

Mathematical Semiotics in Primary Learning: Helping Prospective Teachers Understand Mathematical Representations

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ABSTRACT

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Mathematics learning in elementary schools is often faced with the challenge of bridging abstract concepts with participants' concrete experiences. One of the reasons is the difficulty in understanding and using mathematical signs and symbols appropriately. Semiotic representation of mathematics, which involves symbols, diagrams and notations as visual and conceptual aids, is an important approach in overcoming these challenges. This study aims to explore the ability of prospective elementary school teachers to understand and use semiotic representations of mathematics, especially in the context of number patterns. A qualitative approach with a hermeneutic phenomenological design was chosen to understand prospective teachers' experiences, views, and thought processes in interpreting and using mathematical symbols. The researcher used written tests and semi-structured interviews as data collection techniques to explore the semiotic representations used in mathematical problem solving. The participants were grouped based on the results of the initial ability test, and six of them were selected as research subjects. Data analysis used the Interpretive Phenomenology Analysis (IPA) method with the help of NVivo 14 Plus software, which allows researchers to systematically manage and analyze qualitative data. The results of this study provide insight into prospective teachers' understanding of symbolic representation in mathematics learning and the challenges they face, which can be the basis for designing more effective learning strategies in the context of mathematics education in primary schools. Using a descriptive qualitative approach, data were obtained through observation and analysis of problem solving activities involving symbolic and visual representations. The results of the study are expected to provide insight into the importance of semiotic training in prospective teacher education, so that they are able to convey mathematical concepts meaningfully and contextually.



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A. INTRODUCTION

Mathematics is one of the important subjects taught early on in primary school because of its role in building the foundation of participants' understanding of various basic concepts that will be useful in everyday life and further education. As a discipline, math does not only focus on mastering numeracy skills, but also serves to hone logical thinking, analytical, and problem-solving skills. This process is important to help participants face the challenges of an increasingly complex world, where mathematical understanding is a key tool in decision-

making and systematic problem solving (Bessoondyal, 2017; Durrance, 2019; Pramesti, 2017; Rigelman, 2020; Rigelman & Rhodes, 2023).

However, in practice, learning mathematics often encounters various challenges. One of the main challenges is bridging the abstract concepts that characterize mathematics with the way of thinking of participants who still depend on concrete things. Many participants find it difficult to understand the material being taught because these abstract concepts are not always provided with adequate representations, such as relevant visualizations, symbols or illustrations (Fauzi & Arisetyawan, 2020). This causes math learning to be perceived as difficult and uninteresting, thus affecting participants' motivation to learn (Purwasih, 2023). On the other hand, teachers as learning facilitators also often face obstacles in choosing the right methods and tools to convey these abstract concepts. Without an effective approach, the learning process tends to be one-way, so participants are not actively involved in building their own understanding (Pinnock, 2021; Ünal & Çil, 2023). Thus, a learning strategy is needed that can help participants connect abstract concepts with concrete experiences, so that mathematics learning becomes more meaningful and easy to understand.

In this context, semiotic representations of mathematics become very relevant. With an approach that focuses on signs and symbols, teachers can utilize various tools such as diagrams, graphs, tables, or visual illustrations to explain mathematical concepts. These representations not only help participants understand the material more deeply, but also facilitate them in applying the concepts in everyday life. The ability to interpret mathematical signs and symbols is not only limited to memorizing or applying certain rules, but also requires a deep understanding of the meanings contained in them (Palayukan, 2022). To understand and use these symbols appropriately. This includes the skill to relate mathematical symbols to broader concepts or ideas, as well as understanding the context in which they are used (Claudia et al., 2021). This ability not only enables participants to apply formulas or procedures, but also helps them build a deeper and more meaningful understanding of mathematics, thus encouraging creative and critical thinking skills in problem solving (Khalid & Embong, 2020). To illustrate, the symbol π (pi) is often known as the number 3.14 or used in calculating the circumference of a circle. However, a deeper understanding of π includes the notion that it represents the ratio between the circumference of a circle and its diameter, has endless values, and is a fundamental constant in mathematics (Presmeg, 2006). Therefore, this concept has been introduced since elementary school, with the aim that students have a strong foundation of conceptual knowledge to support learning at the next level. Understanding itself is a process that is dynamic and systematically organized (Hidayah et al., 2020).

Mathematical activities involve the process of interpreting and transforming signs to develop mathematical knowledge (Hoffmann, 2006; Hundeland et al., 2014). When participants encounter a mathematical problem, they tend to perform mathematical thinking activities to provide ideas or solutions. By utilizing signs, participants can connect mathematical concepts with objects around them (Suryaningrum et al., 2019). For example, beads can be used as signs. Teachers can bring in a large number of beads and have participants arrange them into a triangular pattern, where each bead represents one unit in the triangular number pattern. In addition, Quinnell & Carter (2012) explained that in mathematical thinking, participants' ideas communicated in writing can only be achieved through the use of symbols.

Symbols or signs play an important role in helping participants understand the thinking process, symbolize concepts, and communicate them (Ostler, 2011; Radford et al., 2019). In learning mathematics, participants are not only expected to be able to use signs or symbols, but also to be able to provide reasons and explain the meaning behind the use of these signs. Signs such as words, symbols, diagrams, graphs and schemes serve as mediators that connect external reality with internal mental processes, known as semiotic mediation processes (Purwasih et al., 2024). Therefore, cognitive processes in mathematics are essentially semiotic processes involving complex relationships between signs (Santi, 2011).

Previous research shows that participants often have difficulty in recognizing patterns and expressing them in symbolic language (Swaftord & Langrall, 2000). When trying to write symbolic representations, they tend to focus on less relevant aspects of number patterns, especially on recursive relationships between consecutive terms in a sequence (Orton et al., 2016). This confirms that mathematics as a scientific discipline has a close relationship with signs or symbols. Signs, defined as objects that represent something else, either physically or mentally, are at the core of mathematical understanding, because through signs, participants can associate abstract concepts with specific meanings. The understanding of mathematical concepts is highly dependent on the representation and interpretation of signs used in learning (Mudaly, 2014). Proper interpretation of signs can provide deep understanding for participants. However, in practice, participants often experience difficulties in understanding various mathematical signs, such as signs in integers (Bishop et al., 2014), positive and negative signs (Vlassis, 2008; Bofferding, 2014), signs in number sequence relations (Schindler et al., 2017), signs in number sequence relations (Schindler et al., 2017), and signs in number sequence relations (Bofferding, 2014), operation signs (Eichhorn et al., 2018), angle signs in geometry (Biber et al., 2013), as well as representation through algebraic notation and signs in visualization (Widjaja et al., 2011).

This difficulty in understanding signs often results in an inappropriate understanding of the process of solving mathematical problems (Khalid & Embong, 2020). These studies show that there is a close relationship between the ability to understand signs or symbols and participants' success in solving mathematical problems. Given these challenges, it is important to conduct an in-depth analysis of how participants think in, which are an important part of the curriculum in junior high school (Rosikhoh & Abdussakir, 2020). This analysis is expected to provide greater insight into strategies that can be applied to improve participants' understanding of complex mathematical concepts.

The ability of prospective elementary school teachers to understand and teach mathematical concepts is very important to prevent participants from experiencing the same difficulties as found in previous studies. Pre-participants need to master the skills of recognizing patterns, interpreting signs and representing symbols accurately (Santi, 2011). This understanding will help them explain mathematical concepts in a simpler and more contextualized way. For example, they should be able to explain the use of negative and positive signs, number order relations, and mathematical operations in a child-friendly manner (Arzareello & Sabena, 2011). With these skills, prospective teachers can guide participants to develop a deep understanding of mathematical concepts rather than simply memorizing rules. This learning approach that prioritizes symbolic understanding can also help participants

relate abstract concepts to concrete objects in their environment, making learning more relevant and easy to understand (Suryaningrum & Agustina, 2021).

To achieve this goal, education programs for prospective teachers need to pay more attention to training in sign interpretation and symbol representation in mathematics. Prospective teachers should be equipped with semiotic mediation skills, i.e. connecting mathematical symbols with real-world meanings and participants' thought processes. In addition, case-based training, such as analyzing common errors that participants make in understanding signs, can help prospective teachers anticipate problems that may arise in the classroom. With a focused approach on conceptual understanding and the ability to convey material clearly, prospective teachers can create a better learning experience for participants. Ultimately, pre-service teachers who are skilled in teaching mathematical signs and symbols will be able to build a strong foundation for participants, so that they not only understand mathematics technically, but also develop critical thinking and creative problem-solving skills.

In education, especially at the primary school level, the teacher's ability to effectively convey mathematical concepts is key in building the foundation of participants' understanding. Math is not only about numbers and operations, but also about how symbols and signs are used to represent abstract concepts. A deep understanding of mathematical signs, such as operation symbols, number relations, or algebraic notation, is necessary for clear and meaningful learning. However, research shows that participants often face difficulties in understanding these signs, which can impact on their ability to solve mathematical problems effectively. As future educators, the participants of the elementary teacher education study program have a great responsibility to overcome this challenge. They need to be equipped with the ability to understand and interpret mathematical signs well, so that they can teach these concepts to participants in a contextual and relevant manner. For this reason, an introduction to mathematical semiotics is an important step in preparing prospective teachers. With a strong understanding of semiotic representations, prospective teachers can help participants connect abstract concepts with everyday reality, creating more effective and meaningful learning. This study aims to explore the role of introducing mathematical semiotics in helping prospective elementary school teachers understand mathematical representations, so that they are able to teach these concepts in a better way and prevent common difficulties experienced by participants.

B. METHODS

This research was conducted using a qualitative approach. This approach was chosen to obtain results that are relevant to the research focus, namely understanding how prospective elementary teachers develop an understanding of mathematical representation through semiotic recognition. This approach allows researchers to explore in depth the experiences, views, and thought processes of prospective teachers in understanding and using mathematical signs or symbols in learning. Thus, the data obtained not only illustrates their level of understanding, but also provides insight into the challenges they face and how they overcome these difficulties. This approach is in line with the views of Creswell (2017) and Wicaksono et al. (2021), which state that qualitative research is a research method that produces descriptive data in the form of words, writings, or observable behavior of individuals or subjects who are

the focus of research. This method aims to explore the meaning and deep understanding of certain phenomena through detailed exploration of the experiences, views, or actions of the subject. In the context of this study, researchers sought to understand and interpret social phenomena related to the introduction of mathematical semiotics to prospective elementary school teachers.

The analysis was done descriptively and interpretatively to explain how they understood and applied semiotic concepts in mathematics learning. This approach allows the researcher to not only observe the subject's behavior, but also explore the thought process and context behind their actions, resulting in comprehensive and meaningful findings. This research design uses a hermeneutic phenomenological approach. Phenomenology aims to describe the general meaning of an individual's experience of a phenomenon, but has not yet reached the stage of in-depth interpretation (Creswell, 2017). This methodology focuses on how individuals personally experience and give meaning to experienced phenomena, without conducting in-depth analysis of their meaning (Borg, 2014). Hermeneutics complements phenomenology by providing a deeper interpretation, so that the meaning of the subject's experience can be understood in more richness and detail. The phenomenon in this study is how prospective elementary school teachers understand and interpret mathematical symbols, especially in the context of introducing mathematical semiotics, and how they use symbolic representations in learning mathematics. This study focuses on prospective teachers' experiences in identifying and applying mathematical signs, as well as the challenges they face in the process. This phenomenon is important to understand because it can provide insights into prospective teachers' readiness to teach mathematical concepts effectively to participants in primary schools.

This study was conducted among the 2024/2025 academic year participants, which consisted of one class with 43 students. The participants were then grouped into three groups based on the results of the initial ability test, namely low, medium and high groups. This group division aims to see the differences in understanding and application of semiotic representation of mathematics among the participants with different ability levels. After successfully grouping the participants based on their respective ability levels, the researcher proceeded to the next stage, namely the selection of 6 participants as research subjects with the following considerations. Subject determination is based on the principle of saturated data, which is a condition where the addition of subjects no longer produces significant new information. This means that the data obtained is sufficient to answer the research questions. Six students were selected because they were able to convey information verbally fluently and had sufficient time to be interviewed. The data collection techniques in this study included two main methods, namely written tests and interviews. The written test instrument used consisted of problem solving descriptions, which aimed to get an overview of the semiotic representations of mathematics used by the participants in solving problems. This test provides insight into how the participants translate mathematical symbols into a form of representation that is easier to understand and use in a mathematical context.

In addition, interview guidelines were used to dig deeper and further explore semiotic representations of mathematics that may not have been revealed during the process of solving written problems. The interviews aimed to obtain additional information related to the

participants' understanding of the use of mathematical signs and symbols that they applied in solving problems. The interviews used in this study were semi-structured interviews, which allowed the researcher to follow a more flexible flow of conversation and adjust the questions to the conditions or characteristics of the respondents. With this approach, the researcher can ask open-ended questions that can develop according to the answers given by the participants, as well as dig deeper into their thought processes and understanding of the material taught.

Data analysis in this study used the Interpretive Phenomenology Analysis (IPA) method, which is specifically designed for hermeneutic phenomenological (Miller, 2015; Smith & Rayfield, 2017). IPA not only serves as a method of analysis, but also involves sensitivity, reflective mindset, and deep perspective in the process of analysis. This approach requires researchers to understand the nuances and complexities of participants' experiences, and adopt a reflective and interpretive way of thinking. The IPA perspective requires researchers to approach the data with an open attitude, ready to explore meanings that may be hidden in participants' narratives. This sensitivity is important not only to understand the minute details of individual experiences, but also to ensure that the analysis stays true to the meanings that participants are trying to convey. By combining sensitivity, a reflective mindset, and an interpretive perspective, IPA enables researchers to uncover the deep meanings of individuals' subjective experiences of the phenomenon under study. Technically, the data analysis process in this study was carried out by utilizing NVivo 14 Plus software. NVivo is one of the leading software specifically designed for qualitative data analysis, which is very useful in managing and analyzing large and complex data (Edwards-Jones, 2014; Liebe & Camp, 2019). The use of NVivo allows researchers to organize, code, and categorize data in a structured and systematic way. With this tool, researchers can easily find important patterns, identify emerging themes, and explore the relationships between concepts in the data. In addition, NVivo facilitates more in-depth analysis through features such as keyword search, data visualization, and the ability to generate reports that support data interpretation. This greatly assists researchers in exploring the deeper meaning of the data collected, as well as ensuring that the analysis process remains organized and efficient. The data analysis process using NVivo was conducted in three stages: open coding, axial coding, and selective coding (Creswell, 2017). In the open coding stage, the researcher coded the written tests and interview transcripts. In the axial coding stage, the resulting codes were grouped into interrelated categories. Furthermore, at the selective coding stage, the researcher selected and mapped the coding to answer the research questions. NVivo played an important role in each of these stages, helping to ensure the analysis process was systematic and the results could be accounted for.

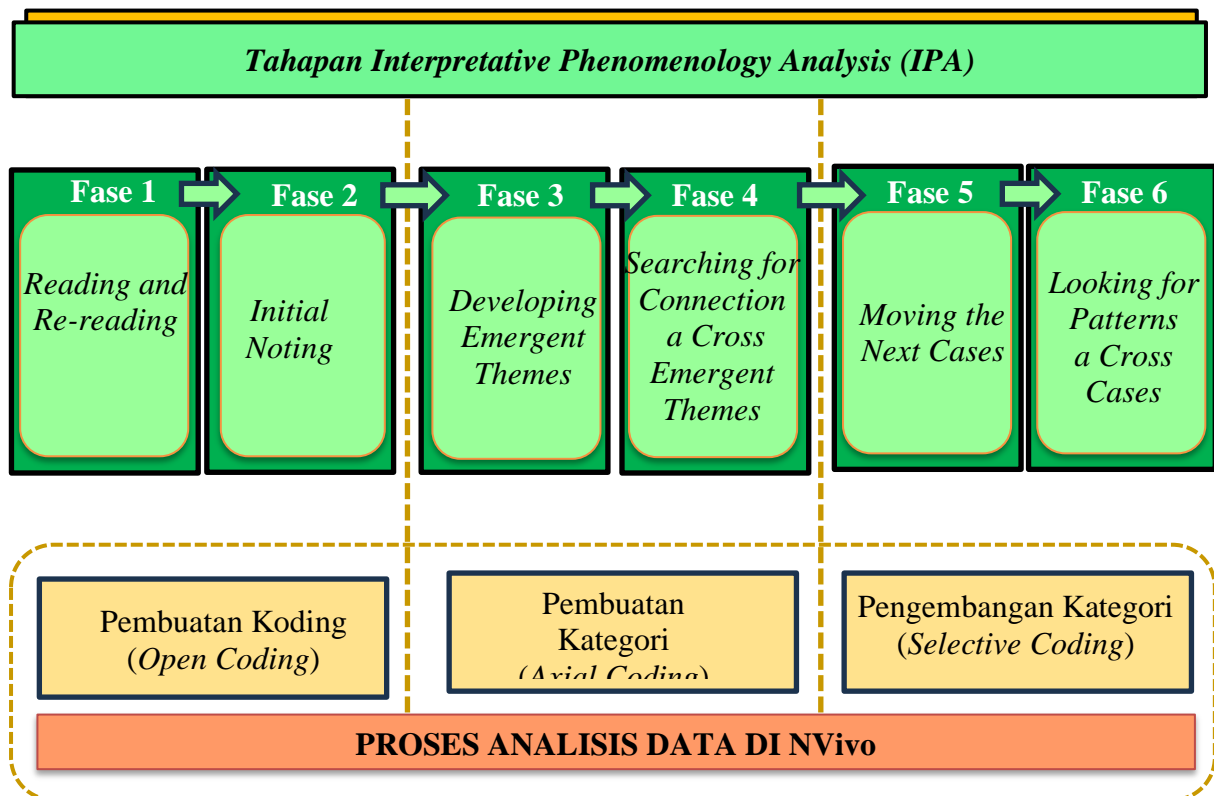


Figure 1. Stages of Interpretative Phenomenology Analysis (IPA) and Data Analysis Process NVivo

C. RESULT AND DISCUSSION

As an initial effort to look at the semiotic of mathematics of prospective elementary school teachers, various keywords that emerged from the interviews were identified. Key identification was carried out using the "*Word Frequency*" facility available in NVivo. The results of the identification are presented in the following Figure 2.



Figure 2. Keyword Results of Participants' Semiotic Mathematics Interviews

Figure 2 above shows that the keywords that often appear are patterns, images, problems, mathematics, structures, terms, problems, concepts, decomposing, and numbers. The various keywords that appeared as an initial guide to mapping the findings of mathematical semiotics

in research participants in solving basic mathematics problems based on participants' written test answers and interviews. Based on the results of the coding conducted using NVivo, a map of the relationship between mathematical semiotics and research participants was obtained.

1. Exposure of Low Ability Participants' Mathematical Semiotic Findings

Low ability participants (P1) in working on math problems usually have difficulty in understanding basic concepts, compiling solution steps, and choosing the right strategy. P1 participants tend to be confused when facing story problems, often make calculation errors, lack confidence, and give up easily. This causes the problem-solving process to be unsystematic and the final result is less precise. In this section, the findings of the semiotic description of mathematics of low ability participants are presented. The results of P1's work can be seen in Figure 3.

untuk menentukan jumlah bola hitam pada urutan ke-6
bisa menggunakan cara seperti $(2 \times n + 3)$
 $(2 \times 6 + 3)$
 $=(12 + 3)$
 $= 15$

untuk menentukan jumlah bola putih pada urutan
ke-10 bisa menggunakan cara $= n \times 5 + (n : 2)$
 $= 10 \times 5 + (10 : 2)$
 $= 50 + 5$
 $= 55$

Figure 3. Problem solving results of Participant P1

In analyzing the arrangement of the ground ball, participant P1's answer in solving the problem seems to include a fairly good understanding. Participant P1 can find the problem when writing down the known and questionable information. Participant P1 saw the pictures showing the number of arrays of soil balls from one picture to the next (6, 10, and 15 arrays, and so on). This shows that participant P1 can recognize the pattern of increasing the number of arrays of soil balls from one picture to the next. Furthermore, participants showed an arrangement of black soil balls in a pattern of 5, 7, 9, indicating the addition of two numbers each time. This shows how participants understood the addition pattern in the black ground ball arrangement. In addition, participant P1 was able to show an arrangement of white ground balls with patterns of 1, 3, and 6, ...etc. These patterns show a more complex pattern of addition. Since the way in which the numbers were derived was not entirely clear, there were patterns that were not clearly visible which led to participant confusion in understanding the subsequent arrays.

The picture above shows that participant P1 at the pattern recognition stage in solving problem 1 by writing a method such as " $(2 \times n + 3)$ " for the black ball and " $n \times 5 + (n : 2)$ " for the white ball. Then participant P1 substituted the value of $n = 6$ and produced a value of 15 black balls and substituted $n = 10$ for white balls. For the black ball rule, participant P1 had correctly formulated the formula and the participant tried to make a formula for the white ball, but there was still an error in looking at the arrangement pattern of the white ball. In addition, participant P1 also made no effort to do pattern recognition well. Participant P1 had only used mathematical notation and tried to find the answer directly with the general formula. Although

the formula written down was not entirely correct according to the actual lineup pattern. At this stage, the research data of participant P1 's interview in solving math problems is presented. The excerpts of the results of the P1 participant 's interview with the researcher are as follows.

- R : *Can you find the pattern structure of the number pattern from your answer?*
 P1 : *There is that regularity pattern. Like the black ball adding two to the next black ball, the white one sharing two minus one.*
 R : *Why did you use the picture to find the structure of the pattern?*
 P1 : *The problem is that drawing takes up space or consumes paper. So the same me is not in the picture.*
 R : *Can you tell me how the picture in the problem relatesto the math concept?*
 P1 : *Maybe, I actually looked for an approximation through the picture first, then looked for the formula with the approximated number. Pictures can help to visualize the problem. If it's just through the story, it's dizzying, if the picture helps with visualization.*
 R : *Why did you answer like that!*
 P1 : *Because I knew only that for that question at that time.*
 R : *Can you verbally describe the structure of the pattern?*
 P1 : *Yes ma'am, that the pattern in the problem increases with each subsequent arrangement.*
 R : *What the ball looks like increase?*
 P1 : *Yes, as in my answer, the black ball is $2 \times n$ plus 3. For the white ball $n \times 5$ then add the result of n divide by 2.*

Participant P1 can be seen in the answer sheet above using a mathematical formula (in the form of arithmetic symbols) to calculate the number of black balls in the 6th and 10th order. The rule compiled by participant P1 is as follows. To determine the number of black and white balls in the sequence is: Number of black balls in the 6th order.

$$\text{6th term} = (2 \times n + 3 = (2 \times 6 + 3) = 15$$

For the black balls in the 10th order is:

Number of black balls in 10th order

$$\text{10th term} = n \times 5 + (n:2) = 10 \times 5 + (10:2) = 50 + 5 = 55$$

Participant P1 represented concepts, operations, and variables symbolically (R2). Participant P1's answer also used mathematical symbols to show certain pattern or sequence relationships in number patterns (R3). The rules or formulas expressed by P1 participants to express the mathematical relationships involved in determining the number of number patterns sought. So that the ability of P1 participants can be said to use mathematical symbolic language to express ideas and conceptual relationships (R3). This is in line with Wicaksono et al (2021) that participant P1 solved the problem in detail by using the correct formula. In addition, participants also understand the problem well before looking for solutions, and prefer to find simple and effective solutions (Furqon et al., 2021).

Participant P1 can also be said to make a mathematical model (R4). Formulating the rules written down, participant P1 can create a mathematical model that can certainly be implemented on problem 1 to find the 6th pattern on the black ball and the 10th pattern on the

white ball. Finally, participant P1 not only presents mathematical formulas in the form of symbols, but can also find numbers in his writing and can also express in verbal form related to how the written formulas can be used to solve problem 1. This reflects the ability of participant P1 to write concepts with language that is mathematical symbols (R5). Participant P1 used a mathematical formula, namely: Number of black balls in the 6th order = $(2 \times n + 3) = (2 \times 6 + 3) = 15$. Number of black balls in the 10th order = $n \times 5 + (n:2) = 10 \times 5 + (10:2) = 50 + 5 = 55$. Participant P1's answer illustrates the application of mathematical formulas in the form of mathematical symbols used to calculate the number of black balls and white balls in the 6th and 10th order. This analysis means that participant P1 can articulate mathematical relationships and at the same time solve problems involving number patterns. Participant P1 worked on the problem by using logical, systematic, objective, and analytical thinking and analyzing the factors involved, and developing an appropriate solution in a structured and effective manner (Azrai et al., 2017).

Finally, participant P1 can use symbols related to problem 1 with prior knowledge. Through the use of formulas that are in accordance with the problem in question, this P1 participant is able to connect the mathematical knowledge with the new concept being taught. This refers to the participant's understanding of mathematical variables and operations with the basic concepts of multiplication (\times), division ($:$), multiples ($2n$) and addition ($+$). These symbols make it easier for students to make calculations and predict the next terms in the sequence, as well as help understand the relationship between variables and operations in number patterns (Rivera, 2010). In this way, participant P1 could identify and utilize the symbols deeply in the concept of number patterns. Participant P1 can explain the meaning that connects the representation with the object being studied. From the formula written down, participant P1 displayed a representation of mathematical symbols that reflected the relationship between the variable n and the number of black or white balls. Through the application of this formula, participants interpreted the meaning and significance of number patterns. Participant P1 certainly did not just mechanically use this formula but understood the meaning and relevance in the number pattern being studied. This means that by substituting the value of $n = 6$ or $n = 10$, P1 participants obtained the correct answer and understood the meaning of $n = 6$ or $n = 10$. This shows that P1 has implemented number patterns in a broader mathematical context. Using arithmetic symbols to represent concepts and operations helps students simplify and generalize number patterns (Purwasih et al., 2023). The components and indicators of mathematical semiotics seen in P1 are as follows.

Table 1. Findings of Participant P1's Mathematical Semiotic Components and Indicators

No	Semiotic Components	Indicators	Simbol
1	Representation (Semiotics refers to ways of conveying ideas or concepts through spoken language, writing, symbols, pictures, diagrams, models, graphs, and written text).	Write down known and questionable information	R1
		Create math models related to number patterns	R2
		Represent concepts, operations, and variables symbolically	R3
		write concepts in language with mathematical symbols	R4

No	Semiotic Components	Indicators	Simbol
2	Mathematical Object (Semiotics refers to the use of mathematical concepts, symbols, images, graphs, or structures)	use symbols related to problem 1 with prior knowledge	M1
		able to connect mathematical knowledge with the new concept being taught	M2
		solve problems involving number patterns using mathematical symbols (problem solving)	M3
3	Interpretant (Semiotics refers to the process of making meaning. interpretation or concluding from mathematical concepts).	connects the representation to the object being studied.	I1
		explain the meaning that connects the representation to the object being studied	I2
		Establish the formula that has been found.	I3

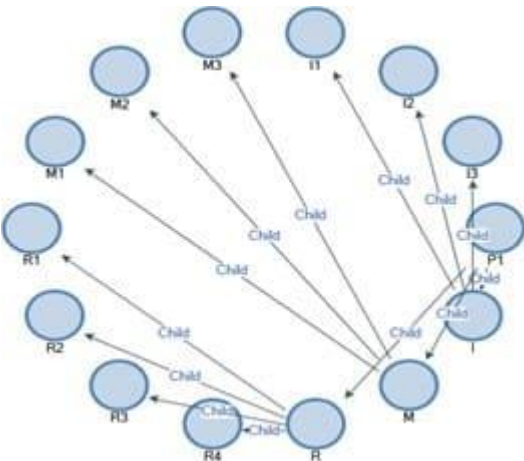


Figure 4. Visualization of Participant P1's Semiotic Components of Mathematics with Nvivo

2. Exposure of Semiotic Mathematics Findings on Moderate Ability Participants

This presentation shows that participants with moderate mathematical ability (P2) have a basic understanding of the semiotic system of mathematics, but still face limitations in flexibility and consistency in the use of representations. They were able to connect symbols with the meaning of concepts, although sometimes had difficulty in switching between forms of representation. Participants were also able to use diagrams or drawings to aid understanding, although not always effectively. Efforts to explain the meaning of mathematical signs were apparent, but their conceptual understanding was not yet fully mature. Overall, they have the potential to be further developed in terms of connections between representations and deeper symbolic meaning, as shown in Figure 5.

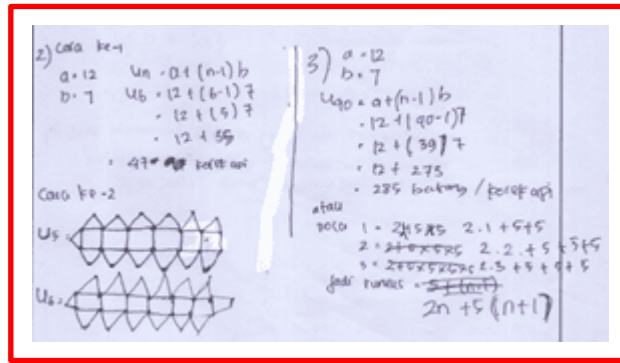


Figure 5. Problem solving results of Participant P2

Participant P2's answer in number 2 is to present the general formula, $U_n = a + (n-1)b$, where a is the first term, n is the number of terms, and b is the difference between terms to answer the question as well as representing concepts, operations, and variables symbolically. P2 participants have not identified the existing pattern. P2 participants can solve math problems by trying various ways directly, rather than just reading about the concept (Soraya et al., 2020). This is in line with the research of Atkins et al (2001) who revealed that P2 can determine the known and questionable elements in the problem, but is less able to restate the original problem by describing it well through their own language. Although P2 participants wrote problem solving in two ways. P2 participants chose to directly use the general formula without doing the numbers of deeper pattern understanding showed that participants had not done the process of recognizing or identifying similar or different patterns or rules to make the relationship contained in the number pattern. Participant P2 can identify the existence of an arithmetic pattern in a sequence of numbers by seeing whether this difference is consistent or constant from one number to the next. By looking at the relationships that may exist between the numbers, P2 participants try to multiply or add several consecutive numbers to see if the results of the multiplication or summing form a certain pattern, or even look for an arithmetic pattern where the difference between consecutive numbers is constant. Not only can the participant find a solution for a given sequence of numbers, but P1 can also generalize a general rule that applies to each term in the sequence. This is in line with (Lannin et al., 2006; Vale & Barbosa, 2015; Zhang & Rivera, 2021) who revealed that visual patterns or patterns found from visualization results make different ways of thinking from one another in solving number pattern problems. Excerpts of interviews between researchers and P2 participants are presented to add information about pattern recognition that is not yet in the written test. The following is an excerpt of the interview:

- R : Can you find the pattern structure of the number pattern from your answer?
- P2 : There is a number pattern ma'am. As seen from 12 to 19, 19 to 26, the addition is still 7. So the pattern is regular.
- R : Why did you use the picture to find the structure of the pattern?
- P2 : Through the picture can find the number of matchsticks in question Replace.
- R : Why do you make pictures like this?
- P2 : Because to make it easier for mom to find the pattern

- R : *Did you find it easier to solve the problem when deciphering the symbols!*
 P2 : *Yes 1 did.*
 R : *Can you tell me how the picture in the problem relates to the math concept?*
 P2 : *To determine the next terms and find the sum of the terms of the pattern*

From the interview excerpt above, P2 wrote down two ways, namely using the algorithm row formula and continuing to draw the match arrangement pattern. Participant P2 used the mathematical formula ($U_n = a + (n-1) \times b$). While the other way is to draw by paying attention to the relationship between the elements in the given image and trying to find patterns or rules that apply to the image. Participant P2 verbally explained the numbers that can be used to describe the number pattern sequence, although it did not include a picture or diagram. Participant P2 tried to explain the relationship between each term in the sequence, where each term is the sum of consecutive positive integers, using the notation. Participant P2 connects each term with the result of consecutive addition, the participant explains the meaning of the notation. Utilizing arithmetic symbols such as parentheses, plus signs, and variable n , P2 participants can provide a visual representation of a clear and structured mathematical number pattern sequence of visible number patterns. This is a form of representing concepts, operations, and variables symbolically.

P2 participants create mathematical models that describe the addition pattern and conceptualize how the sequence develops. P2 used verbal language to explain the concept of the number pattern by detailing each term as a consecutive sum of the pattern sequence. Although no language variations were explicitly included, the way P2 participants communicated ideas through words and mathematical notations showed an understanding of number patterns. In other words, participant P2 used words and mathematical symbols to communicate the concept of number patterns orally and in writing. Although "different language" does not refer to language variation, the participant's ability to use verbal language and mathematical notation shows P2's ability to convey and explain mathematical concepts in a clear and structured manner. This mathematical notation also helps in creating mathematical models that explain the concept of consecutive addition in the n th term of the sequence, showing P2 participants' efforts in understanding number patterns through mathematical language.

Participant P2 explains how the addition pattern changes from one term to the next and makes predictions about the gradual development of the pattern along the row. P2 not only explains the changes that occur from one term to the next, but also makes predictions about the development of the pattern along the number pattern line. Participant P2 is able to project how the sum will continue to change according to the identified pattern, forming a gradual pattern. Participant P2 understood the number patterns in the sequence, such as consecutive addition from the first term to the n th term. Participant P2 successfully solved the problem by identifying and explaining the number pattern, as well as understanding the structure of the pattern and using mathematical notation to show the pattern.

P2 participants used a formula to find the number of 6th terms and 40th terms. P2 participants also used pictures to find the number of 6th terms from the arrangement of the pictures. Participant P2 looks for the 40th term in the image sequence by applying the general

rule, namely $U40 = a + (n-1) b$. P2 is not only able to understand number patterns at a basic level, but also able to apply them in broader mathematical situations or problems. By explaining each term as the sum of consecutive numbers, P2 participants created a mathematical model that can be applied in a broader situation. The components and indicators of mathematical semiotics seen in P2 are as shown in Table 2.

Table 2. Findings of Components and Indicators of Participant P2's Mathematical Semiotics

No	Semiotic Components	Indicators	Simbol
1	Representation (Semiotics refers to ways of conveying ideas or concepts through spoken language, writing, symbols, pictures, diagrams, models, graphs, and written text).	Recognize or identify similar or different patterns or rules	R1
		Drawing number patterns	R2
		Drawing with attention to the relationship between elements	R3
		Creation of a mathematical model that explains the concept of consecutive addition of the nth term of a sequence,	R4
2	Mathematical Object (Semiotics refers to the use of mathematical concepts, symbols, images, graphs, or structures)	Make predictions about the gradual development of the pattern along the row	M1
		Solve problems by identifying and explaining number patterns	M2
		Understand the structure of patterns and use mathematical notation to show patterns	M3
3	Interpretant (Semiotics refers to the process of making meaning, interpretation or concluding from mathematical concepts).	Apply it to broader mathematical situations or problems	I1
		Apply the general rule that $U40 = a + (n-1) \times b$	I2
		Use the formula to find the number of 6th terms and 40th terms.	I3

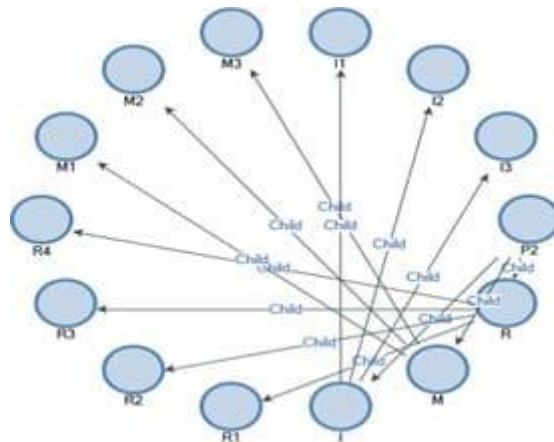


Figure 6. Visualization of Semiotic Components of Participant P2's Mathematics with Nvivo

3. Exposure of Mathematical Semiotic Findings on High Ability Participants

This finding shows that participant (P3) is able to connect various representations flexibly, use mathematical symbols appropriately, and utilize visualization and language to build deep conceptual understanding. This reflects the ability to think abstractly and transition between representations efficiently, as shown in Figure 7.

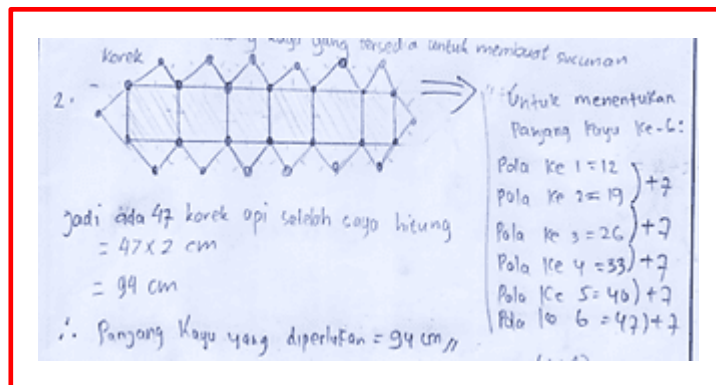


Figure 7. Problem solving results for participant P3

Participant P3's answer shows an attempt to recognize patterns using a certain way to solve problems through drawings and iterations to find the required length of wood. Participant P3 decomposed the 1st pattern to the 6th pattern to find a regular pattern. Participant P3 has also identified a sequence pattern based on constant addition. In this pattern, each subsequent term is obtained by adding 7 to the previous term. Participant P3 successfully identified the pattern or rule that governs the sequence of numbers given. By observing the pattern of constant addition between each consecutive term, participant P3 can understand the pattern and apply it to generate the next value in the sequence. Therefore, participant P3 reflects the ability to recognize patterns. Mathematics is known for its unique and complex language, one of the most prominent features of which is the use of symbols (Quinnell & Carter, 2012). Obtain *computational thinking* to solve problem 2, the results of the P3 participant interview related to the pattern recognition indicator in problem 2 are presented:

- R : Can you find the pattern structure of the number pattern from your answer?
- P3 : I have found a pattern.
- R : Why did you use the picture to find the structure of the pattern?
- P3 : Because when working on number two I had not found the pattern, then I used a picture, making it easier to find the pattern for that problem. So that you can easily see the pattern by drawing.
- R : Why would you make symbols like this?
- P3 : The symbol n indicates the order of the terms in the pattern. The symbol n can be substituted by the values 1,2,3,...etc.
- R : Did you find it easier to solve the problem when deciphering the symbols?
- P3 : Yes, I do. I decomposed the problem to understand the problem. I found the pattern rule, so it's easier to solve the problem when the n th term is asked, let's say the 90th term.
- R : Can you tell me how the picture in the problem relates to the math concept?

P3 : *Maybe, I actually looked for an approximation through the picture first, then looked for the formula with the approximated number. Pictures can help to visualize the problem. If it's just through the story, it's dizzying, if the picture helps visualize it.*

From the interviews conducted, it can be seen that participant P3 managed to identify pattern structure based on the image analysis contained in the problem. Although they did not directly use the picture, P3 participants managed to describe the pattern of order in which the number of matchsticks. Participant P3 can be seen in the answer sheet above using a picture to find the 6th order of matchsticks. Participants visualize number patterns in the form of pictures or diagrams, mathematical symbols, or verbally. P3 participants represented concepts, operations, and variables symbolically. In addition, P3 participants' answers have also used mathematical symbols which show the relationship of certain patterns or sequences in number patterns. The rule or formula expressed by participant P3 is a proof to express the mathematical relationship involved in determining the number of number patterns sought. So that the ability of P3 participants can be said to use conceptual language.

P3 participants can also be said to make mathematical models (R4). Through the iterations written down, participant P3 looked for the number of matchsticks in the 6th pattern and the 40th pattern. Participant P3 also translates the numbers in writing and can also express in verbal form regarding how the iteration written down can be used to solve problem 2 in number 2. This means that the ability of participant P3 can write concepts with language that symbolizes mathematics. This process involves understanding the underlying mathematical relationships and the ability to simplify or generalize these concepts (Maharani, 2020).

Participant P3 can identify patterns or rules that affect the change from one number or iteration to the next number or iteration. Participant P3 can predict that the pattern is the addition of 7 to each number, so that the next number will be 19, then 26, and so on. Participant P3 can use symbols related to problem 2 with prior knowledge. Participant P3 predicted how the pattern changed from one iteration to the next. Participant P3 consistently wrote down the sequence of number patterns involving addition and multiplication to form the next number sequence. Participant P3 identified the rule or pattern that formed the number sequence and then the rule was used to solve problems related to the general rule of the sum of the n terms in question, namely $U_n = 3 + 2n$. Mathematical symbols facilitate the statement of the relationship between the terms in pattern (Torigoe & Gladding, 2011). P3 participants were able to connect their mathematical knowledge with the new concepts taught. This refers to the participant's understanding of mathematical variables and operations with basic concepts, namely multiplication (\times), division ($:$), multiples ($2n$) and addition ($+$). Through this method, P3 participants can identify and use these symbols in depth on the concept of number patterns. This is in line with Bofferding & Wessman-Enzinger (2017) that the various representations of symbols or images made by students are ways for students to present and visualize mathematical problems involving whole numbers. This is relevant to the results of research Winarti et al (2017) that participants have the ability to solve problems by using formulas correctly.

Participant P3 in his answer was able to express the meaning that linked the representation to the object studied. Participants predicted how the pattern changed from one iteration to the

next showing the use of mathematical symbols that reflect the relationship between variables. Participant P3 managed to get the correct answer by describing the number of matchsticks in the 6th order. Hu (2011) dan Mgova (2018) revealed that solving math problems step by step requires abstraction to choose which parts of the data to omit or keep, as shown in Table 3.

Table 3. Findings of Components and Indicators of Participant P3's Mathematical Semiotics

No	Komponen Semiotik	Indikator	Simbol
1	Representation (Semiotics refers to ways of conveying ideas or concepts through spoken language, writing, symbols, pictures, diagrams, models, graphs, and written text).	Solving problems through drawings and iteration	R1
		Describe patterns of order	R2
		represent concepts, operations, and variables symbolically	R3
		Using conceptual language	R4
		Write down concepts with the language of mathematical symbols	R5
2	Mathematical Object (Semiotics refers to the use of mathematical concepts, symbols, images, graphs, or structures)	Make connections between mathematical knowledge and new concepts	M1
		Use of formulas that are appropriate to the problem	M2
		Understand the structure of patterns and use mathematical notation to show patterns	M3
3	Interpretant (Semiotics refers to the process of making meaning. interpretation or concluding from mathematical concepts).	Apply it to broader mathematical situations or problems	I1
		Applying general rules	I2
		Using the formula to find the sum of the 6th term and the 40th term	I3
		Expresses meaning that relates the representation to the object being studied	I4

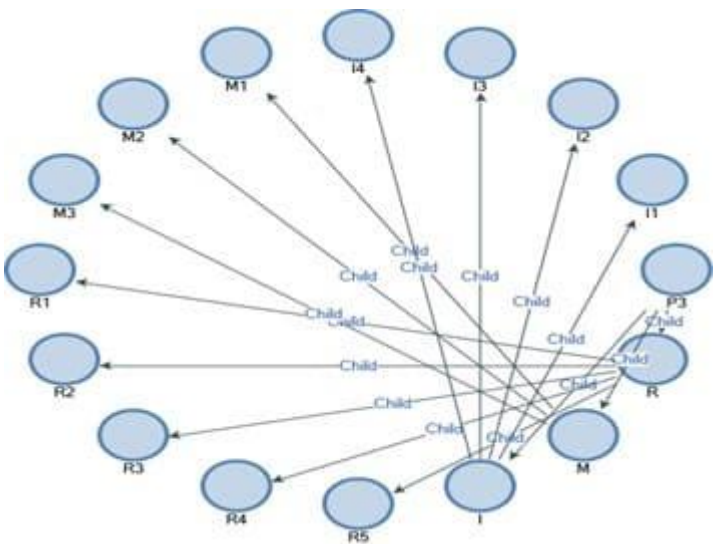


Figure 8. Visualization of Semiotic Components of Mathematics of Participant P3 with Nvivo

Participant P3 present numbers sequentially and systematically when solving problems, but there are errors in applying the algorithm process. This is in line with (Syahputra, 2018; Winarti et al., 2017) that P3 students have not been able to show problem-solving strategies that will be used to obtain solutions, because diverger students have difficulty in detecting problems early on. P3 students have not fully abstracted and generalized but these P3 students

can do algorithms. P3 students are able to identify and write down the information known from the question and without including questions or things asked.

D. CONCLUSION AND SUGGESTIONS

Participant P1 in the mathematical semiotic review was able to recognize, draw, and model number patterns reflecting the process of meaning through mathematical signs. Each element in the rows, notations, and formulas such as $U_n = a + (n-1) \times b$ function as *signs* that connect abstract concepts with symbolic representations. This process shows how P1 constructs mathematical meaning through the transformation of various forms of visual, symbolic, and conceptual representations so as to support the understanding of patterns and their application in problem solving. For P2 participants, they are able to develop understanding of number patterns through identifying and analyzing similarities or differences in patterns, drawing patterns by paying attention to the relationship between elements, and building mathematical models. Students are encouraged to predict the development of patterns, solve problems based on number patterns, and use mathematical notation to represent pattern structures. In addition, students learn to apply the general formula of arithmetic sequence, such as $U_n = a + (n-1) \times b$, to determine a particular term and the number of terms in the sequence, and relate it to a broader contextual situation. Participant P3 showed good ability in solving mathematical problems through visual and iterative approaches, such as drawing and recognizing patterns of regularity. He was able to represent mathematical concepts symbolically, use conceptual language appropriately, and write concepts in the form of mathematical symbols. P3 also shows understanding in connecting existing mathematical knowledge with new concepts. In practice, P3 is able to select and use formulas that are appropriate to the context of the problem, and apply general rules such as $U_n = a + (n-1) \times b$ to determine the 6th and 40th terms of a sequence. P3 can understand the structure of the pattern, use it in a broader situation, and express the meaning that links the mathematical representation with the object being studied. The participants P1, P2, and P3 have similarities in terms of the ability to recognize patterns, use mathematical representations, and apply the general formula of the arithmetic sequence $U_n = a + (n-1) \times b$ in problem solving. All three showed an understanding of the pattern structure and could relate mathematical concepts to a broader context. The difference lies in the approach used: P1 emphasizes the construction of meaning through the transformation of various forms of representation semiotically, P2 focuses on the analysis and development of patterns visually and contextually, while P3 relies on a visual-iterative approach and strengths in the use of symbols and conceptual language to connect new concepts with prior knowledge. With a deeper understanding of how mathematical symbols are understood and interpreted by students, semiotic research can play a significant role in the development of mathematics curriculum. This research allows educators to identify the best ways to convey mathematical concepts clearly and meaningfully. Thus, the resulting curriculum can be more oriented towards conceptual understanding, ensuring that students do not simply memorize procedures, but truly understand the essence of each mathematical concept being taught. In addition, a curriculum based on semiotic research can encourage more effective learning methods, helping students develop critical thinking and problem-solving skills. This, in turn, can increase student engagement and make learning mathematics more interesting and relevant.

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