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The Students Thinking Process in Constructing Evidence with Mathematics Induction Reviewed from Information Processing Theory

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ABSTRACT

This study describes the student's thought process in constructing evidence that begins with a stimulus. The stimulus is then entered into the sensory register through the senses of sight and hearing. The attention that occurs is focused on the complete problem as indicated by the emergence of perceptions about stimuli following the information given, namely solving the problem by mathematical induction. In short-term memory, the construction of proofs by mathematical induction begins in retrieving the concepts of mathematical induction principles. The research subjects were six students of mathematics education; namely, two people with high abilities or the upper group, two people with low skills or the lower group, and two people with medium abilities or the middle group. The retrieval process in students belonging to the upper group runs smoothly. In the long-term memory of the issues of this group, the knowledge needed by working memory is stored. Proof of truth by mathematical induction is interpreted correctly, proving the truth for n=1 to n=k+1. The assumption of truth for n=k is the basis for establishing the truth for n=k+1 by upper group subjects. This is different from what happened to the topics of the middle and lower groups. The assumption of truth for the written value of n=k is not involved in proving the truth for n=k+1. The encoding process that occurs in students is in the form of strengthening some concepts that have been retrieved from long-term memory.

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A. INTRODUCTION

In mathematics, the primary object studied is abstract, which is often also called a mental object. Because mathematical objects are so abstract, students need high thinking and reasoning skills in learning them (Saputra et al., 2019). One of the essential components in mathematics that requires high thinking skills in students is the proof construction process (Aini et al., 2019). Constructing evidence requires an understanding and sufficient experience, and proof requires a high level of ability that requires significant effort to acquire (Harjo et al., 2019).

According to Follow, mathematical proof is the main barrier for students, and many try to ignore it by avoiding it. However, in mathematics courses, few materials inevitably require students to deal directly with proof, as in the number theory course taken in the first semester. In the chapter on integers (sub-chapters of basic principles of mathematics) and a chapter on division, students must construct proofs by mathematical induction (Nelson et al., 2018). Mathematical induction is one of the essential proof techniques/methods in mathematics that must be mastered/understood by students from the start because this proof principle will be used in subsequent mathematics courses (Even & Kvatinsky, 2010).

In the NCTM document, it is written, "Proof is complicated for undergraduate mathematics students. Maybe... because their experience in writing proofs is only found in geometry lessons (at the high school level)"(Aini et al., 2019; Maskur et al., 2020). Proving is difficult for students because constructing it requires skill in choosing strategies and extracting knowledge in memory long ago (Auria, 2019; Gordon & Ramani, 2021; Maskur et al., 2020). Moreover, in terms of creating evidence by induction, even though the proof strategy is known, the problem is whether or not the concept of the strategy is stored in the student's memory to be recalled when they need it.

Therefore, it is not only previous experience that makes it challenging to construct evidence for students, but also prior knowledge is needed where this knowledge is permanently stored in memory (Khoiriyah & Husamah, 2018; Maskur et al., 2020). One of the theories used in assessing students' thinking processes in constructing evidence in this research is the information processing theory. Information processing theory is a cognitive learning theory that describes the processing, storage, and tracking of knowledge from the brain or mind (Nückles et al., 2020). Information processing theory is concerned with visible changes in behavior and information processing within how people enter information and use various information (Janssen & Kirschner, 2020). In evidence construction, information obtained by a person from the environment is in the form of questions/problems to be solved, and students' thinking processes will be analyzed based on the components contained in information processing theory.

Explains that information processing begins with the information received by humans in the sensory register. Some of the information (relevant information) is given attention, which raises the perception of the information and is brought to short-term memory (Kirschner et al., 2018; Tomporowski & Qazi, 2020). When attention continues to be given, and there is frequent repetition of the information, then the information that has been perceived will go into long-term memory, which at any time (albeit for an extended period) can be recalled when needed (Zazkis & Zazkis, 2010).

It is essential to describe students' thinking process in constructing evidence to know the flow of thinking, difficulty level, weakness, and understanding of a concept/knowledge. Therefore, this study intends to describe students' thinking processes in constructing evidence by mathematical induction in terms of information processing theory. The research provides an overview of the thinking processes of mathematics Tadris students IAIN Parepare in creating proof by mathematical induction in terms of information processing theory and becomes an evaluation material for evidence learning that has taken place to date.

B. METHODS

This type of research is exploratory research because the investigation is intended to reveal facts about students' thinking processes in constructing evidence. In this study, the researcher did not have a proposed hypothesis. This follows the characteristics of exploratory research, whose answers are still being sought and are difficult to predict. Of course, it is difficult to expect anything or even impossible to hypothesize. The data collected is descriptive, which describes participants' actual conditions and practices in constructing evidence.

The research subjects were students of mathematics at IAIN Parepare who had taken number theory courses. Research subjects consider communication skills, academic abilities, and students' willingness to devote time to research activities. The research subjects were born as many as six people, namely S1-S6. Upper group students who are the subject of S1 and S2 are subjects who can carry out the process of constructing evidence from the receipt of the stimulus until the response is found in their working memory. Middle group students who are the subject of S3 and S4 are subjects who can think quite systematically. Still, the arguments presented in the construction of evidence are not clear, and they doubt themselves so that the answers they get are also less confident. Lower group students who are the subject of S5 and S6 are subjects in applying mathematical induction steps quite systematically. Still, the arguments presented to obtain evidence are unclear and often appear suddenly without connection with the previous statement.

Data collection is done by giving students problems to solve as for the questions you gave to the subject: (1) $\sum_{i=1}^{n} i = 1+2+3+...+n = \frac{1}{2}n(n+1)$ for any $n \in N$. Prove it!; (2) 6 divide $7^n - 1$ for all positive integers n. Prove the statement's truth!; and (3) Prove that $4n < n^2 - 7$, $n \geq 6$ with $n \in \mathrm{N}$!

In solving these problems, students express aloud what they are thinking. The researcher recorded the students' verbal expressions and behavior (expressions), including the unique things that students did when solving the problem. After completing one student, the same thing is done again for other students until data on a predetermined number of subjects is obtained. This kind of data collection belongs to the TOL / Think Out Loud method. For the same problem, other researchers use the term Think Alouds. This method asks the research subject to solve the problem and tell what he thinks. The data in this study were obtained from student work and audiovisual recordings.

The process of data analysis in this study was carried out with the following steps: (1) transcribing the collected data, (2) analyzing the available data, namely from the results of think-aloud, the results of written evidence construction, and recordings of student expressions (3) conducting data reduction, namely selecting, focusing and classifying similar data, then simplifying it by removing unnecessary things (4) arranging in units which are further categorized by coding, (5) analyzing thinking processes, (6) describing students' thinking diagrams in completing evidence and (8) concluding.

C. RESULT AND DISCUSSION

1. Analysis of the Thinking Process of Upper Group Subjects in terms of Information Processing Theory

Students who are the subject of the upper group are S1 and S2. The thinking process of the upper group subjects can be seen from the evidence construction process carried out, from the receipt of the stimulus until the response is found in its working memory. Upper group subjects are very sure of the arguments presented and their answers. The upper group subjects felt they were familiar with the problem, and there were no wrong interpretations of the meaning of the given stimulus. The basic concepts in mathematics needed by working memory are correctly stored in the LTM of upper group subjects so that it helps the smooth process of proof construction that is carried out. The answers were almost correct on questions 1 and 3 by S1 and number 2 by S2.

Thinking Process S1

The construction of evidence carried out by S1 on the three questions illustrates that the thinking process of S1 is strongly influenced by whether or not the stimulus that is captured is complete. Whether or not the stimulation captured is done will affect the perception of S1 in solving the problem. The previous knowledge needed to process the stimulus in working memory is stored well in S1's long-term memory. It is constructive for S1 to obtain the required response even though not all questions are resolved according to the correct answer. The concept of mathematical induction is mastered by S1, but the essence of proof by mathematical induction is poorly understood.

The encoding that occurs in constructing this evidence strengthens all concepts in LTM (long-term memory) S1. Images retrieved and processed in working memory to solve the problems at hand become stronger in S1 memory. Such as the principles of mathematical induction, multiplication of factors, addition, etc. In detail, the concepts coded by S1 can be seen in the S1 thought process diagrams. In the process of evidence construction, individuals do not acquire new knowledge because evidence construction is a problem-solving process that requires knowledge that has been stored in long-term memory.

Thinking Process S2

Of the three questions constructed by S2, confusion occurs at the same proof stage, namely proving the truth of n=k+1. This is because there is a misperception of the stimuli in the master's mind. The perception error is due to the master's lack of understanding of mathematical induction principles. S2's thought in solving the three problems faced was to use the principle of mathematical induction. Later, the desired result was that if n were replaced with a natural number, the final result would be a natural number. So that when proving the truth for the value of n=k+1, S2 always tries to substitute the value of n with an accurate number before concluding the validity of the response obtained. If the final result is a natural number, he immediately says proven. The essence of proof by mathematical induction is not like that. The encoding in S2 is the same as what happened in S1, namely the strengthening of concepts in long-term memory.

The S1 Thinking Process is almost the same as the S2 process in Constructing Evidence for Question Number 1. Due to space limitations, the S1 thinking process is described in detail. The S1 thinking process can be seen from the evidence construction process. When a question (stimulus) is given, S1 sees and reads the question as follows:

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{1}{2}n(n+1)$$

Next, start S1 by realizing what has been perceived. The previous one is solving problems by mathematical induction. S1 retrieval (recall) knowledge from long-term memory of the principle of mathematical induction, the concept of substitution, and arithmetic (multiplication and addition), respectively. The S1 statements and writings related to this retrieval process are as follows:

$$n = 1$$
$$1 = \frac{1}{2}\cdot 1\cdot(1+1)$$
$$1 = \frac{1}{2}(2)$$
$$1 = 1 \quad \ldots \quad \text{Terbukti}$$

This S1 statement was followed by his writing as follows:

$$n = k.$$
$$1+2+3+\cdots+k = \frac{1}{2}k(k+1).$$
$$n = k+1$$
$$1+2+3+\cdots+k+k+1 = \frac{1}{2}(k+1)(k+1+1)$$

When it comes to the step of applying the value of n = k, S1 immediately states as follows:

$$\underbrace{1+2+3+\cdots+k}+k+1 = \frac{1}{2}(k+1)(k+1+1)$$
$$\frac{1}{2}k(k+1)+k+1 = \frac{1}{2}(k+1)(k+2).$$

While constructing the evidence for question number 1, S1 never did rehearsals. This can be seen from the smooth process of evidence construction carried out by S1, and S1 never raises questions. S1 metacognition can also be said to work well because it is he who regulates all one's thought processes, such as when a concept should be called, what image should be used, what are the next steps, and so on. Due to the proper functioning of the information processing components in S1, it is clear and correct that S1's answers are as follows:

$$\frac{1}{2}(k^2+3k+2) = \frac{1}{2}(k^2+3k+2)$$

2. Analysis of the Thinking Process of Middle Group Subjects in terms of Information Processing Theory.

The subjects of the middle group consisted of S3 and S4. The thinking process of the central group subjects was quite systematic. Still, the arguments presented in the construction of the evidence were unclear, and they doubted themselves, so the answers they obtained were less believable. This group understands the systematics of proof by mathematical induction well, but the principle of mathematical induction is poorly understood.

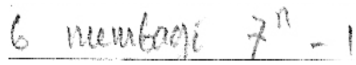
Thinking Process S3

The construction of the evidence carried out by S3 on the three problems given encountered difficulties at the same stage. S3 is not able to prove the induction step for n=k+1. The occurrence of these difficulties in S3 was caused by the incomplete concept of the principles of mathematical induction stored in S3's memory. One forgotten step from the principle of induction by S3 is the assumption of the truth of n=k. Due to the incomplete concept in S3's memory, S3 has reached a dead end and is confused in proving the truth of n=k+1. Because, in principle, the process of verifying the validity for n=k+1 must be based on the assumption of truth for the value of n=k. The most fundamental thing in constructing a proof by mathematical induction is to understand the principle of mathematical induction. Without this understanding, the proof construction process will not work as expected. This understanding is not found in S3.

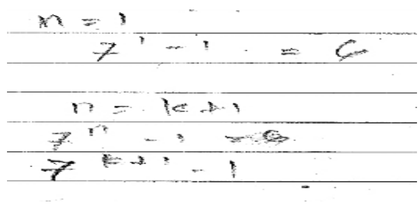
Thinking Process S4

From the results of the construction of evidence carried out by S4 on the three questions above, it can be seen that S4 memorized the principles of mathematical induction, including the steps for proving it. However, this memorization is not accompanied by understanding. The assumption that S4 confirms n=k is not fully understood as the basis for establishing the truth for n=k+1, so there is confusion in showing the truth for n=k+1. In addition to chaos, the arguments are invalid, and he is unsure about the ideas. The encoding that occurs in the proof construction process carried out by S4 from the problems solved strengthens the concepts in long-term memory. These concepts have been called upon to help him solve problems in working memory, including the principles of mathematical induction, arithmetic, substitution, and so on (see diagrams, section LTM).

The S3 Thinking Process is almost the same as the S4 thinking process in Constructing Evidence for Question Number 2. Due to space limitations, the S3 thinking process is described in detail. The evidence construction steps taken by S3 in question number 2 are the same as the steps in constructing evidence in question number 1. When the stimulus comes, it goes straight into the sensory register through the senses of sight and hearing. The incoming inspiration is complete, and that is what then becomes the doctor's attention which can help S3 to obtain a stimulus solution to the given problem, as stated in the following S3 expressions and writings:



The perception in S3's mind is also the same as when solving the problem in number 1. Namely, the problem is solved using mathematical induction, which can be seen in the answers' characteristics because the perception is not expressed directly by S3. The same problem related to the retrieval process of the concept of mathematical induction also occurs in the construction of the proof in problem number two. However, the substitution concept is correct, as can be seen in the results of the S3 answers that have been written below:



The retrieval process in the construction of proof number 2 is the retrieval of mathematical induction, substitution, and subtraction concepts. The return of the idea of induction and substitution is the same as what happened in the construction of question number 1. At the same time, the subtraction operation occurs in steps 71-1 which results in 6. Next, S3 is confused about how else to prove it, as he said, "How will this happen, how do you prove it?" whether it is confirmed or not." So, the proof stops at the final answer that he doesn't believe in. S3 thinking structure in constructing question number 2 can be described in the following diagram:

3. Analysis of the Thinking Process of Lower Group Subjects from the Stimulus Processing Theory.

Students who are the subject of the lower group are S5 and S6. The thinking process of lower group subjects applying mathematical induction steps was quite systematic and middle group subjects. The arguments presented to obtain evidence are unclear and often appear suddenly without connecting with the previous statement. The systematics of proof by mathematical induction related to the relationship between evidence of the truth of n=k and n=k+1 is poorly understood. Basic concepts in mathematics such as the definition of sigma, exponents, number sequences, and arithmetic are not stored well in the LTM of this group, resulting in errors in the proof construction process. Due to the lack of understanding of the subject with the essence of the principle of mathematical induction, the resulting evidence is not valid.

Thinking Process S5

Of the three questions constructed by S5, the thinking process in creating proof by mathematical induction that occurred in S5 experienced difficulties and dead-ends at the same stage, starting from the second step of mathematical induction related to the assumption of truth for the value of n=k. The statements presented by S5 in each set of the proof are not based on the correct concept/theory, so it causes the resulting evidence to be less valid. From the answers produced, it can be seen that S5 lacks mastery of the concept of mathematical induction and the essence of proof of the principle of mathematical induction itself. S5 does not explain the evidence of the truth for n=k+1 in problem number 1 but only writes the final result in an equation that cannot be acknowledged as valid. While in questions number 2 and 3, the proof of the truth for n = k + 1 is done by substituting the value of k = 1 as the treatment given to the value of n in the initial step. In fact, in the principle of mathematical induction, this does not apply to k. Still, the proof fork serves to generalize the validity of a statement to all natural numbers by doing a general proof, not by replacing/substituting the value of k with a number. S5 poorly understands the concept of mathematical induction, but other ideas, such as the meaning of the sigma symbol, are also poorly understood.

Thinking Process S6

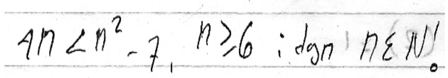
The results of the evidence construction carried out by S6 on the three questions given show that S6 knows the systematics of proof steps by mathematical induction, starting from revealing the truth of n=1 to n=k+1. Of the three constructed questions, S6 himself was unsure about his results.

The assumption that S6 proves n=k is not fully understood as the basis for establishing the truth for n=k+1. However, the premise of the reality of n = k is considered the final form that must be obtained to prove the truth of n = k+1. S6's perception in constructing the proof of the three questions remains consistent with the principle of mathematical induction. Of course, his attention is focused on the concept of mathematical induction and the complete form of stimulus.

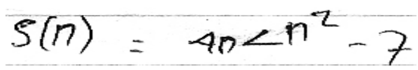
The encoding that occurs in the proof construction process carried out by S6 from the problems solved strengthens the concepts in long-term memory. These concepts have been called upon to help him solve problems in working memory, including the principles of mathematical induction, arithmetic, substitution, and so on (see diagram, section LTM). The knowledge that is called S6 from his LTM is also minimal, resulting in his inability to complete the construction of evidence correctly. The following will describe the S6 thinking process in constructing proof from each of the questions given.

The S5 Thinking Process is almost the same as the S6 thinking process in Constructing Evidence for Question Number 3. Due to space limitations, the S5 thinking process is described in detail.

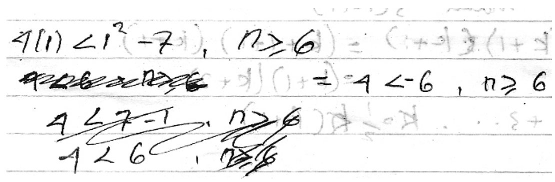
In constructing the evidence for question number 3, S5 first reads the question while writing it down as follows:



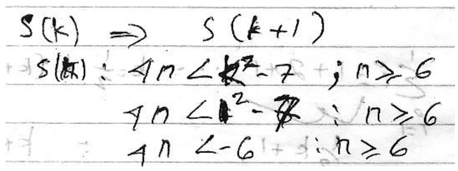
After reading the stimulus, S5 writes the form S(n), which indicates what is the focus of his mind as follows:



Next, S5 proves the truth for n=1 and substitutes it for the problem. After that, S5 explained the purpose of the evidence construction that would be carried out related to this step in his statement, which said, "He wants this n to be greater than the final six results". After substituting the value of n=1 into 4n<n2-7, S5 gets the result 4<6, and he immediately concludes that he is right. However, after doing the repetition, it turns out that S5 miscalculated. The result is not 6 but -6, while it is wrong if the statement is four <-6. S5 thinks about how to get the information accurate; then, he reverses the fact from 4(1)<12-7 to 4<7-1 to produce statement 4<6. But in the end, S5 concluded the truth of S(1) based on the operation results found, namely four <- six, because it was by what was explained earlier that n would later be greater than 6. The results of the researcher's question and answer with S5 and the written answer were related to the construction. Areas follow:



In this process, there is a retrieval of the main steps of induction, substitution, multiplication, subtraction, and number order operations so that S5 can conclude the truth of the proof. The retrieval of the principle of mathematical induction can be seen in the example for n=1; substitution is made for the value of n=1 into the form 4n<n2-6 so that we get 4(1)<(1)2-7. The multiplication and subtraction operations are carried out to operate 4 with one to become 4 and 1 with seven, resulting in -6. To say that 6>-6 is confirmed, a concept of sequence of numbers is needed, which has also been retrieved by S5. The next step is to prove the truth for n = k, which results in true for n = k + 1 until finally, the final result is obtained as in the answer written below:



The concept retrieval process that occurs in this step includes the steps of mathematical induction, substitution, and subtraction operations. The retrieval of the principle of mathematical induction can be seen in the example for n=1; substitution is made for the value of n=1 into the form 4n<n2-6 so that we get 4(1)<(1)2-7. The multiplication and subtraction operations are carried out to operate 4 with one to become 4 and 1 with seven, resulting in -6. To say that 6>-6 is confirmed, a concept of sequence of numbers is needed, which has also been retrieved by S5.

Judging from the construction of S5's answer to question number 3, S5's perception remains consistent with the principle of mathematical induction, and his attention is focused on the complete stimulus. The encoding that occurs in S5's mind is contained in the concepts of mathematical induction, subtraction operations, multiplication operations, substitution concepts, and number sequences. The following diagram illustrates the structure of S2 thinking in constructing evidence in question number 3.

4. Analysis of Differences in the Thinking Process of Upper, Middle, and Lower Group Subjects

The differences in the thought processes of the three groups of subjects can be seen in the structure of thinking. The thinking structure of each subject group is analyzed based on the similarity of thought processes between each subject in one group. The colored arrows in the diagram, both arrows that indicate the occurrence of the retrieval process and the indicators of retrieval results, indicate the beginning of errors in the cognitive process components.

Meanwhile, the shaded box in the diagram shows a mistake in the stimulus storage component. From the thought structure diagram, we can see how the flow of thinking of each subject group is.

The thinking flow of the upper group subjects showed a smooth thinking process from the time the stimulus was received until the response was found. Stimulus recorded in their sensory register is understood correctly so that attention and perception occur precisely. The cognitive processing component runs smoothly because their working and long-term memory work quite well. The concepts required by working memory are stored in their long-term memory. This includes the idea of the principles of mathematical induction.

Proving the truth in the principle of mathematical induction, from verifying the validity for the value of n natural numbers to n=k+1, is well interpreted in their minds. The application of the n=k induction form occurs as it should in the proving process for the value of n=k+1, and the upper group subjects quite understand the meaning and purpose of the assumption of truth for n=k are. The proving process that happened was fast, without taking a long time. The concepts in LTM relevant to the form to be proven are pretty well fulfilled. The arguments presented by the top group subjects are obvious and well-founded.

The thinking structure of the subjects of the middle and lower groups shows the occurrence of a flow of thinking processes that are not smooth. The principle of mathematical induction that they have in mind is not interpreted correctly. The application of the n = k induction form does not occur appropriately in proving the value of n = k + 1, and they do not understand the meaning and purpose of the assumption of truth for n = k are. Even the lower group subjects did not understand the first step of proving the principle of mathematical induction, namely establishing the truth for the substantial value of n. The concepts required by working memory are less fulfilled because of the limited images stored in their LTM. Many of the ideas in LTM are misapplied, as can be seen in the shaded boxes in the diagram.

5. Discussion

Information processing theory is concerned with visible changes in behavior and information processing within how people enter information and use various information (Warner & Kaur, 2017; Widana, 2018). In every average person, there must be an information processing component that will function automatically when dealing with data from their environment (Janssen & Kirschner, 2020; Lavie & Dalton, 2014). In constructing evidence with mathematical induction, all information processing elements must work well to produce valid proof, especially attention, perception, and long-term memory. The thinking process has three steps, namely the formation of understanding, the construction of opinions, and concluding. These three steps of the thinking process will work correctly as expected if the information processing components that exist from the stimulus to the long-term memory in a person function accurately and adequately.

The thinking process in the top group of subjects happened quite well, and this is due to the proper functioning of the information processing components. Attention and perception in the upper group function correctly and process the existing stimulus accurately. The concepts stored in the long-term memory of upper group subjects needed in constructing evidence are also quite numerous and good. These concepts are beneficial for upper group subjects in concluding and are pretty sure of the truth of these conclusions. The arguments presented by the subject of the upper group in the construction of the proof are pretty straightforward and understandable because these arguments are obtained through a logical process and can be justified and known, and accepted by the community of people who guarantee the truth of a mathematical statement (Diani et al., 2020; Saputra et al., 2019).

The validity of the evidence generated in this construction is strongly influenced by understanding the principle of mathematical induction itself. In this case, the upper group subject has a pretty good experience of the principle of induction. The upper group subjects understood the application of the induction steps, especially for applying the assumptions for n=k into n=k+1. However, the upper group subjects understand the real essence of proof by mathematical induction, namely the general description for n=k+1, not replacing the value of k with a number. Therefore, it can be said that the thinking structure of upper group subjects in constructing proofs by mathematical induction is complete.

As for the thinking structure of the middle group subject, it was incomplete. Existing information processing components are not functioning correctly. Attention and perception in the middle group subjects function correctly and process the existing stimulus accurately (Anggraini et al., 2018; Hanum et al., 2021; Pathoni et al., 2020). Still, the LTM does not work correctly because the concepts needed are not stored properly and are limited. Thus, the arguments presented by the middle group subject in constructing the evidence are less clear and less understandable.

The subjects of the middle group did not understand the principle of mathematical induction and the nature of proof by the actual installation. They memorize the induction step without knowing and understanding its true meaning. Thus, the application of the assumption of n=k does not occur at step n=k+1, and the proof of n=k+1 in the process is limited to the ability of the concepts stored in the LTM. Therefore, it can be said that the thinking structure of the middle group subject in constructing evidence by mathematical induction is incomplete.

Meanwhile, the structure of thinking in the lower subjects was also incomplete, and this is due to the poor functioning of the information processing component. The concepts stored in the long-term memory of lower group subjects needed in constructing evidence are minimal (Ishabu et al., 2019; Sari Nurza et al., 2021; Seruni et al., 2020). So that, in conclusion, the lower group subjects are not sure of the truth. The arguments presented by the lower group subjects in constructing the evidence are unclear and incomprehensible because the discussions were obtained suddenly for no apparent reason and without going through a logical process (Diani et al., 2020; Hasanah et al., 2021; Widana, 2018).

The lower group subject's understanding of the principle of mathematical induction is also lacking. The steps of mathematical induction that are written in the construction of the proof are only obtained from rote results. The lower group subjects did not sufficiently understand the application of the induction steps; moreover, the application of assumptions for n=k into n=k+1. The written belief of n=k is meaningless and has nothing to do with the proof of n=k+1. The lower group subjects themselves are not sure about the conclusions they make. Therefore, it can be said that the thinking structure of lower group subjects in constructing evidence by mathematical induction is incomplete.

D. CONCLUSION AND SUGGESTIONS

Based on the research that has been done, it can be concluded that the thinking process of IAIN Parepare students in constructing evidence begins with a stimulus, which is a matter of proof. The inspiration in this study was in the form of a proof question consisting of three questions, namely, six divided and, which were ordered to prove using mathematical induction. The stimulus is then entered into the sensory register through the senses of sight and hearing. Students enter the complete stimulation into their sensory log.

The attention that occurs in students is focused on the complete stimulus, namely the S(n) form of the three questions and the command to use mathematical induction, which is indicated by the emergence of perception of the stimulus that is by the inspiration that has been given attention, namely the completion of the push by mathematical induction. In short-term memory (working memory), the construction of proof by mathematical induction begins by retrieving the concepts of mathematical induction principles and other ideas until a response is found.

The encoding process that occurs in students is in the form of strengthening several concepts that have been retrieved from long-term memory. In the answers that are believed to be correct, there is the encoding of the images that have been called from the previous long-term memory. However, for solutions that are not accurate, coding does not occur because mistakes in steps or previous concepts cause students' insecurity.

Some recommendations that researchers want to put forward related to the research results that researchers have done include. There is a need for more in-depth research on the thinking process of other mathematical objects based on information processing theory. There is a need for a learning model that can help students/students acquire new knowledge so that the ability can be stored permanently in their long-term memory.

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