differences

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ABSTRACT

This study investigates the dynamics of fluid flow through a narrow strait connecting two large water bodies with different densities using numerical simulations. The research focuses on understanding how density-driven currents develop and interact in a confined channel, particularly the role of lateral density contrasts and the influence of gravitational and geostrophic forces. A semi-implicit numerical method is employed to efficiently model the complex flow dynamics while ensuring stability. The simulation results are analyzed using visualizations of the flow fields, which highlight the evolution of density-driven currents, vortex formation, and geostrophic adjustments over time. The findings reveal that denser water from the western basin flows toward the eastern basin, lowering the sea surface in the west and raising it in the east. Over time, the Coriolis force causes the bottom flow to deflect southward and the returning surface flow to shift northward, leading to geostrophic equilibrium. Transient vortices emerge within the strait, while stationary vortices form in the outflow regions, underscoring the interplay between gravitational forces, density contrasts, and rotational effects. These findings offer important insights into the hydrodynamic behavior of narrow straits, which are common in nature. The results can help improve the understanding of flow patterns in similar environments, such as fjords, estuaries, and channels, and may contribute to studies on sediment transport, nutrient mixing, and renewable energy potential in density-driven systems.

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A. INTRODUCTION

Previous research has explored various hydrodynamic scenarios, but comprehensive studies on fluid flow through narrow straits with density differences remain limited, particularly regarding the effects of lateral density gradients and Coriolis forces. Straits play a crucial role in connecting water bodies, facilitating the exchange of water, heat, biota, and sediments, which significantly impacts regional and global climate dynamics (Rossi et al., 2023a). The unique bathymetry of straits, combined with local air-sea interactions and remote forcing, influences their dynamics and necessitates an integrated approach using in situ observations, remote sensing, and numerical models to understand their biogeochemical impacts (Haid et al., 2020; Liu et al., 2021). Exchange flows in these straits, driven by gravitational forces due to density differences, often lead to bi-directional flows where denser water moves beneath lighter water layers. These flows are typically controlled at the narrowest cross-sectional point, influenced by both horizontal and vertical constrictions (Dharmawan et al., 2024; Pudjaprasetya & Swastika, 2023).

Topographic features within straits further complicate the dynamics, influencing internal mixing, secondary circulations, and pressure gradients. For example, the interaction of tides and topography in the Seto Inland Sea enhances mixing and induces time-mean eddies, which reduce throughflow (Kurogi & Hasumi, 2019). Additionally, Coriolis forces introduce geostrophic adjustments, which suppress turbulent mixing and form Ekman layers, leading to secondary cross-channel circulations. The depositional characteristics of straits also play a significant role, as these environments act as individual depositional systems with predictable sediment transport pathways and bedform changes (Rossi et al., 2023b). Experimental studies on turbulent hydrodynamics in narrow channels have revealed significant variations in flow velocity and Reynolds stresses across vertical and horizontal profiles, indicating complex flow behavior (Pal et al., 2017). Furthermore, the interaction of hydrodynamic forces between floating structures in narrow waterways can create traffic flow lumps, reducing flux and requiring specialized flow management strategies (Sun et al., 2015). Despite progress in understanding strait dynamics, further research is needed to examine the intricate interplay of lateral density gradients, Coriolis effects, and bi-directional exchange flows, particularly their implications for broader hydrodynamic and climatic systems.

Recent advances in numerical methods, particularly semi-implicit methods (SIM), have significantly enhanced the accuracy of fluid flow modelling under complex conditions, yet challenges remain in their effective application to narrow straits with substantial density differences. SIM combine implicit and explicit time discretization, making them unconditionally stable and highly accurate for solving advection equations, even with variable velocities and numerical parameters (Frolkovič et al., 2022). For example, SIM-based finite volume schemes have proven effective for geophysical flows involving free surface shallow water flow and floating rigid structures (Brutto & Dumbser, 2023). Similarly, the use of mixed precision arithmetic in SI time-stepping schemes for ocean and atmosphere models has reduced computational costs while preserving solution quality, which is essential for large-scale applications(Ackmann et al., 2022).

Despite these advancements, applying SIM methods to narrow straits with substantial density differences poses unique challenges. Narrow channels, such as the Kerch Strait, often feature significant depth variations and complex bottom topography that can affect model stability and accuracy (Chistyakov et al., 2020). Advanced methods, like the velocity-vorticity formulation of Navier-Stokes equations combined with the Boundary-Domain Integral Method (BDIM), have shown promise for handling boundary conditions but require careful implementation to manage computational complexity (Tibaut et al., 2023). Additionally, meshless algorithms using radial basis functions (RBF) demonstrate excellent accuracy and stability for fluid flows in complex domains, even under high Courant numbers(Shahane & Vanka, 2022).

These methods are particularly relevant for capturing intricate hydrodynamic phenomena, including free-surface and mixed flow regimes in hydraulic systems (Hu et al., 2019). Numerical treatments of Reynolds-stress gradients are equally important, as inaccuracies can lead to unrealistic results and solution instability (So, 2022). Applications such as artificial upwelling (AU) processes in oceanic flows further illustrate the need for robust numerical models capable of resolving temperature, salinity, and nutrient transport interactions (Kemper et al., 2022).

Finally, integrating numerical simulations with experimental validation, as seen in semi-circular breakwater designs, underscores the value of accurate models for solving complex fluid dynamics problems (Gomaa et al., 2023).

Numerous advances in fluid flow modelling have significantly improved our understanding of density-driven currents and related phenomena. However, studies focusing specifically on narrow channels with lateral density differences remain limited, particularly regarding the interplay between gravitational and geostrophic forces. This gap is critical because narrow straits often feature complex flow dynamics influenced by density gradients, sea surface adjustments, and vortex formations. While previous research has explored general fluid flow behavior, the lack of targeted analysis on how these forces interact in confined channels hinders our ability to fully understand their effects.

The aim of this study is to address this research gap by analyzing the behavior of density-driven currents in narrow channels and investigating the interaction between gravitational and geostrophic forces. Using a numerical model, this work visualizes the formation of transient vortices and stationary barotropic vortices, providing new insights into their role in shaping flow dynamics and sea surface levels. The findings offer a deeper understanding of flow structures in narrow straits and highlight practical applications, such as optimizing locations for renewable energy exploration.

B. METHODS

In this research, the model used is the Navier-Stokes equation with details as follows. Momentum equation (Ikhwan et al., 2021):

$$\begin{aligned} &\frac{\partial u}{\partial t}+\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-fv=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+\left(\frac{\partial}{\partial x}\left(A\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A\frac{\partial u}{\partial y}\right)\right),\\ &\frac{\partial v}{\partial t}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)+fv=-\frac{1}{\rho_0}\frac{\partial P}{\partial y}+\left(\frac{\partial}{\partial x}\left(A\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(A\frac{\partial v}{\partial y}\right)\right). \end{aligned} \tag{1}$$

Continuity equation (Haditiar et al., 2019):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2}$$

where u and $\boldsymbol{v}$ are dependent variables or fluid flow velocities, $\boldsymbol{x}$ and $\boldsymbol{y}$ are independent variables and spatial coordinates, $\boldsymbol{\rho}$ is the density, $\boldsymbol{P}$ is the pressure, $\boldsymbol{f}$ is the external force, and $\boldsymbol{A}$ is the amplitude (Setiawan et al., 2020). This study was initiated with a comprehensive literature review to thoroughly understand the differential equations relevant to the research. Specifically, the focus was on the two-dimensional Navier-Stokes equations, employing a semi-implicit FTCS (Forward Time Centered Space) scheme. The literature review involved an extensive examination of previous studies, research papers, and authoritative texts to gather insights into the theoretical foundations and practical applications of the Navier-Stokes equations. This phase was crucial in building a solid understanding of the mathematical and physical principles underlying fluid dynamics, as well as the various numerical methods used for solving these equations. By delving deeply into the existing body of knowledge, the study aimed to identify the most effective approaches and techniques for tackling the complex problem of fluid flow in narrow straits with varying densities.

Following the literature review, the next step involved solving the Navier-Stokes equations using the finite difference method. The Navier-Stokes equations describe the motion of fluid substances and are fundamental to fluid dynamics (Kämpf, 2009, 2010). To solve these equations, a series of methodical steps were undertaken. Initially, the precise form of the Navier-Stokes equations and the associated boundary conditions were determined. This involved defining the velocity components $\boldsymbol{u}$ and $\boldsymbol{v}$, which are the dependent variables representing fluid flow velocities, and identifying the spatial coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ as the independent variables. Additionally, parameters such as density ($\boldsymbol{\rho}$), pressure ($\boldsymbol{P}$), external forces ($\boldsymbol{f}$), and amplitude ($\boldsymbol{A}$) were incorporated into the equations. Once the form and boundaries were established, the next task was to discretize the Navier-Stokes equations. This was achieved by substituting various finite difference approximations for the differential terms, thereby transforming the continuous equations into a set of discrete algebraic equations. The discretization process was carried out with careful consideration to ensure accuracy and stability. The chosen method, the semi-implicit FTCS scheme, was selected for its ability to efficiently handle the complex interactions within the flow while maintaining computational stability. After the discretization, the resulting semi-implicit FTCS scheme for the Navier-Stokes equations was examined. This scheme was then used to substitute the boundary values into the Navier-Stokes equations, completing the discretization process, as shown in Figure 1.

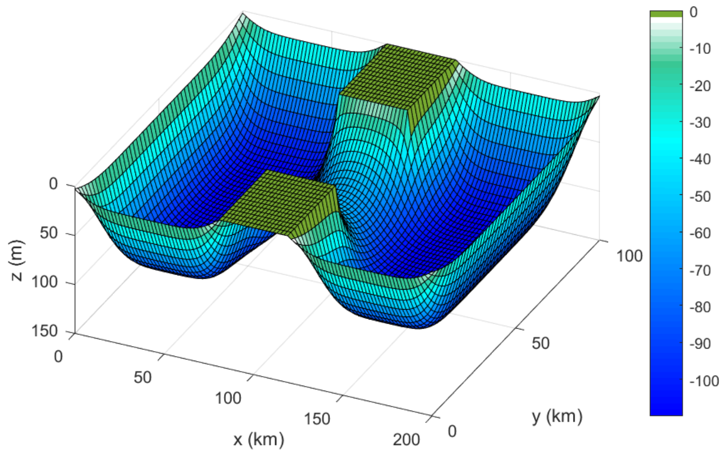


Figure 1. The bathymetric form is made into two dimensions

The subsequent phase involved implementing the discretized Navier-Stokes equations into a computational program. This was accomplished using Matlab R2016a software, a powerful tool for numerical computation and visualization. The discretized equations were translated into a Matlab script, allowing for the numerical simulation of fluid flow in the defined domain. The program was designed to handle the specified grid dimensions, boundary conditions, and initial conditions, thereby enabling a realistic simulation of the fluid dynamics within the narrow strait. The final phase of the study was the analysis of the simulation results.

C. RESULT AND DISCUSSION

1. Numerical Solution

The advection equation describes the transport of a substance or quantity due to the flow of a fluid. In this case, it is represented as:

$$\frac{\partial B}{\partial t}+\frac{\partial (uB)}{\partial x}+\frac{\partial (vB)}{\partial y}=0 \tag{3}$$

where: B represents the quantity being transported (such as momentum, temperature, or a passive scalar); u and v are the velocity components in the x and y-directions, respectively; $\frac{\partial B}{\partial t}$ is the time derivative of B, showing how B changes over time; $\frac{\partial (uB)}{\partial x}$ and $\frac{\partial (vB)}{\partial y}$ represent the advection (transport) of B in the x and y-directions. This equation is the basis for the control volume formulation used for momentum equations.

a. Control Volume for Momentum u

The control volume for momentum u is derived from the general advection equation and shows how momentum is transported in the x-direction (momentum component u). The control volume balance involves changes due to advection, which can be written as:

$$\frac{\partial (uB)}{\partial x}-\frac{\partial (vB)}{\partial y}=(\,C_{ue}^{+}B_{ue}^{+}\;+\;C_{ue}^{-}\,B_{ue}^{-})-(C_{uw}^{+}B_{uw}^{+}+\;C_{uw}^{-}B_{uw}^{-}) \tag{4}$$

Where: C_{ue}^{+}, C_{ue}^{-}, C_{uw}^{+}, and C_{uw}^{-} are coefficients that account for the advection and flow directions in the control volume; B_{ue}^{+}, B_{ue}^{-}, B_{uw}^{+}, and B_{uw}^{-} are the corresponding values of B (the transported quantity) on the eastern and western sides of the control volume. The balance shows how the quantity B (such as velocity or momentum) is transported and redistributed across the control volume due to the flow in the x-direction. That formula also applies to the momentum v.

b. Control Volume for Momentum v

For momentum v (the vertical momentum component), the control volume equation is similar to that of momentum u but applies to the y-direction. The control volume equation for momentum v is written as:

$$\frac{\partial (vB)}{\partial x}-\frac{\partial (vB)}{\partial y}=(\,C_{ve}^{+}B_{ve}^{+}\;+\;C_{ve}^{-}\,B_{ve}^{-})-(C_{vw}^{+}B_{vw}^{+}+\;C_{vw}^{-}B_{vw}^{-}) \tag{5}$$

Where: C_{ve}^{+}, C_{ve}^{-}, C_{vw}^{+}, and C_{vw}^{-} are the coefficients for momentum transport in the y-direction; B_{ve}^{+}, B_{ve}^{-}, B_{vw}^{+}, and B_{vw}^{-} are the values of \(B\) at different points in the control volume along the y-direction. This equation accounts for the advection of momentum in the vertical direction and describes how the momentum component v is redistributed within the control volume. The constants B and C for momentum v help define the advection and transport of the momentum component v in the vertical y-direction.

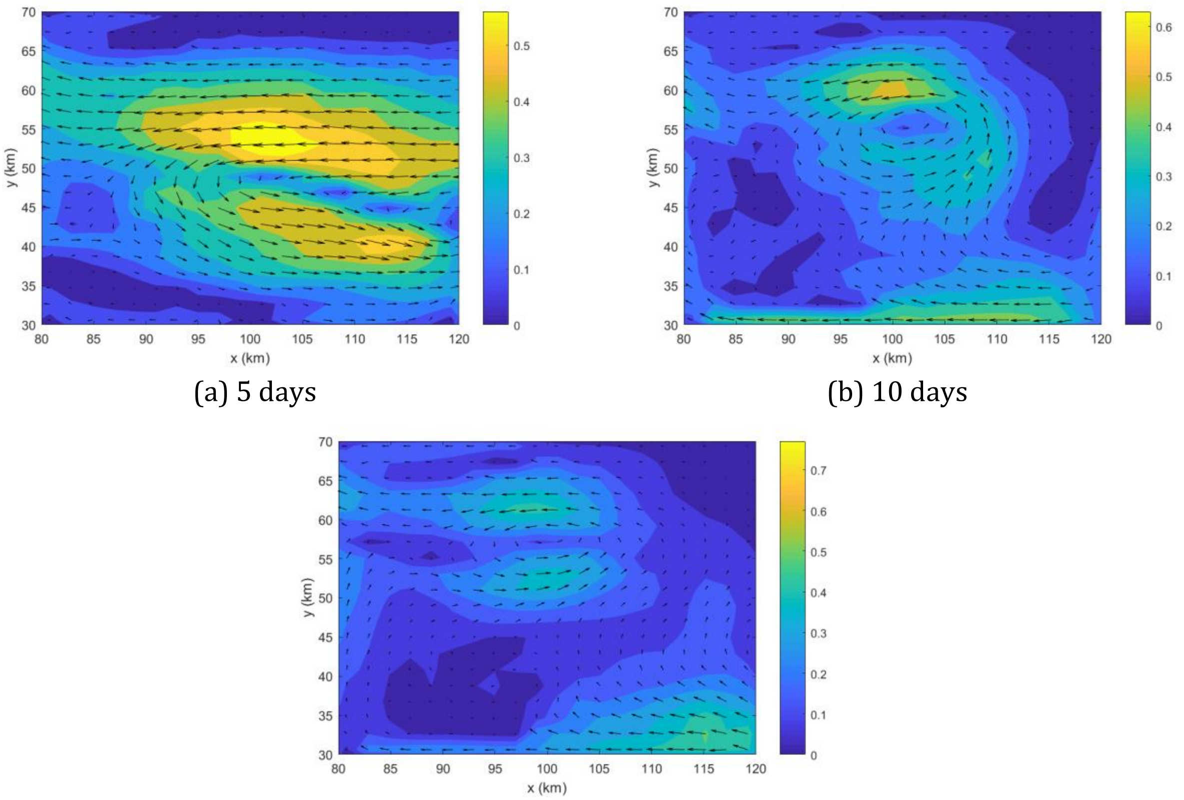
1) Constant C for momentum v is used to account for the effect of the velocity v on the variation in the y-direction, similar to the Courant number used for momentum u:
$C^{+}_{j,k} = 0.25\ (\ |v_{j,k}| + v_{j,k} + v_{j,k+1} + |v_{j,k+1}|)\ /\ \Delta y$
$C^{-}_{j,k} = C^{+}_{j,k-1}$
$C^{+}_{vw} = C^{+}_{j,k-1}$
$C^{-}_{vw} = C^{+}_{j,k}$
$C^{+}_{ve} = 0.25(|v_{j,k}| + v_{j,k+1} + v_{j+1,k} + |v_{j+1,k+1}|)\ /\ \Delta y$

These constants C describe the variation of the vertical velocity v along the y-axis and are computed at different grid points (j, k).
2) Constant B for momentum v describes the change in the advected quantity B due to the influence of the v-component on the control volume grid. The values of B are updated using a Total Variation Diminishing (TVD) approach, particularly with the Superbee scheme, which ensures stability in the advection process. The formulas are as follows:
$B^{+}_{vw} = B^{n}_{j,k} +\ 0.5\ \psi\ (\tau^{n}_{j,k})\ (1\ -\ C^{+}_{vw})\ \ (B^{n}_{j,k} - B^{n}_{j,k-1})$
$B^{-}_{vw} = B^{n}_{j,k} - 0.5\ \psi\ (\tau^{n}_{j,k})\ (1\ +\ C^{-}_{vw})\ (B^{n}_{j,k}\ - B^{n}_{j,k-1})$
$B^{+}_{ve} = B^{n}_{j,k} + 0.5\ \psi\ (\tau^{n}_{j,k})\ (1\ -\ C^{+}_{ve})\ \ (B^{n}_{j+1,k} - B^{n}_{j,k})$
$B^{-}_{ve} = B^{n}_{j,k}\ - 0.5\ \psi\ (\tau\ ^{n}_{j,k})\ (1\ +\ C^{-}_{ve})\ (B^{n}_{j+1,k} - B^{n}_{j,k})$

B^{+}_{vw} and B^{-}_{vw}: these represent the advected values of B at the western boundary of the control volume for the v-momentum. The values are updated based on the vertical velocity v and the difference between B at neighboring grid points (j, k) and (j, k-1); B^{+}_{ve} and B^{-}_{ve}: these represent the advected values of B at the eastern boundary of the control volume for v-momentum, updated similarly based on the velocity v and the grid points (j+1, k) and (j, k). The control volume formulation for momentum v involves calculating the changes in the advected quantity B due to changes in the vertical velocity v along the y-axis. The constants C and B for momentum v assist in discretizing the vertical velocity component on the numerical grid and updating the advected quantities using the TVD method and the Superbee scheme for stability.

2. Simulation

Simulation results are shown for the middle part of the strait only. Over a 20-day simulation period, the behavior of dense water outflow and its interaction with ambient water undergoes significant changes, evolving through distinct phases: early, mid, and late in the simulation. In the initial phase of the simulation, the dense water outflow from the east basin to the west basin encounters an equal density horizon at a specific depth. This horizon represents a layer in the water column where the density of the outflowing water matches that of the surrounding ambient water. Initially, the dense water flows along the seabed due to its higher density. However, upon reaching this equal density layer, the water can no longer sink and instead begins to rise into the overlying water column. This upward movement disrupts the existing stratification, leading to an initial stage of mixing between the outflowing dense water and the ambient water. The interaction at this stage is critical as it sets the stage for subsequent changes. The density gradients start to evolve, and preliminary mixing begins to alter the surface circulation patterns. The changes observed during these first five days are fundamental, as they influence the initial distribution of density and the development of circulation features that will evolve in the later stages of the simulation, as shown in Figure 2.



(a) 5 days (b) 10 days

(c) 20 days

Figure 2. Surface outlook in 20 days simulation

As the simulation progresses into the mid-phase, spanning days 6 through 15, the mixing process intensifies. By now, the dense water that was injected into the ambient water column has begun to form submerged mesoscale eddies, commonly referred to as "meddies." These eddies are large, coherent vortices that result from the interaction of the dense outflow with the surrounding water. The simulation results indicate that the meddies primarily form near the western boundary of the strait where the denser bottom currents encounter lateral density contrasts and bathymetric features, causing water masses to rotate and organize into vortices. The formation of meddies is a crucial outcome of the continued mixing and circulation changes that have been taking place. During this period, the system experiences enhanced turbulence and more pronounced changes in density distribution. The meddies can vary in size and intensity, depending on the strength of the outflow and the prevailing oceanographic conditions. As shown in Figure 2, the development of these meddies coincides with significant lateral and vertical density gradients, highlighting their role in redistributing heat, salinity, and nutrients both horizontally and vertically across the basin. The presence of these eddies affects the vertical and horizontal transport of properties like heat, salinity, and nutrients. The mid-phase is characterized by a dynamic interplay between the meddies and the surrounding water masses, leading to more complex and variable surface circulation patterns. The visualizations illustrate that the density-driven currents cause a clockwise rotation of water masses near the dense outflow zones, while the surrounding areas experience turbulence and secondary circulations, further emphasizing the impact of meddies on local energy and property. This period is critical for observing the development and evolution of the meddies and their impact on the overall circulation and mixing processes within the basin.

In the final phase of the simulation, from days 16 to 20, the system approaches a more stable state after the intense mixing and eddy formation of the earlier stages. During this period, the meddies that formed in the mid-phase continue to interact with the surrounding water, but their shapes and influence may evolve as the system seeks equilibrium. The simulation results show that the meddies gradually lose energy due to turbulent dissipation and friction with surrounding water masses, leading to reduced rotational speeds and a slight elongation of their structure. This process facilitates the gradual homogenization of density within the eastern basin. The circulation patterns established earlier become more pronounced, and the surface features stabilize. The dynamics of the surface circulation reflect the adjustments made by the system to the ongoing mixing and eddy formation. Figure 2 further supports this stabilization phase, showing that surface currents in the eastern basin exhibit a counterclockwise flow pattern driven by density gradients and residual meddy activity, which align with geostrophic adjustments observed in real-world systems. The surface circulation in the eastern basin often exhibits a counterclockwise pattern, which aligns with the overall influence of the meddies and the changes in density gradients. By the end of the simulation, the system exhibits a more organized circulation pattern, though still influenced by the residual effects of the meddies and the previous mixing events. Importantly, the remaining meddy structures continue to play a role in nutrient and energy distribution by enhancing vertical mixing and facilitating the transport of dense water into the deeper basin layers. This mechanism underscores the broader implications of meddy dynamics for ecosystem processes and energy fluxes in natural channels. This final phase provides insights into the long-term behavior and stability of the circulation patterns and the ongoing role of meddies in shaping the dynamics of the water masses.

D. CONCLUSION AND SUGGESTIONS

In conclusion, the 20-day simulation provides valuable insights into the behavior of dense water outflows and their subsequent interactions with the surrounding water column, addressing the research objectives outlined in the introduction. The simulation begins by modeling the dense water flowing from the east basin to the west, where it encounters an equal density horizon, triggering a transition from bottom-layer flow to upward injection. This shift initiates complex mixing processes, which contribute to changes in density distribution and circulation patterns. The mid-phase of the simulation reveals the formation of submerged mesoscale eddies, or "meddies," which play a crucial role in the vertical and horizontal transport of oceanic properties, such as heat and nutrients. These meddies significantly affect surface circulation patterns and contribute to overall mixing within the basin. In the final phase, the system approaches a stable state, with residual effects from the meddies continuing to influence circulation patterns. The stable circulation observed in this phase emphasizes the long-term impact of density-driven currents on ocean dynamics, providing critical information on the role of such phenomena in regulating ocean circulation. The simulation results contribute to a deeper understanding of the complexities of high-density water flow dynamics, filling gaps in prior research on the interaction between geostrophic and density-driven forces in narrow channels. Comparisons with previous studies highlight the significance of this work in advancing the study of mesoscale processes, with implications for oceanographic modeling and energy exploration. This research provides new perspectives on the dynamic behavior of dense water outflows and their role in shaping ocean circulation in narrow straits.

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