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Dynamic Analysis of COVID-19 Model with Quarantine and Isolation

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ABSTRACT

This study discusses the dynamic analysis of the COVID-19 model with quarantine and isolation. The population in this model is divided into seven subpopulations: subpopulation of susceptible, exposed, asymptomatic, symptomatic, quarantine, isolated and recovered. Two equilibrium points were obtained based on the analysis results, namely the disease-free and endemic equilibrium points. The existence and local stability of the equilibrium point depends on the value of the basic reproduction number R_0. Then, the point of disease-free equilibrium always exists, and the point of endemic equilibrium exists when it meets $R_0 > 1$. The point of disease-free equilibrium is locally asymptotically stable when it satisfies $R_0 < 1$ and the endemic equilibrium point is locally asymptotically stable with conditions. Furthermore, numerical simulations are carried out to determine the model's behavior using the fourth-order Runge-Kutta method. The numerical simulation obtained supports the dynamic analysis results. Finally, the graphical results are presented. The findings here suggest that human-to-human contact is a potential cause of the COVID-19 outbreak. Therefore, quarantine of susceptible and exposed subpopulations can reduce the risk of infection. Likewise, isolation of infected subpopulations can reduce the risk of spreading COVID-19.

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A. INTRODUCTION

Coronavirus is a virus that can cause respiratory infections ranging from the common cold to SARS (Severe Acute Respiratory Syndrome) and MERS (Middle East Respiratory Syndrome) (WHO, 2020a). The three significant outbreaks due to the Coronavirus that have occurred are the SARS outbreak in China (2003), the MERS (camel flu) outbreak in Middle Eastern countries and was reported in Saudi Arabia (2012), and the MERS outbreak in South Korea (2015) (Kucharski et al., 2020; Rao et al., 2021; Tahir et al., 2019; Usaini et al., 2019). In addition, the newest type of Coronavirus found was named Novel-Coronavirus or 2019n-Cov as the cause of the COVID-19 disease.

COVID-19 first appeared and was identified in Wuhan-Hubei Province, China, around December 2019 (Rao et al., 2021). Furthermore, the virus spreads to various countries rapidly through individuals who have had a history of travel to Wuhan (Chen et al., 2020; Tang et al., 2020; WHO, 2020a; Yousefpour et al., 2020). Before being reported and informed to the public, a doctor named Li Wenliang had provided information about the emergence of this virus because seven patients from the local seafood market were diagnosed with a SARS-like disease and were quarantined in a hospital (Nainggolan, 2020).

On January 10, 2020, the World Health Organization (WHO) released various guidelines or temporary guidelines for all countries, such as observing potentially infected people, collecting and testing samples (tracing), treating patients, controlling and reducing the burden of COVID infection in hospital (Rois et al., 2021b).

The symptoms experienced are usually mild and appear slowly. However, humans can be infected with mild or no symptoms and become infected with serious symptoms, namely experiencing fever or shortness of breath or coughing accompanied by difficulty breathing and chest pain (WHO, 2020a). COVID-19 is spread through respiratory droplets secreted by an infected individual either with mild or no symptoms and serious symptoms or with symptoms. WHO also stated that individuals without symptoms can transmit the Coronavirus through the air and even from the environment.

The government has made all efforts to control COVID-19, such as social distancing, tracing, working from home, and stringent territorial restrictions (Rois et al., 2021a). Then, how to control every time an outbreak occurs in an area without vaccines or treatment, individual isolation, and quarantine is the most effective way (Usaini et al., 2019). According to WHO (2020b), quarantine is defined as restricting activities or segregating susceptible individuals as long as there is no essential need to leave the house. Then, individuals who have a history of contact with individuals infected with COVID-19 or have a history of travelling to an area where local transmission has occurred and separate themselves by staying at home during the incubation period (2 weeks) are also included in the quarantine group. Furthermore, isolation is defined as the separation of a sick or infected individual from other individuals either directly in the hospital or at home (self-isolation) with medical personnel monitoring.

WHO recognizes mathematical models play an important role in informing decisions or solutions to health decision-makers (doctors or health professionals) and policymakers (government) (Tang et al., 2020). One approach to explaining problems in the real world is to formulate real problems into mathematical models. The assumptions used on problems that exist in the environment can be transformed into a mathematical model. After the mathematical model is obtained, it can be solved mathematically and applied again to real problems. Therefore, researchers are moving together to research COVID-19 from all different aspects according to the area studied with mathematical modeling. Some solutions can be taken to destroy or reduce the increase in the number of infected.

The mathematical model that can describe the spread of disease is the *SIR* model. Kermack and McKendrick first introduced the SIR model (1927), then became a reference source and played an important role in developing mathematics about disease transmission (Müller & Kuttler, 2015; Murray, 2002). Furthermore, science has made *SIR* models a reference for many scientists to make mathematical models of disease spread more specifically. This mathematical model is used as quantitative information in disease and provides benefits for policy-making and disease outbreaks. Several studies related to the disease spread, such as a study on the Coronavirus, resulted in SARS (Feng, 2007) and MERS (Tahir et al., 2019; Usaini et al., 2019). Then the Coronavirus developed into a virus known as the COVID-19 virus and became a hot topic in 2020.

Soewono (2020) models the initial spread of COVID-19 by applying the *SEIR* model, which consists of four subpopulations: S (susceptible), E (exposed), I (symptomatic), and R (recovered). Furthermore, Belgaid et al. (2020) added subpopulation A (asymptomatic), so that the population is divided into five subpopulations: S, E, A, I, and R. The cause of the subpopulation being infected without symptoms is due to WHO information that individuals infected with COVID-19 can be infected by showing symptoms and some who do not show symptoms. Another study, Zeb et al. (2020), added the isolation subpopulation (H) and divided the population into five subpopulations: S, E, I, H, and R. This is based on the latest information that infected individuals will spread to surrounding individuals because they are not given isolation measures. A study on COVID-19 was also carried out by Jia et al. (2020) involving quarantine (Q) and isolation (H) subpopulations so that the model presented divides the population into seven subpopulations: S, E, A, I, Q, H, and R. The model made is also based on the latest information from WHO, that susceptible individual should be quarantined first to reduce further spread.

In this study, the COVID-19 model was constructed by combining research by Belgaid et al. (2020), Zeb et al. (2020), and Jia et al. (2020), which aims to refine the model such as the conditions reported by WHO. Some reports from WHO are that infected individuals without symptoms can transmit COVID-19, healthy individuals and exposed individuals are better off quarantined, and this can also reduce the burden of labor in hospitals, individuals who are quarantined during the incubation period with no symptoms can be said to be recovered, and if there are symptoms, isolation is carried out, and infected individuals, both symptomatic and asymptomatic, in order to recover, are isolated first. Population in this model is divided into seven subpopulations: S, E, A, I, Q, H, and R. The model that has been formed is then carried out dynamic analysis such as determining the positivity and limitations of the solution, the point of equilibrium, finding the basic reproduction number R_0 and analyze its stability. Furthermore, numerical simulation is carried out using the fourth-order Runge-Kutta method. Finally, the graphical results are presented. The findings here suggest that human-to-human contact is a potential cause of the COVID-19 outbreak. Therefore, quarantine of susceptible and exposed subpopulations can reduce the risk of infection and isolation of infected subpopulation can reduce the risk of spreading COVID-19.

B. METHODS

In this study, several stages were carried out as follows:

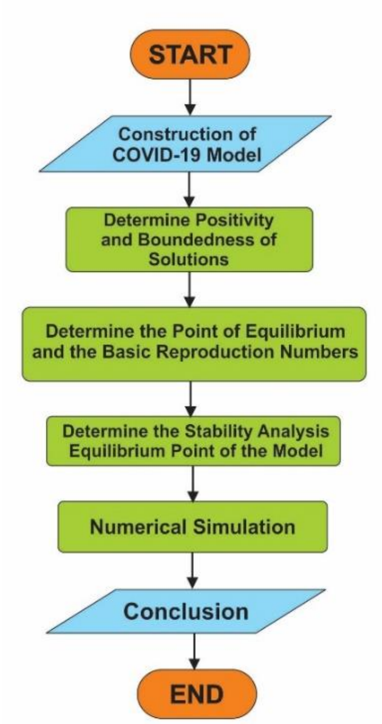

Figure 1. Flowchart of discussion of the dynamic analysis

The explanation of the stages of the research are as follows:

1. Construction of COVID-19 Model

   This model consists of seven subpopulations: S (susceptible), E (exposed), A (asymptomatic), I (symptomatic), Q (quarantine), H (isolation), and R (recovered). $SEIAQHR$ model is based on conditions in Indonesia and a combination of research by Belgaid et al. (2020), Jia et al. (2020), and Zeb et al. (2020).

2. Point of equilibrium

   Definition 1. (Layek, 2015) The point $\vec{x}^* = (x_1^*, \dots, x_n^*)$ is called the critical point of the system of equations $\frac{d\vec{x}}{dt} = \vec{g}(\vec{x}), \vec{x} \in \mathbb{R}^n$ if it satisfies $\vec{g}(\vec{x}) = 0$.

3. The Basic Reproduction Number (R_0)

   R_0 is the average number of newly infected individuals caused by one infected individual in a susceptible population. R_0 is used to determine the spread of disease and predict a population can endanger or not. The conditions that arise are among the following possibilities (Heffernan et al., 2005):

   a. If $R_0 < 1$ means that an infected individual can transmit the disease on average less than one newly infected individual, so it can be predicted that the infection will disappear and there will be no spread of disease, or it is called disease-free.

   b. If $R_0 > 1$ means that an infected individual can transmit the disease on average more than one newly infected individual, causing it to spread the disease easily.

   Furthermore, the next-generation matrix (NGM) is one method for determining R_0. In the NGM method, the compartment model used is the infected compartment (Brauer & Castillo-Chavez, 2012). Furthermore, the infected compartment model can be expressed as:

$$\frac{dx}{dt} = f - v \tag{1}$$

   So, obtained

$$F = \frac{\partial f(E_0)}{\partial x},\ V = \frac{\partial v(E_0)}{\partial x}. \tag{2}$$

   NGM is defined as $M = FV^{-1}$ and R_0 can be obtained from $R_0 = \rho(M)$, where $\rho(M)$ is the spectral radius of the matrix M, which is the largest modulus of the eigenvalues of the matrix M.

4. Stability analysis

   Theorem 1. (Layek, 2015) Let $\lambda_1, \lambda_2, \dots \lambda_n$ be the eigenvalues of the matrix M. The stability criterion are:

   a. Asymptotically stable, if all eigenvalues have a negative real part,

   b. Unstable if there is at least one eigenvalue that has a positive real part.

   Furthermore, the Routh-Hurwitz criterion is an alternative for determining eigenvalues, apart from the jacobian matrix at the equilibrium point. The characteristic equations system in the form of a polynomial is given as follows (Murray, 2002):

$$f(\lambda) = a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0, \tag{3}$$

with coefficients a_i, $i = 1, 2, \ldots, n$ and $a_n \neq 0$. The Routh-Hurwitz criterion was used to determine the real part of the characteristic root equation of the matrix M. The root of the characteristic equation (3) has a negative real part if and only if $a_n > 0$ and

$$\Delta_1 = |a_1| > 0,$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0,$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0,$$

$$\Delta_k = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & a_{2k-1} \\ a_0 & a_2 & a_4 & \cdots & a_{2k-2} \\ 0 & a_1 & a_3 & \cdots & a_{2k-3} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & a_k \end{vmatrix} > 0, k = 1, 2, n, \text{ dan } a_i = 0 \text{ untuk } i > n.$$

Furthermore, numerical simulations used in this study used the fourth-order Runge-Kutta method in the Matlab R2017a software.

C. RESULT AND DISCUSSION

1. Model Formulation

COVID-19 model can be described in the compartment diagram in Figure 2 as follows

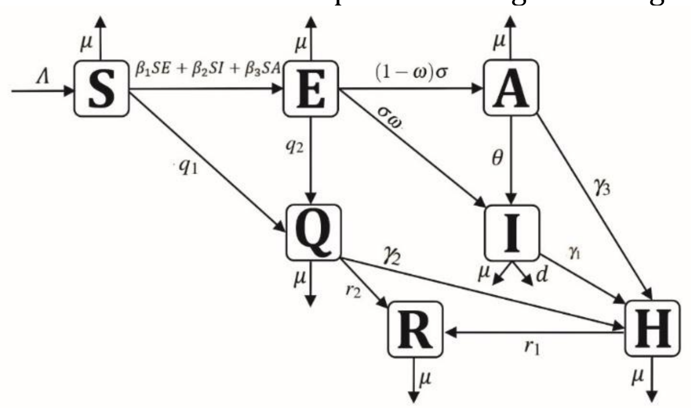

Figure 2. COVID-19 compartment model

Based on Figure 2, the following COVID-19 model is obtained

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - S\left(\beta_1 E + \beta_2 I + \beta_3 A + \mu + q_1\right),\\
\frac{dE}{dt} &= S\left(\beta_1 E + \beta_2 I + \beta_3 A\right) - E\left(\mu + \sigma + q_2\right),\\
\frac{dA}{dt} &= (1-\omega)\sigma E - A\left(\gamma_3 + \theta + \mu\right),\\
\frac{dI}{dt} &= \sigma\omega E + \theta A - I\left(\gamma_1 + \mu + d\right),\\
\frac{dQ}{dt} &= q_1 S + q_2 E - Q\left(\mu + \gamma_2 + r_2\right),\\
\frac{dH}{dt} &= \gamma_1 I + \gamma_2 Q + \gamma_3 A - H\left(\mu + r_1\right),\\
\frac{dR}{dt} &= r_1 H + r_2 Q - \mu R.
\end{aligned} \tag{4}
$$

Description
Λ : Human recruitment rate,
β_1: Contact rate of susceptible subpopulations with exposed,
β_2: Subpopulation's contact rate is susceptible to symptomatic,
β_3: Subpopulation's contact rate is susceptible to asymptomatic,
μ : Natural death rate,
d : Death due to COVID-19 rate,
q_1: Rate of susceptible individuals quarantined,
q_2: Rate of exposed individuals quarantined,
α : Rate of return of individuals to susceptible subpopulations after quarantine,
γ_1: Isolation rate from symptomatic subpopulations,
γ_2: Isolation rate from quarantine subpopulation,
γ_3: Isolation rate from asymptomatic subpopulation,
θ : Rate of asymptomatic individuals becomes symptomatic,
r_1: Recovery rate after isolation,
r_2: Recovery rate after quarantine,
σ : Rate of progression from exposed to symptomatic,
ω: Proportion of becoming infected is symptomatic.

2. Model Analysis

a. Basic properties

If N is the total population, then $N = S + E + A + I + Q + H + R$. Thus obtained

$$
\begin{aligned}
\frac{dN}{dt} &= \frac{dS}{dt} + \frac{dE}{dt} + \frac{dA}{dt} + \frac{dI}{dt} + \frac{dQ}{dt} + \frac{dH}{dt} + \frac{dR}{dt},\\
&= \Lambda - dI - \mu N,\\
&\le \Lambda - \mu N,
\end{aligned}
$$

and obtained

$$N(t) \le \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right)e^{-\mu t}.$$

As a result, for $t \to \infty$ then $\lim_{t\to\infty} N(t)\frac{\Lambda}{\mu}$. From the solution above, it can be concluded that the model has a limited solution with the solution area is

$$\Omega = \left\{ (S, E, A, I, Q, H, R) \mid N(t) \le \frac{\Lambda}{\mu} \right\}.$$

Theorem 2. (Positivity) Suppose the solution is related to the initial conditions $(S(0), E(0), A(0), I(0), Q(0), H(0), R(0)) \in \mathbb{R}_+^7$ is $(S(t), E(t), A(t), I(t), Q(t), H(t), R(t))$. So that for model (4), the positively invariant set is $\mathbb{R}_+^7$.

Proof. Suppose that $\varepsilon = \beta_1 E + \beta_2 I + \beta_3 A$, the first equation of model (4) results in

$$\frac{dS}{dt} = \Lambda - S(\varepsilon + \mu + q_1). \qquad (5)$$

Suppose a solution exists from the model (4) for the interval $T \in [0; +\infty]$, then equation (5) can be solved, $\forall t \in T$, as

$$\frac{dS}{dt} + S(\varepsilon + \mu + q_1) = \Lambda,$$

$$\frac{d}{dt}\left(e^{(\mu+q_1)t+\int_0^t \varepsilon\, ds} S \right) = \Lambda e^{(\mu+q_1)t+\int_0^t \varepsilon\, ds},$$

which results in

$$e^{(\mu+q_1)t+\int_0^t \varepsilon\, ds} S - S(0) = \int_0^t \Lambda e^{(\mu+q_1)w+\int_0^w \varepsilon\, dz} dw,$$

$$e^{(\mu+q_1)t+\int_0^t \varepsilon\, ds} S = S(0) + \int_0^t \Lambda e^{(\mu+q_1)w+\int_0^w \varepsilon\, dz} dw,$$

$$S = S(0) e^{-(\mu+q_1)t-\int_0^t \varepsilon\, ds} + e^{-(\mu+q_1)t-\int_0^t \varepsilon\, ds} \times \int_0^t \Lambda e^{(\mu+q_1)w+\int_0^w \varepsilon\, dz} dw \ge 0.$$

Therefore, $\forall t \in T, S(t) \ge 0$. Then, the second equation of model (4) is obtained

$$\frac{dE}{dt} = S(\beta_1 E + \beta_2 I + \beta_3 A) - E(\mu + \sigma + q_2) \ge -E(\mu + \sigma + q_2),$$

or

$$\frac{dE}{dt} \ge -E(\mu + \sigma + q_2),$$

and can be written as

$$\frac{dE}{E} \ge -(\mu + \sigma + q_2)dt \qquad (E \ne 0).$$

then integrate, so get

$$\ln E \ge -(\mu + \sigma + q_2)t + c,$$

$$E \ge e^{-(\mu+\sigma+q_2)t+c},$$

$$E \ge Ce^{-(\mu+\sigma+q_2)t},$$

when $t = 0$,

$$E \ge E(0)e^{-(\mu+\sigma+q_2)0} \ge 0.$$

Hence, $\forall t, I(t) \ge 0$. Similarly, the third equation of model (4) is shown

$$\frac{dA}{dt}=(1-\omega)\sigma E-A(\gamma_3+\theta+\mu)\geq -A(\gamma_3+\theta+\mu),$$

or

$$\frac{dA}{dt}\geq -A(\gamma_3+\theta+\mu),$$

and can be written as

$$\frac{dA}{A}\geq -(\gamma_3+\theta+\mu)dt \qquad (A\neq 0).$$

then integrate, so get

$$\ln A\geq -(\gamma_3+\theta+\mu)t+c,$$

$$A\geq e^{-(\gamma_3+\theta+\mu)t+c},$$

$$A\geq Ce^{-(\gamma_3+\theta+\mu)t},$$

when $t=0$,

$$A\geq A(0)e^{-(\gamma_3+\theta+\mu)0}\geq 0,$$

Hence, $\forall t, A(t)\geq 0$. Furthermore, in the same way, it can be shown that $I(t)$, $Q(t)$, $H(t)$, $R(t)$ are positive in the given interval.

b. Equilibrium point and the basic reproduction number $(\boldsymbol{R_0})$

Equilibrium point for the system of equations obtained when $\frac{dS}{dt}=0, \frac{dE}{dt}=0, \frac{dA}{dt}=0, \frac{dI}{dt}=0, \frac{dQ}{dt}=0, \frac{dH}{dt}=0, \frac{dR}{dt}=0$.

$$\begin{aligned}
&\Lambda-S(\beta_1E+\beta_2I+\beta_3A+h_1)=0,\\
&S(\beta_1E+\beta_2I+\beta_3A)-h_2E=0,\\
&(1-\omega)\sigma E-h_4A=0,\\
&\sigma\omega E+\theta A-h_3I=0,\\
&q_1S+q_2E-h_5Q=0,\\
&\gamma_1I+\gamma_2Q+\gamma_3A-h_6H=0,\\
&r_1H+r_2Q-\mu R=0,
\end{aligned} \tag{6}$$

with $h_1=\mu+q_1$, $h_2=\mu+\sigma+q_2$, $h_3=\gamma_1+\mu+d$, $h_4=\gamma_3+\theta+\mu$, $h_5=\mu+\gamma_2+r_2$, and $h_6=\mu+r$.

Some manipulations of algebraic obtain two solutions of the system (6). The disease-free equilibrium point (K^0) is one of them as follows:

$$K^0=(S^0,E^0,A^0,I^0,Q^0,H^0,R^0)=\left(\frac{\Lambda}{h_1},0,0,0,\frac{q_1\Lambda}{h_1h_5},\frac{\gamma_2q_1\Lambda}{h_1h_5h_6},\frac{r_1\gamma_2q_1\Lambda+r_2q_1h_6\Lambda}{h_1h_5h_6\mu}\right),$$

Next, calculate R_0 of the model (4). After that, the second point of equilibrium will be discussed. R_0 is obtained using the NGM method.

Suppose $x=(E,I,A)$ and can be expressed to be

$$\frac{dx}{dt}=f-v, \tag{7}$$

where

$$f=\begin{bmatrix} S\left(\beta_1 E+\beta_2 I+\beta_3 A\right) \\ 0 \\ 0 \end{bmatrix},\quad v=\begin{bmatrix} h_2 E \\ -\sigma\omega E-\theta A+h_3 I \\ -(1-\omega)\sigma E+h_4 A \end{bmatrix}. \qquad (8)$$

Then, the partial derivatives of matrices f and v at point K^0 are

$$F=\begin{bmatrix} \Lambda\beta_1/h_1 & \Lambda\beta_2/h_1 & \Lambda\beta_3/h_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad V=\begin{bmatrix} h_2 & 0 & 0 \\ -\sigma\omega & h_3 & -\theta \\ -(1-\omega)\sigma & 0 & h_4 \end{bmatrix}. \qquad (9)$$

The NGM by calculating $M=FV^{-1}$. Furthermore, the basic reproduction number is always positive because all given parameters are positive, and it is obtained from the spectral radius of the M matrix as follows

$$R_0=\rho(M)=\frac{\Lambda}{h_1h_2}\left(\beta_1+\frac{\beta_2\sigma\left(h_4\omega+\theta(1-\omega)\right)}{h_3h_4}+\frac{\beta_3\sigma(1-\omega)}{h_4}\right). \qquad (10)$$

Theorem 3. For model (4), if $R_0>1$ then there exists a unique positive endemic equilibrium point K^*.

*Proof***.** Using some manipulations of algebraic, and the second point of equilibrium of model (6) is obtained

$$S^*=\frac{\Lambda}{h_1R_0},\ E^*=\frac{\Lambda}{h_2}\left(1-\frac{1}{R_0}\right),\ A^*=\left(\frac{\Lambda\sigma(1-\omega)}{h_2h_4}\right)\left(1-\frac{1}{R_0}\right),$$

$$I^*=\left(\frac{\Lambda\left(\left(h_4\sigma\omega+\theta\sigma(1-\omega)\right)\right)}{h_2h_3h_4}\right)\left(1-\frac{1}{R_0}\right),\ Q^*=\frac{\Lambda}{h_5R_0}\left(\frac{q_1}{h_1}+\frac{q_2}{h_2}(R_0-1)\right),$$

$$H^*=\frac{\Lambda\sigma}{h_2h_4h_6}\left(1-\frac{1}{R_0}\right)\left(\frac{\gamma_1h_4\omega}{h_3}+\frac{(1-\omega)}{h_3}(\gamma_1\theta+h_3\gamma_3)\right)+\frac{\Lambda\gamma_2}{h_5h_6R_0}\left(\frac{q_1}{h_1}+\frac{q_2}{h_2}(R_0-1)\right),$$

$$R^*=\left(1-\frac{1}{R_0}\right)\frac{\Lambda\sigma r_1}{\mu h_2h_4h_6}\left(\frac{\gamma_1h_4\omega}{h_3}+\frac{(1-\omega)}{h_3}(\gamma_1\theta+h_3\gamma_3)\right)+\frac{\Lambda\gamma_2r_1}{\mu h_5h_6R_0}\left(\frac{q_1}{h_1}+\frac{q_2}{h_2}(R_0-1)\right)$$
$$+\frac{\Lambda r_2}{\mu h_5R_0}\left(\frac{q_1}{h_1}+\frac{q_2}{h_2}(R_0-1)\right),$$

It is clear from the values of $S^*, E^*, A^*, I^*, Q^*, H^*,$ and R^* that is, if $R_0>1$ then there exists a unique positive endemic equilibrium point K^*.

c. Stability analysis

Based on the linearization process in the model (4), the Jacobi matrix is obtained as follows:

$$J=\begin{bmatrix} -(\beta_1E+\beta_2I+\beta_3A+h_1) & -\beta_1S & -\beta_2S & -\beta_3S & 0 & 0 & 0 \\ \beta_1E+\beta_2I+\beta_3A & \beta_1S-h_2 & \beta_2S & \beta_3S & 0 & 0 & 0 \\ 0 & \sigma\omega & -h_3 & \theta & 0 & 0 & 0 \\ 0 & (1-\omega)\sigma & 0 & h_4 & 0 & 0 & 0 \\ q_1 & q_2 & 0 & 0 & -h_5 & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_3 & \gamma_2 & -h_6 & 0 \\ 0 & 0 & 0 & 0 & r_2 & r_1 & -\mu \end{bmatrix}. \quad (11)$$

First, the Jacobi matrix equation (11) at the point K^0 is

$$J(K^0)=\begin{bmatrix} -h_1 & \frac{-\beta_1\Lambda}{h_1} & \frac{-\beta_2\Lambda}{h_1} & \frac{-\beta_3\Lambda}{h_1} & 0 & 0 & 0 \\ 0 & \frac{\beta_1\Lambda}{h_1}-h_2 & \frac{\beta_2\Lambda}{h_1} & \frac{\beta_3\Lambda}{h_1} & 0 & 0 & 0 \\ 0 & \sigma\omega & -h_3 & \theta & 0 & 0 & 0 \\ 0 & (1-\omega)\sigma & 0 & h_4 & 0 & 0 & 0 \\ q_1 & q_2 & 0 & 0 & -h_5 & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_3 & \gamma_2 & -h_6 & 0 \\ 0 & 0 & 0 & 0 & r_2 & r_1 & -\mu \end{bmatrix}. \quad (12)$$

Then from equation (12), it can be concluded that the eigenvalues are $\lambda_1=-h_1<0$, $\lambda_2=-h_5<0$, $\lambda_3=-h_6<0$, and $\lambda_4=-\mu<0$. Therefore, the stability of the point K^0 depends on

$$M_1=\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & -a_{22}-\lambda & a_{23} \\ a_{31} & 0 & -a_{33}-\lambda \end{vmatrix}=0, \quad (13)$$

with
$a_{11}=\frac{\beta_1\Lambda}{h_1}-h_2$, $a_{12}=\frac{\beta_2\Lambda}{h_1}$, $a_{13}=\frac{\beta_3\Lambda}{h_1}$, $a_{21}=\sigma\omega$, $a_{22}=h_3$, $a_{23}=\theta$, $a_{31}=(1-\omega)\sigma$, and $a_{33}=h_4$.

So that it is obtained

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0, \quad (14)$$

where

$$a_1=h_2(1-R_0)+\frac{\Lambda\beta_2\sigma(h_4\omega+\theta(1-\omega))}{h_1h_3h_4}+\frac{\Lambda\beta_3\sigma(1-\omega)}{h_1h_4}+h_3+h_4,$$

$$a_2=h_2h_4(1-R_0)+h_2h_3(1-R_0)+h_3h_4+\frac{\Lambda\beta_2\sigma(h_4\omega+\theta(1-\omega))}{h_1h_3}+\frac{\Lambda\beta_2\sigma\theta(1-\omega)}{h_1h_4}$$
$$+\frac{\Lambda\beta_3\sigma h_3(1-\omega)}{h_1h_4},$$

$$a_3=h_2h_3h_4(1-R_0),$$

and

$$a_1a_2 - a_3 = h_2\left(1-R_0\right)^2\left[h_2h_4 + h_2h_3 + 2h_3h_4 + h_3^2 + h_4^2\right] + \frac{\Lambda\sigma h_2\left(1-\omega\right)\left(1-R_0\right)}{h_1h_4}\left(\beta_2\theta + 2\beta_3h_3 + \beta_3h_4\right)$$
$$+\frac{\Lambda\beta_2\sigma\left(h_4\omega+\theta\left(1-\omega\right)\right)}{h_1}\left[2+Y+\frac{h_4}{h_1h_3}\right]+\frac{\Lambda\beta_2\sigma h_2\left(h_4\omega+\theta\left(1-\omega\right)\right)\left(1-R_0\right)}{h_1}\left(\frac{2}{h_3}+\frac{1}{h_4}\right)$$
$$+h_3^2h_4 + h_3h_4^2,$$

where

$$Y = \frac{\Lambda\beta_2\sigma\left(h_4\omega+\theta\left(1-\omega\right)\right)}{h_1h_3^2h_4} + \frac{\Lambda\sigma\left(1-\omega\right)}{h_1h_3h_4}\left(\frac{\beta_2\theta}{h_4} + \frac{\beta_3h_3}{h_4} + \beta_3\right).$$

The characteristic equation (10) has real roots that are negative if it meets the Routh-Hurwitz criteria. If $R_0 < 1$, then $a_1 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$. Based on Theorem 1, if $R_0 < 1$ then the point K^0 is obtained as locally asymptotically stable. If $R_0 > 1$ then the value of $a_3 < 0$, so the Routh-Hurwitz criterion is not met. As a result, if $R_0 > 1$ then the point K^0 is locally unstable.

Second, the Jacobi matrix equation (10) at the point K^* is

$$J\left(K^*\right) = \begin{bmatrix} -\left(\beta_1E^* + \beta_2I^* + \beta_3A^* + h_1\right) & -\beta_1S^* & -\beta_2S^* & -\beta_3S^* & 0 & 0 & 0 \\ \beta_1E^* + \beta_2I^* + \beta_3A^* & \beta_1S^* - h_2 & \beta_2S^* & \beta_3S^* & 0 & 0 & 0 \\ 0 & \sigma\omega & -h_3 & \theta & 0 & 0 & 0 \\ 0 & \left(1-\omega\right)\sigma & 0 & h_4 & 0 & 0 & 0 \\ q_1 & q_2 & 0 & 0 & -h_5 & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_3 & \gamma_2 & -h_6 & 0 \\ 0 & 0 & 0 & 0 & r_2 & r_1 & -\mu \end{bmatrix}. \quad (15)$$

So that the eigenvalues are obtained $\lambda_1 = -h_5 < 0$, $\lambda_2 = -h_6 < 0$, and $\lambda_3 = -\mu < 0$. Therefore, the stability of the point K^* depends on

$$M_2 = \begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ 0 & a_{32} & a_{33}-\lambda & a_{34} \\ 0 & a_{42} & 0 & a_{44}-\lambda \end{vmatrix} \quad (16)$$

with $a_{11} = -(\beta_1E^* + \beta_2I^* + \beta_3A^* + h_1)$, $a_{12} = -\beta_1S^*$, $a_{13} = -\beta_2S^*$, $a_{14} = -\beta_3S^*$, $a_{21} = \beta_1E^* + \beta_2I^* + \beta_3A^*$, $a_{22} = \beta_1S^* - h_2$, $a_{23} = \beta_2S^*$, $a_{24} = \beta_3S^*$, $a_{32} = \sigma\omega$, $a_{33} = -h_3$, $a_{34} = \theta$, $a_{42} = (1-\omega)\sigma$, and $a_{44} = -h_4$.

So that it is obtained

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \quad (17)$$

where

$$a_1 = h_1 R_0 + \frac{\Lambda\beta_2\sigma\left(h_4\omega+\theta(1-\omega)\right)}{h_1h_3h_4R_0} + \frac{\Lambda\beta_3\sigma(1-\omega)}{h_1h_4R_0} + h_3 + h_4,$$

$$a_2 = h_1\left(R_0-1\right)\left(h_2+h_3+h_4\right)+h_1h_3+h_1h_4+h_3h_4+\frac{\Lambda\beta_2\sigma\left(h_4\omega+\theta(1-\omega)\right)}{h_3R_0}\left[\frac{1}{h_4}+\frac{1}{h_1}\right]$$

$$+\frac{\Lambda\beta_3\sigma(1-\omega)}{h_4R_0}\left[\beta_3+\frac{\beta_2\theta}{h_1}+\frac{h_3\beta_3}{h_1}\right],$$

$$a_3 = h_1\left(R_0-1\right)\left(h_2h_3+h_2h_4+h_3h_4\right)+h_1h_3h_4+\frac{\Lambda\beta_2\sigma\left(h_4\omega+\theta(1-\omega)\right)}{h_3R_0}+\frac{\Lambda\sigma h_1(1-\omega)}{h_1h_4R_0}\left[h_3\beta_3+\beta_2\theta\right],$$

$$a_4 = \left(1-\frac{1}{R_0}\right)\left[\Lambda\beta_1h_3h_4+\Lambda\beta_2\sigma\left(h_4\omega+\theta(1-\omega)\right)+\Lambda\sigma\beta_3h_3(1-\omega)\right].$$

It can be concluded that if $R_0 > 1$ then $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0$ and if $R_0 < 1$ then $a_4 < 0$. Based on the characteristic equation (16), it is not easy to get the root value. Therefore, the stability properties of the point K^* were obtained using the Routh-Hurwitz criterion. Furthermore, the point K^* is asymptotically stable if and only if it meets the following criteria:

1) $a_1 > 0$,
2) $a_4 > 0$,
3) $a_1a_2 - a_3 > 0$,
4) $a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0$.

Criteria (1) and (2) are met, so that point K^* is locally asymptotically stable if the following criteria:

1) $a_1a_2 - a_3 > 0$,
2) $a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0$.

d. Numerical simulations

Numerical simulation of the solution model (4) is carried out to illustrate the analysis results obtained. The parameter values used in this simulation are shown in Table 1 and Table 2.

Table 1. Parameters used for numerical simulation

Parameter	Parameter Value	Source
Λ	1.685	Assumed
μ	$3.9139 \text{ X } 10^{-5}$	(Aldila et al., 2020)
σ	0.196	Assumed
θ	0.01	(Aldila et al., 2020)
ω	0.4	(Aldila et al., 2020)
d	0.087	(Sasmita et al., 2020)
q_1	0.15	Assumed
q_2	0.1	(Rois et al., 2021a)
β_1	0.0067	Assumed
β_2	0.0012203	Assumed
β_3	0.036	Assumed

Parameter	Parameter Value	Source
γ_1	0.083	(Aldila et al., 2020)
γ_2	0.01	(Aldila et al., 2020)
γ_3	0.2435	(Aldila et al., 2020)
r_1	0.1	(Aldila et al., 2020)
r_2	0.125	(Aldila et al., 2020)

1) Numeric simulation for $R_0 < 1$

In this simulation, it is shown that the stability of the point K^0 from the parameter values given in Table 1, is obtained $R_0 = 0.910223138 < 1$.

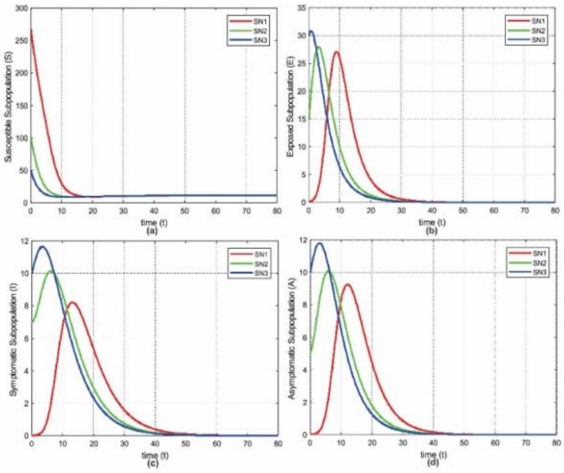

Figure 3. Solution graph for $R_0 < 1$

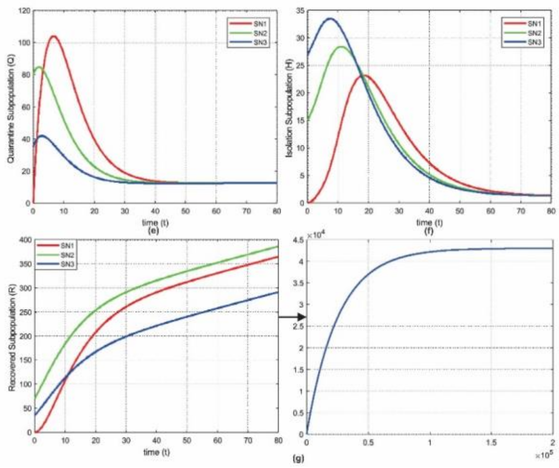

Figure 4. Next solution graph for $R_0 < 1$

In Figure 3 and Figure 4, the graph of the solution is shown when $R_0 < 1$ with three different initial values, namely

$SN_1 = (267.6976, 0.0642, 0.0106, 0.0317, 0.0102, 0.0082, 0.0016)$,

$SN_2 = (100, 15, 7, 5, 80, 15, 70)$,

$SN_3 = (50, 30, 10, 10, 35, 27, 35)$.

The simulation results with several initial values show that the solution graphs to the point K^0. This means that after a long period, no one has been infected with COVID-19. The results of the numerical simulation support the results of the analysis, if $R_0 < 1$, then K^0 is locally asymptotically stable.

2) Numeric simulation for $R_0 > 1$

Shows the stability of the point K^*, the parameter values in the table above are used except $q_1 = 0.09, \beta_1 = 0.01, \beta_2 = 0.1, \beta_3 = 0.1$, and $\sigma = 0.0196$, so that it is obtained the value $R_0 = 3.05362439 > 1$.

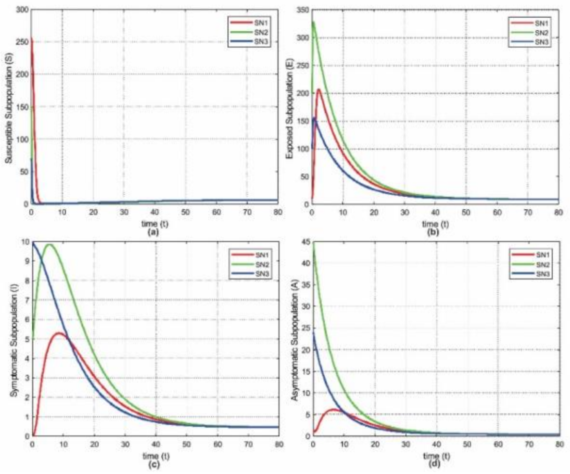

Figure 5. Solution graph for $R_0 > 1$

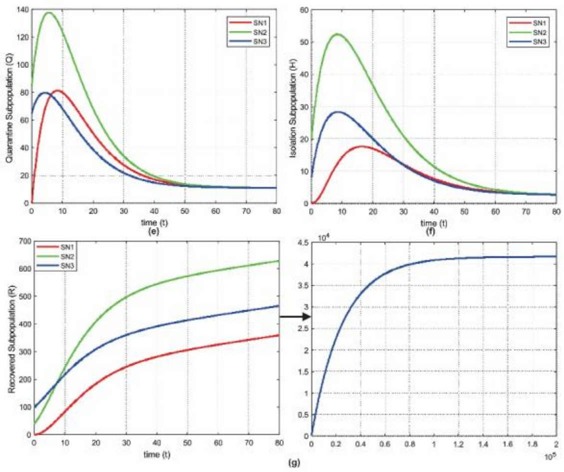

Figure 6. Next solution graph for $R_0 > 1$

Based on the parameter values given, the following values were obtained

a) $a_1a_2 - a_3 = 0.1287464752 > 0$,
b) $a_1a_2a_3 - a_3^2 - a_1^2a_4 = 0.002333422891 > 0$.

Here, it shows that the Routh-Hurwitz criteria are met, and the root of the characteristic equation (17) has a negative real part. Therefore, the point K^* is locally asymptotically stable as it satisfies Theorem 1. In figure 4 shows the graph of the solution for $R_0 > 1$ with three different initial values, namely

$SN_1 = (256.5839, 10.1401, 0.0106, 1.0551, 0.0245, 0.0082, 0.0016)$,
$SN_2 = (150, 200, 5, 45, 85, 20, 40)$,
$SN_3 = (70, 100, 10, 24, 65, 8, 100)$.

The results of the simulation with several initial values given the graph of the solution to the point K^*And this means that there is a spread of disease due to COVID-19. The numerical simulation results obtained are in accordance with the analysis results, namely if $R_0 > 1$, hence the point of K^* is locally stable asymptomatic. Based on the given parameter values, we get $R_0 > 1$. It means that there is an outbreak of disease due to COVID-19. This shows that quarantine and isolation are not enough to suppress the overall spread because quarantine and isolation are not carried out properly and are less effective. Therefore, the government needs to take more control to reduce the spread of more severe diseases. The controls taken are increasing quarantine and isolation, issuing regional restriction policies, providing socialization to maintain health protocols, not traveling if not too important, improving hospital care and management, and promoting vaccines and others.

D. CONCLUSION AND SUGGESTIONS

There are two equilibrium points in the COVID-19 model with quarantine and isolation: the point of the disease-free equilibrium (K^0) and the point of the endemic equilibrium (K^*). The point K^0 always exists, whereas the point K^* exists if it satisfies $R_0 > 1$. Furthermore, the point K^0 is asymptotically stable if it meets $R_0 < 1$, whereas the point K^* is asymptotically stable with the conditions.

This research examines the problem of solving the COVID-19 model with quarantine and isolation. For further study, it is advisable to add compartments such as a subpopulation of lifestyle changes, preventive controls such as preventive measures through education, treatment for infected individuals, vaccination, and tightening traveling out of town by showing the results of rapid antigen or swabs to reduce of COVID-19 infections. In addition to the control measures given, it is also suggested to discuss cost-effectiveness analysis related to optimal control problems.

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