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# Application of Beddington DeAngelis Response Function in Ecological Mathematical System: Study Fish Endemic Oliv Predator Species in Merauke

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**ABSTRACT**

Predator-prey type fishery models Oliv fish is a trans-endemic predator species that inhabits freshwater swamps and brackish water in Merauke, Papua. Maintaining the survival of the Oliv fish species is the main reason for compiling a mathematical model, so that it can be considered by local governments in making ecological policies. Method on model discussed is assembled with the growth of predator-prey populations following the growth of logistics. The response or predatory function corresponding to the behavior of endemic Oliv fish is the Beddington DeAngelis type. The growth of predatory species uses the concept of growth with stage structure, are divided into mature and immature. Research results show there are four equilibrium points of the mathematical model, but only one point becomes the asymptotic stable equilibrium point without harvesting $W_4(x^*, y^*, z^*) = 92.823, 1311.489, 525.957$ and equilibrium point with harvesting $W_4(x^*, y^*, z^*) = 95.062, 92.639, 160.466$. Harvesting exploitation efforts are carried out by the community so that the harvesting variables are added with a proportional concept. Simulation of the results of the study shows a stable scheme and harvesting conducted can maintain the number of populations that continue.

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## A. INTRODUCTION

Research on resource management is very important and has been done a lot, including in agricultural, fishery, and forestry resources. Fishery resources become one form of exploitation in renewable resources. Fishery resources consisting of fish organisms, shellfish, reptiles, amphibians, and marine mammals can produce biologically surplus in a given time (Ang & Safuan, 2019). Prudent species management and exploitation of harvesting without compromising future species productivity led to fishery resources being classified as renewable resources (Vijaya Lakshmi, 2020).

Efforts to exploit and sustain species in the ecosystem must be carried out in a balanced manner. Fishery management urgently needs to be formulated appropriately (Belkhodja et al., 2018). The result of fishery management formulation can be a recommendation of government policymakers in making laws or regulations on fishery exploitation. Fishery management measures are usually divided into catch control, growth control, and species sustainability technical measures (Chakraborty et al., 2013). The utilization of sustainable resources means that those resources must be used as such so that they are not exhausted or not permanently damaged.

The concept of sustainable balance of ecosystems, the economic productivity of fish populations, ecosystems, and population growth increases if fish population growth is not harvested (Pratama et al., 2019). Such a concept is very unlikely to happen, because the consumption and economic needs of the community are very high. Ecosystem-based fisheries management is an approach that takes the structural and functional roles of ecosystem components and services (Kaushik & Banerjee, 2021). The main objective is to achieve sustainable capability through appropriate fisheries management policies.

Olive fish is a species of fish that inhabits the freshwater swamp fishery area of Merauke. As a typical predatory fish, Oliv fish do not prey on all types of prey fish. Availability factors, nutritional needs, and opportunities for predation are the causes of Oliv fish tend to be picky about their prey (Walters et al., 2016). As an endemic species, Olive fish are strongly supported by environmental forces on food availability, reproduction, and climate change. The influence of the carrying capacity of the environment causes an explosion in the number of species at a certain time. Exploitation or catching is carried out a lot, in line with the demand for exports of dry species from the Merauke area. Meanwhile, modern sophisticated exploitation or fishing efforts rarely catch only one type of fish. Economic short-term profit, into consideration of economic concepts. Meanwhile, many fish species experience a decline in population growth, which causes the ecosystem to grow unstable.

The concept of harvesting like this is widely opposed by researchers who are oriented towards ecosystem sustainability, while it is supported by researchers who are profit-oriented (Ghosh et al., 2020). Therefore, the harvesting balance that is carried out must consider the two sectors (Liu & Huang, 2020). Consumer needs to meet economic and nutritional needs are the biggest factors in supporting harvesting. Mathematical modeling in harvesting or fishery exploitation behavior has been explained by research that is very concerned (Manna et al., 2018).

Mathematical modeling systems, especially in predator-prey population dynamics, are highly dependent on predator density, reciprocal interactions, predation functions, et all. These factors play an important role in the sustainability of the modeling system cycle (Zhao & Zeng, 2019). An important assumption in the predator-prey population dynamics model is that if the growth of the number of predators is high, then the density of the population plays an important role in the stability of the mathematical model system (Datta et al., 2019).

Study, the interaction function or response function used is the Beddington DeAngelis type functional. The response function assumes that there is interference between predators. The response function model that characterizes Beddington DeAngelis's predator predation is an extension of the Holling type II response function (Lu et al., 2017). Characteristics that stand out in the predatory behavior of Olive fish are actively looking for prey, handling prey, and disturbing each other between competitors. The Beddington DeAngelis response function represents a complete description of predator characteristics rather than a prey-dependent response function (Ghanbari et al., 2020). With the abundance of prey populations or in other cases when population densities are high, the Beddington DeAngelis response function can represent a better mathematical model system.

Meanwhile, in its development, predators with stage-structure are more efficient at drawing the actual state in the ecosystem. Predators with a stage-structure have been studied by many researchers (Jia et al., 2017). Describing immature and mature predators will facilitate the form of clustering of the predatory systems that occur. The age development of species, especially fish, really needs to be taken into account (Wang & Huang, 2014). There are species characteristics whose growth rate is very immature depending on the mature age group. It is common for systems of mutual protection to occur in these species (Kundu &

Maitra, 2018). Things like this will be more realistic to be taken into account in mathematical models (Yanni & Zulfahmi, 2019).

The case of harvesting carried out in the predator-prey population model system, adopting rules prohibiting harvesting because it is included in the conservation area. Swamp areas in conservation areas are not allowed to harvest. This assumption will be the control area for harvesting in the context of population sustainability. Furthermore, the problem of harvesting control is carried out with an iterative numerical approach to describe the actual conditions in the ecosystem environment. In addition, an analysis was carried out regarding harvesting the results of the control system intervention and without intervention with the same model without optimal control.

## B. METHODS

Research conducted is a literature study on mathematical modeling of endemic Oliv fish populations in Merauke. Characteristics considered in a mathematical model prey-prey system with a response function of the Beddington DeAngelis type. Assuming variables used $x(t)$, $y(t)$, and $z(t)$, respectively, are the size of the prey population, immature predators, and mature predators at time $t$. Populations grow in a homogeneous environment, freshwater swamp area, and follow a logistical type of growth (Malard et al., 2020). The dynamic system of the mathematical model is described in differential equations, as follows,

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\beta xz}{a+bx+cz} \\ \frac{dy}{dt} &= \frac{w\beta xz}{a+bx+cz}-my-ny \\ \frac{dz}{dt} &= my-\rho z \end{aligned} \tag{1}$$

where $x(t)0, y(t)0$, and $z(t)0$.

The parameter $r(k)$ is the intrinsic growth rate (carrying capacity) of the prey population, $\beta$ is the interaction coefficient between prey and mature predators, $w\beta$ represents the conversion efficiency of consumed prey into new predators and $\sigma$ is the conversion rate assuming that $0 < \sigma < 1$, because the interaction results in these prey and predators do not all become the rate of growth of new predators. Parameters $a, b$ are positive constants that describe the rate of prey capture and handling time during the predation process. Parameter $c$ is a positive constant that describes the amount of disturbance that occurs between predators. Parameter $m$, is the rate of change of immature predators to mature predators, while parameter $n$, is the natural death rate of predators, respectively. The assumption of the death rate in predators can occur due to natural factors such as natural disasters, diseases, birth defects, and other factors that cannot be intervened.

Functional forms of harvesting actions generally use the concept *Catch-Per-Unit-Effort* (CPUE). The assumption is that CPUE is proportional to the level of availability of fish species to be exploited. The intended harvesting function is written as follows,

$$\pi(t) = \delta dEz \tag{2}$$

## C. RESULT AND DISCUSSION

### 1. Predator-Prey Model

Parameter $\delta$ is the catch coefficient of mature predatory species. While the combined efforts arrests used for the process of harvesting mature predator fish species is $E$. Parameters $d$ $(0 < d < 1)$ part of the available population for harvesting, this means that it is possible to control the harvesting area by considering the numerical assumption that parameter $d$ is in the range 0 to 1. Parameter that simplifies subsitusion $w\beta$ and harvesting concepts from the model (1) and (2),

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{k}\right)-\frac{\beta xz}{a+bx+cz}, \\ \frac{dy}{dt} &= \frac{\sigma xz}{a+bx+cz}-my-ny, \\ \frac{dz}{dt} &= my-\rho z-\delta dEz, \end{aligned} \tag{3}$$

where the initial conditions of the model meet the assumptions of $x(0) \geq 0, y(0) \geq 0, z(0) \geq 0$ and $0 < d < 1$. The predator-prey population dynamics model with stage-structure (3), the operational approach uses a differential equation solution by taking into account the dimensions of the parameters that have been taken. The dimensions of each parameter are taken based on actual assumptions in the ecosystem environment.

**Table 1.** Definition of parameters in the model

| Parameter | Definition | Dimension |
|---|---|---|
| $r$ | The intrinsic growth rate of prey | $[T^{-1}]$ |
| $k$ | *Carrying capacity* | $[N^{-1}]$ |
| $\beta$ | Predation rate by mature predators | $[T]^{-2}$ |
| $\sigma$ | The efficiency of prey-to-immature predator conversion | $[T]^{-2}$ |
| $a$ | Catch rate on predation function | $[N][T]^{-1}$ |
| $b$ | Handling time on predation function | $[N][T]^{-1}$ |
| $c$ | Interference interaction rate in mature predators | $[N][T]^{-1}$ |
| $m$ | Rate of change from Immature predator to mature predator | $[T^{-1}]$ |
| $n$ | Natural death of the immature predator | $[T^{-1}]$ |
| $\rho$ | Natural death of the mature predator | $[T^{-1}]$ |
| $\delta$ | Catch coefficient | $[T^{-1}]$ |
| $d$ | Availability of population size | - |
| $E$ | Harvesting effort of mature predator | - |

### 2. Equilibrium

Analysis point $W(x, y, z)$ was carried out on model (2) by taking into account the rules of differential equations. Differential model (2) by considering $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0$ and $\frac{dz}{dt} = 0$. from model (2) obtained four equilibrium points,

a. $W_1(0,0,0)$,
b. $W_2(k, 0,0)$,
c. $W_3(x, y, z)$,
d. $W_4(x^*, y^*, z^*)$.

Respectively the equilibrium points $W_3(x,y,z)$, and $W_4(x^*,y^*,z^*)$ are positive and negative factors. Taking the positive equilibrium point becomes a very real decision to continue the calculation of the model (2). As for each positive equilibrium point $W_4(x^*,y^*,z^*)$ show as,

$$x^* = \frac{A_2 + \sqrt{A_3}}{2\,A_1},$$
$$y^* = \frac{\big((-b+\sigma)m - bn\rho\big)\sqrt{A_3}}{2A_1cm(m+n)} + \frac{\big((-2aA_1 - bA_2) + \sigma A_2\big)m}{2A_1cm(m\ +\ n)} - \frac{n\rho(2aA_1 +\ bA_2)}{2A_1cm(m\ +)}n,$$
$$z^* = \frac{(-b\rho(m\ +\ n) +\ m\sigma)\sqrt{A_3}}{2A_1c\rho\,(m\ +\ n)} - \frac{2\rho\left(AA_1 + \frac{A}{2}A_2\right)(m\ +\ n) +\ m\sigma A_2}{2A_1c\rho\,(m\ +)}n.$$

where,

$$A_1 = cmr, A_2 = b\,km\rho + b\,kn\rho + ckmr\sigma - \beta km\sigma, A_3 = b^2\,\beta^2k^2m^2\rho^2 +\ 2mnb^2\,\beta^2k^2\rho^2 + b^2\,\beta^2k^2n^2\rho^2 + 2b\beta cr\sigma\rho k^2m^2 + 2b\beta cr\sigma\rho mnk^2 + c^2\,k^2m^2r^2\sigma^2 + 4a\beta ck\sigma r\rho m^2 + 4a\beta ck\sigma r\rho mn - 2\,b\beta^2k^2m^2 - 2\,b\beta^2k^2mn\sigma\rho - 2\beta crk^2m^2\sigma^2 + \beta^2k^2m^2\sigma^2.$$

In point form equilibrium which has been obtained that form $\sqrt{A_3}$ shows a positive result. Therefore the movement of value will go to positive roots. The equilibrium point analysis was determined using the Routh-Hurwitz criteria (Yulida & Karim, 2019). The Jacobi matrix used in the differential equation model (2) in testing the equilibrium point $W_4(x^*,y^*,z^*)$ is as follows,

$$J_{jacob}(W_4) = \begin{bmatrix} j_{11} & 0 & j_{13} \\ j_{21} & j_{22} & j_{23} \\ 0 & j_{32} & j_{33} \end{bmatrix}$$

where,

$$j_{11} = r\left(1 - \frac{x^*}{k}\right) - \frac{rx^*}{k} - \frac{\beta z^*}{bx^* + cz^* + a} + \frac{b\beta x^*z^*}{(bx^* + cz^* + a)^2},$$
$$j_{13} = -\ \frac{\beta x^*}{bx^* + cz^* + a} + \frac{c\beta x^*z^*}{(bx^* + cz^* + a)^2},$$
$$j_{21} = \frac{\sigma x^*}{bx^* + cz^* + a} - \frac{bx^*z^*}{(bx^* + cz^* + a)^2},$$
$$j_{22} = -mn,$$
$$j_{23} = \frac{\sigma x^*}{bx^* + cz^* + a} - \frac{cx^*z^*}{(bx^* + cz^* + a)^2},$$
$$j_{32} = m,$$
$$j_{33} = -\rho.$$

The substitution of equilibrium point $W_4(x^*,y^*,z^*)$ performed on the Jacobian matrix will show the characteristic equation of model (2). The roots of the characteristic equation determine the equilibrium point of equilibrium. The characteristic equation $J_{jacob}(W_4)$,

$$\lambda^3 + N_1\lambda^2 + N_2 + N_3 = 0,$$

Equilibrium point $W_4(x^*,y^*,z^*)$ satisfies the local stable for the Routh-Hurwitz criterion, $N_1 > 0, N_2 > 0, N_3 > 0$ , and $N_1N_2 > N_3$.

## 3. Harvesting and Maximum Profit

Stable equilibrium point will be used to obtain the rate of population harvesting rate and maximum profit. In model (3), consideration of harvesting has been given by adding the harvesting function $dEz$. The equilibrium point $W_4(x^*, y^*, z^*)$ will be the test point for harvesting on the mature predator $(z^*)$. While the profit function given is $TR = pzE$ where $p$ is the price per unit of species biomass (Wang & Huang, 2014). The calculated expenditure function is $TC = cE$, where $c$ is the constant value of the expenditure from each harvesting effort (Pratama et al., 2021). The profit function formed from the effort to harvest the mature prey population is,

$$\pi(E) = pzE - cE, \tag{4}$$

Critical value of the profit function for the harvesting effort $(E)$ will be, the local maximum harvesting effort, $\pi(E) = pz^*E - qE$.

## 4. Numerical Simulation

Simulation conducted in this research is to show the test of the specified parameters. Determination of parameter values is taken from assumptions that are in accordance with the actual conditions of the ecosystem environment (Li et al., 2017). The first simulation was carried out on model (2) without harvesting.

Parameters $r = 1.5, k = 100, = 0.8, = 0.04, a = 10, b = 8, c = 6, m = 0.0008, n = 0.003$ and $\rho = 0.0002$. The resulting equilibrium points $W_4(x^*, y^*, z^*)$ arerespectively $(92.823, 1311.489, 525.957)$. The characteristic equation from the Jacobian matrix of the equilibrium point is,

$$f(\lambda) = \lambda^3 + 1.37688597\,\lambda^2 + 0.005488158\lambda + 0.00000085$$

From the characteristic equation, the Routh-Hurwitz criterion test is carried out and fulfills the form of eigenvalue $\lambda_1 = -1.37189, \lambda_2 = -0.00016, \lambda_3 = -\,0.00384$. Negative eigenvalues obtained from the point of equilibrium indicate that $W_4$ is locally asymptotically stable. The stability of the equilibrium point provides an idea of the continuity of the species population for a long time.

Simulation in model (3) was carried out to obtain stable harvesting data and optimal profit. The parameters taken in the harvesting function are $\delta = 0.2, d = 0.4, p = 100$, and $q = 50$. As for the equilibrium point at $W_4(x^*, y^*, z^*)$ with harvesting is a positive form. The profit function (4) on the harvesting effort of mature predators is shown as follows,

$$\pi(E^*) = (100z^* - 50)E, \tag{5}$$

with the profit function (5) the harvesting effort value is obtained from the differential equation of the profit function, namely $E^* = 0.0033$. The harvesting effort obtained is the local stable point of the profit function.

The maximum local profit obtained is $\pi = 52.359$. The equilibrium point $W_4(x^*, y^*, z^*)$ associated with the critical point of harvesting effort in a model (3) is $(95.062, 92.639, 160.466)$. The characteristic equation of the equilibrium point Jacobian matrix is $f(\lambda) = \lambda^3 + 1.3977032\,\lambda^2 + 0.0059396\,\lambda + 0.0000014$, with eigenvalues of $\lambda_1 = -1.39344, \lambda_2 = -0.00401, \lambda_3 = -0.00025$. The results of the final eigenvalues obtained show that $W_4(x^*, y^*, z^*)$ is an asymptotically stable equilibrium. This condition will show that the prey and predator populations are stable for a long period of time.

The numerical simulation is to take the value around the equilibrium point which will show the population growth curve in a long time. The initial values taken are $x(0) = 20, y(0) = 10$, and $z(0) = 30$. The curves of prey, immature predatory, and mature predator populations of Olive fish species will be shown before and after harvesting.

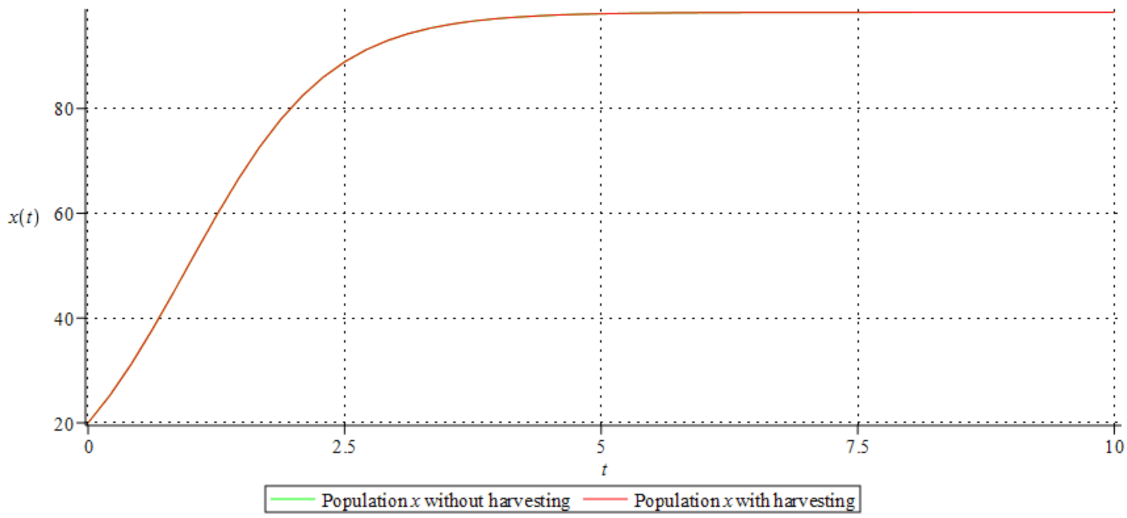


**Figure 1.** Trajectories for population $x$ with and without harvesting.

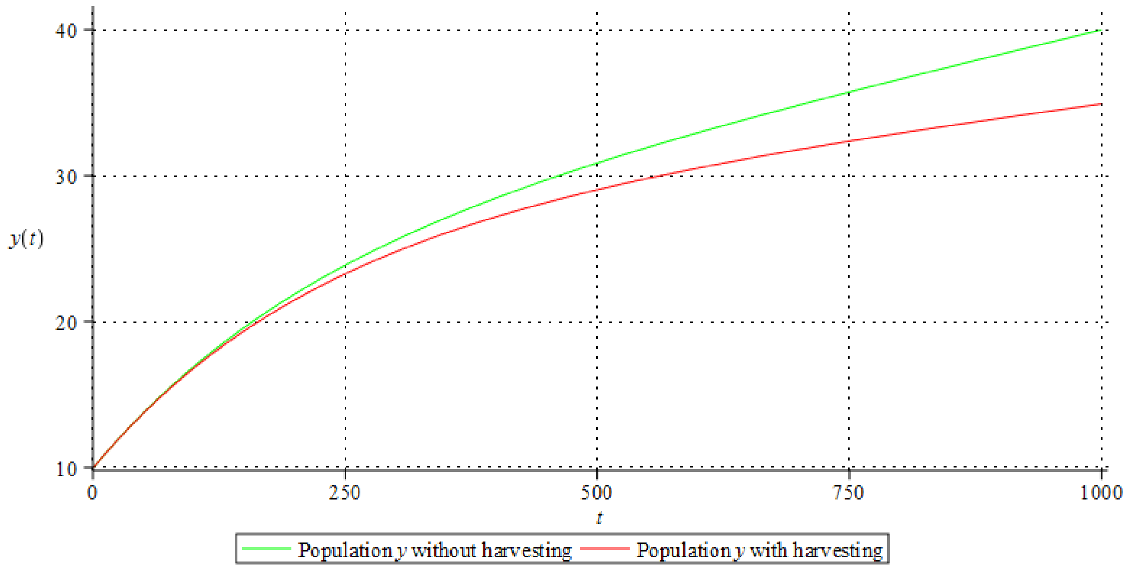


**Figure 2.** Trajectories for population $y$ with and without harvesting.


**Figure 3.** Trajectories for population $z$ with and without harvesting.

Figures 1, 2, and 3 show that the curves of all populations continue to grow upward and persist over a long period of time. Changes in the equilibrium point in the mathematical model of Olive fish without harvesting $W_4 = (92.823, 1311.489, 525.957)$, and the equilibrium point with harvesting $W_4 = (95.062, 92.639, 160.466)$. The value of the equilibrium point in prey fish species increases with harvesting. In populations of immature predatory fish species and matures, the equilibrium point has decreased. The behavior of harvesting or catching carried out on mature Oliv fish species, apparently affects the number of species in the carrying capacity area, but has the survival of the species in the long term.

## D. CONCLUSION AND SUGGESTIONS

One mathematical model of the predator-prey population with step structure for predatory population and Beddington DeAngelis response function has been developed by adding selective harvesting of mature predatory populations. In the mathematical model (3), four equilibrium points are found, but only one is possible to be an interior equilibrium. Taking parameters that match the interior equilibrium point without harvesting local stables.

Mathematical model with constant harvesting effort, a stable equilibrium point is then associated with the problem of maximizing the profit function. The test results of model (3) provide maximum profit and the interior equilibrium point remains stable for a long period of time. This shows that efforts to continue to preserve endemic Olive fish species can be carried out, and harvesting is carried out to produce maximum profits for a long period of time. Of course, this will provide a mathematical picture for policymakers to make decisions. Increase the productivity of people's income economically and maintain the extinction of the Oliv fish species as an endemic fish in Merauke Regency. Future research can pay attention to mathematical models on endemic species, this is done because species that require more attention for sustainability around the environment. Mathematical models can be adopted with prey and super predator variables.

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