Differences in Students' Algebraic Thinking in Online and Offline Learning

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ABSTRACT

Mathematics teachers still have to create their creativity in online and offline learning. Therefore, mathematics teachers must pay attention to the assignments given to their students. One of the higher-order thinking skills that teachers must consider is algebraic thinking. This study aims to describe students' algebraic thinking as impact of online and offline learning. Researchers want to see the difference in algebraic thinking between students who are given online mathematics learning and students who are given offline mathematics learning. This study uses a qualitative research approach. Participants in this study consisted of 30 students taken from 2 junior high schools taken in the city of Ambon. The research procedure carried out in this research process is the stage of giving questions and thinking hard, as well as the interview stage. The interview guide was made based on indicators of algebraic thinking (Herbert & Brown, 1997). The results showed that the algebraic thinking skills of students who were subjects of online learning were said to be incomplete because they experienced construction holes at the stage of looking for patterns and generalizations. In contrast, students who were subjects of offline learning had complete algebraic thinking according to the algebraic thinking process.

Keywords:
Algebraic thinking; Online learning; Offline learning.

A. INTRODUCTION

The Covid-19 pandemic has hit all countries in the world, including Indonesia. According to the latest data from the World Health Organization, on April 24, 2020, as many as 213 countries were infected. One of them is Indonesia. The complexity of handling the outbreak for which there are no vaccines and drugs to cure Covid-19 patients and the limited Personal Protective Equipment (PPE) for health workers has made the government implement strict policies to break the chain of Covid-19 spread (Filip et al., 2022; Hashim et al., 2021; UNICEF, 2020). One way to break the spread of Covid-19 is to limit public interaction applied in terms of physical distancing. However, the physical distancing policy can hamper growth in various fields of life, both in the economic, social and, of course, the field of education (Muhyiddin & Nugroho, 2021). In addition, the government's decision to fire students and move the teaching and learning process from school to home by implementing the Work From Home (WFH) policy has worried many parties (OECD, 2020).
COVID-19 has significantly impacted education because students carry out the online learning process. Some schools use the LMS system, and some use a video conference system (Agustina & Nandiyanto, 2021). However, the impact of online learning is not fully implemented. For example, in Maluku, some lessons are held online (video conference), and some are conducted offline or in small groups at home or at places that the school has recommended. In addition, some schools combine two learning systems, namely online-offline. Online-offline learning combines direct and indirect learning (Koay et al., 2021). Online learning is a form of utilizing internet-based technology that has the potential to improve quality and equalize public access to education and learning (Pei & Wu, 2019).

In online and offline learning, mathematics teachers still have to be creative (Arrieta et al., 2021; Bringula et al., 2021). Mathematics teachers must continue to pay attention to the tasks given to their students (Abdillah et al., 2020; Mastuti & Prayitno, 2023). One of the demands of tasks in this emergency curriculum is assignments that hone students' higher-order thinking skills. The promotion of learning by teachers must explore students' reasoning (Mastuti, Abdillah, & Rijal, 2022). One of the higher-order thinking skills teachers must consider is algebraic thinking (Afifah & Retnawati, 2019; Mastuti, Abdillah, Sehuwaky, et al., 2022).

Some experts define algebraic thinking as a mental process such as reasoning with the unknown, generalizing and formalizing the relationship between quantities and developing the concept of variables (Sibgatullin et al., 2022; Tagle et al., 2016). Algebraic thinking is a mental process with something unknown, generalizing and formulating the relationship between scales and building variable concepts (Blanton et al., 2015; Kusumaningsih et al., 2018a). Teachers must know students' algebraic thinking skills, especially in junior high school mathematics problems (D. Rahmawati, 2018). Teachers must understand students' thinking in algebra (Wahyuni & Herman, 2018). Teacher thinking is important to pay attention to when the teacher provides polyhedron material, numbers, functional relationships, social arithmetic and others. These materials require the ability to use algebraic forms and their solutions in algebraic form (Kusumaningsih et al., 2018b). This is in accordance with the opinion of Wilkinson et al. (2018), which shows that students need to have mathematical reasoning to solve algebraic problems in learning mathematics.

Algebraic thinking consists of generalization, abstraction, dynamic thinking, modelling, analytical thinking, and organization (Hardiani, 2022; A. W. Rahmawati et al., 2019). Algebraic thinking is a process that involves developing a way of thinking using algebraic symbols as a tool but not separate from algebra, and also ways of thinking without using algebraic symbols such as analyzing the relationship between quantitative, paying attention to structure, studying change, generalizing, solving problems, model, conclude, and predict (Kusumaningsih et al., 2018a). Based on this description, it can be concluded that the ability to think algebraically is a thinking activity that involves processing information, generalizing, making hypotheses, and reasoning using mathematical symbols. To achieve the learning objectives, students must have good algebraic thinking skills. Algebraic thinking skills, namely students in problem-solving, representation, and reasoning in algebraic contexts (Mastuti et al., 2022; Sibgatullin et al., 2022).

Research by Fakhrunisa & Hasanah (2020) shows that students’ algebraic thinking skills do not represent ideal conditions. One of the problems experienced by students in algebraic
thinking skills is understanding variables as a useful representation tool for generalizing expressions (Nada et al., 2020). Meanwhile, Ardiansari & Wahyudin (2020), explained that junior high school students used variables without a deep understanding of the flexibility of the symbol system in algebra. In addition to students’ difficulties regarding symbolic and representational variables, another problem related to students' algebraic thinking skills is transitioning arithmetic thinking skills to algebraic thinking (Permatasari et al., 2021). Transitioning to algebraic thinking is one of the most difficult steps experienced by students in learning mathematics (Jupri et al., 2014). The process of transitioning from arithmetic to algebra occurs during elementary and junior high school because, in junior high school, students should ideally enter the formal operations stage; as stated by Piaget that at the formal operations stage, students can think abstractly (Suryadi et al., 2019). The diversity of students' algebraic thinking abilities has its level, which is important for teachers and students to pay attention to (Appah et al., 2020).

In each lesson, students face problems presented in context. They work in pairs or small groups to act out stories, either kinesthetically, visually by drawing, or manipulatively by modelling situations with physical objects (Gurganus, 2017). They are involved in an investigative process to solve problems: (1) they look for patterns in stories, (2) they recognize patterns and describe them using different methods, and (3) they generalize patterns and relate them to stories (Rejeki & Rahmasari, 2022). A broad view of algebraic thinking is taken to show students the real-life use and relevance of algebra. Algebraic thinking uses symbols and mathematical tools to analyze different situations by (1) extracting information from situations; (2) represent the information mathematically in the form of words, diagrams, tables, graphs, and equations; and (3) in interpreting and applying mathematical findings, such as solving the unknown, testing conjectures, and identifying functional relationships, to similar situations and related new situations. The process of investigating algebraic thinking (Herbert & Brown, 1997) is used in the Patterns in Numbers and Shapes unit, as shown in Figure 1.

![Figure 1. Algebraic Thinking Framework](image-url)

This study aims to describe differences in students’ algebraic thinking as a result of online and offline learning. Researchers want to see the difference in algebraic thinking between students who are given online mathematics learning and those who are given offline mathematics learning. An important element of this research is the students’ algebraic thinking process, not the value of the impact of online and offline learning.
B. METHODS

This study uses a qualitative research approach. Qualitative research here describes differences in students' algebraic thinking as the impact of online and offline learning. Participants in this study consisted of 30 students taken from 2 junior high schools taken in the city of Ambon. The first participants were taken from schools that consistently conducted online classes during Covid-19, while the other 15 participants were from schools that carried out small group offline schools which were held at students' homes. All participants were given algebra test questions at the end of the lesson. Based on the results of the first test, two students who met the criteria for algebraic thinking were taken as the subjects of this study. Based on saturated data, one student was a subject in the online class and another from the offline class.

The instruments used in this study are test questions and interview guidelines. The research procedure carried out in this research process is the stage of giving questions and thinking aloud, as well as the interview stage. The interview guide was made based on indicators of algebraic thinking (Herbert & Brown, 1997), as shown in Table 1.

<table>
<thead>
<tr>
<th>The process of investigating algebraic thinking</th>
<th>Indicator</th>
</tr>
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</table>
| 1. Pattern Seeking                            | a. Write down what is known and asked in the questions;  
|                                              | b. Determine the variables of the questions;  
|                                              | c. Make a mathematical model of the variables that have been formed according to the problem. |
| 2. Pattern Recognition                        | a. Apply mathematical models to solve problems;  
|                                              | b. Perform algebraic manipulation; |
| 3. Generalizing                                | a. Articulate the general rules of their pattern in the way they feel most comfortable using words, diagrams, symbols they have come up with, or equations and explain them by relating them to the original situation;  
|                                              | b. Draw logical arguments in the form of conclusions. |

In this study, students were asked to state the results of their thoughts in solving problems given orally. This result is done to think aloud the data. After working on the problem, the both subjects were interviewed to strengthen the research results. Data analysis is data that is compiled, revised, and choreographed (Creswell & Creswell, 2022). The data analysis technique of the research results was carried out through three stages: data reduction, data presentation, and drawing conclusions. In data reduction, the researcher selects important points from the data according to predetermined indicators. Then, researchers present descriptive data that is arranged systematically. Meanwhile, the conclusion is obtained by the researcher taking data from the evidence from the study.

C. RESULT AND DISCUSSION

1. Algebraic Thinking Students Engaged in Online Learning
   a. Pattern Seeking

   The subject of S1 is one of the students who is given test questions to reveal their algebraic thinking process. S1 has written what is known in the problem where it is known that three consecutive positive numbers are 21. While S1 writes down determine
three odd numbers. It appears that S1 makes a notation or performs an example by writing the variable \( x \) as a representative of the first odd number, namely \( x, (x + 2), (x + 4) \), as shown in Figure 2.

**Figure 2.** The results of S1 work in determining the variables of the questions

The subject’s statement when interviewed was, "the next step I took was to make an example by changing the editor of the question into a variable or symbolic form where I assume an odd number with \( x \). Then I had some doubts and remembered that if the difference in consecutive odd numbers is always 2, then I’ll take the odd numbers as \( x + (x + 2) + (x + 4) \)." Thus, showing that S1 performs an example by taking notations as a substitute for existing variables to facilitate understanding and solving the given problem, S1 has determined the variables of the problem to solve the existing problem. After making an example from these notations, make a mathematical model of the problem and determine the equations to solve the problem. This can be seen in Figure 3.

**Figure 3.** S1 work in making mathematical models to solve problems

S1 makes a mathematical model by adding up 3 consecutive odd numbers that have been for example \( x, (x + 2), (x + 4) \) to \( 21 = x + (x + 2) + (x + 4) \). Then simplify the equation to \( 21 = 3x + 6 \). The S1 statement is the same when it is clarified in the interview: "The process is first I write down the value of consecutive odd numbers. Where \( x \) plus \( (x + 2) \) plus \( (x + 4) \) or \( 21 = x + (x + 2) + (x + 4) \). Then to solve the problem, I looked back at the problem; there was the result of the first equation. Based on S1’s information, S1 can make mathematical modelling, where the student changes story questions into equations by writing consecutive odd numbers and shows that S1 has been able to create mathematical models to solve problems. At this pattern-seeking stage, S1 found out what was asked, determined variables in mathematical problems, and made mathematical models.

b. Pattern Recognition

S1 makes story problems in the form of a model or equation, then applies a mathematical model and makes plans using a combined method to solve the problem. For example, this looks like Figure 4.
Figure 4. The results of S1 work in applying mathematical models to solve problems

S1 starts to rewrite the mathematical model like \(21 = 3x + 6\) then S1 finishes it by subtracting both sides by the number 6. Then both sides are divided by 3 to become \(\frac{15}{3} = \frac{3x}{3}\). In addition, based on the interview, S1 explained, "the process to obtain the value of \(x\) is by dividing both sides equally by 3 so that the result is 5 or \(x = 5\)."

Based on the clarification, S1 can make plans by choosing existing problem-solving methods to obtain unknown values. The technique used by S1 in solving the problem is by creating an equation in which both sides are equally divided by the same number, namely 3, so that the final value obtained for the value of \(x\) is 5 or \(x = 5\). This shows that S1 can already solve mathematical modelling problems and perform algebraic manipulation at the pattern recognition stage.

c. Generalizing

In the last step, S1 applies the model to solve the problem and obtains the value of the \(x\) variable, then applies the variable value to determine the final value of the problem. For example, this can be seen in Figure 5.

![Figure 5](image)

**Translation:**
Substitute the value of \(x\) to the initial equation, so:
\[
\begin{align*}
    x &= 5 \\
    &= 5 + (5 + 2) + (5 + 4) \\
    &= 5, 7, 9
\end{align*}
\]

**Conclusion:**
So the odd sequential numbers are 5, 7, 9.

Based on the results of S1 work, as shown in Figure 5 shows that S1 substitutes the value of the \(x\) variable into equation one, such as \(5 + (5 + 2) + (5 + 4)\), hen gets the value of three odd numbers 5, 7, and 9. Then S1 concludes the final value of the problem by writing three consecutive odd numbers is 5, 7, 9. The researcher tried to reconfirm every reason explained by S1 during the interview. S1 seems hesitant when entering the Pattern Seeking phase. “...in the beginning I wanted to assume \(x, x + 1, x + 2\), but I’m not sure if the result is 21. I forgot the odd number pattern at first, but after thinking about it and asking again I just remembered it”.

The researcher claims that S1 experienced a construction hole at the pattern-seeking stage. After repeating the memory or schema by asking questions, S1 again finds the answer. Even though S1’s answer is correct, S1 has a concept error when generalizing,
as shown in Figure 5. S1 made an error when substituting the result which should produce the final result. Once confirmed, S1 realized the error and confirmed it correctly.

2. Algebraic Thinking Students Engaged in Offline Learning
   a. Pattern Seeking
      The initial algebraic thinking process carried out by S2 is almost no different from S1. After being given test questions, S2 understands the problem by reading the questions given then S2 begins to identify and write down the information contained in the test questions or existing problems. S2 can understand the existing problems indicated by writing down information in the form of what is known and asked from the question or problem. S2 then determines the variables or performs an example by taking the variables to make it easier to solve the given problem, as shown in Figure 6.

      \[ x+(x+2)+(x+4) = 21 \]
      \[ 3x + 6 = 21 \]

      Figure 6. S2 work in making mathematical models to solve problems

      Based on the results of S2 work, as shown in Figure 6, S2 has written three positive odd numbers whose sequence is \( x + (x + 2) + (x + 4) \) as equation one. Then S2 simplifies the first equation to \( 3x + 6 = 21 \). At this pattern-seeking stage, S2 found out what was asked, determined variables in mathematical problems, and made mathematical models.

   b. Pattern Recognition
      After making story problems in the form of a model or equation to solve the problem, S2 applies a mathematical model to make plans by choosing a way to solve story problems using substitution. This is as shown in Figure 7.

      \[ \frac{3x + 6 - 6}{3} = \frac{21 - 6}{3} \]
      \[ \frac{3x}{3} = \frac{15}{3} \]
      \[ x = 5 \]

      Figure 7. The results of S2 work in applying mathematical models to solve problems

      Based on the results of S2 work, it seems that algebraic thinking is at the structured pattern-seeking stage. This is reinforced by the interview "the process to obtain the value of \( x \) is by dividing both sides equally by 3 so that the result is 5 as the value of \( x \) or \( x = 5 \). S2 can make plans by choosing ways to solve existing problems to obtain unknown values. The method used by S2 in solving the problem is by making an equation in which both sides are equally divided by the same number, namely 3, so that the final value obtained for \( x = 5 \). This shows that S2 can already solve mathematical modelling problems and perform algebraic manipulation at the pattern recognition stage.
c. Generalizing

At this last stage, S2 does what S1 did: applying a model to solve the problem and obtaining the value of the x variable, then applying the variable value to determine the final value of the question. For example, this can be seen in Figure 8.

<table>
<thead>
<tr>
<th>Translation:</th>
</tr>
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<tbody>
<tr>
<td>For ( x = 5 ), then</td>
</tr>
<tr>
<td>( x + (x + 2) + (x + 4) = 21 )</td>
</tr>
<tr>
<td>( 5 + (5 + 2) + (5 + 4) = 21 )</td>
</tr>
<tr>
<td>( 5 + 7 + 9 = 21 )</td>
</tr>
</tbody>
</table>

| So three sequential odd numbers |
| whose number 21 is 5, 7, 9. |

Based on the results of S2 work, as shown in Figure 8, S2 did a structured proof. S2 also concluded his work correctly. Finally, the researcher explores the algebraic thinking process that was carried out at the final stage by asking the S2 argument from the beginning. “....I’m confident in the separation I’ve chosen. After knowing the answer, I tried to prove the results I got”. Based on the results of S2 work and interviews, the researcher believes that the algebraic thinking process carried out by S2 is very structured. S2 knows how to prove the right and left sides and perform algebraic operations correctly. So it can be concluded that S2 can articulate the general rules of their pattern in the way they feel most comfortable using words, diagrams, symbols they made, or equations and explain them by relating them to the original situation, as well as drawing logical arguments in the form of conclusions.

S1 and S2 have differences in the pattern-seeking stage and the generalizing stage. S1 experienced a construction hole at the pattern-seeking stage, and there were different schemes in the thought process. The construction hole occurs because a scheme does not yet exist in the construction of the problem solving carried out by the subject. Construction holes occur because of logical thinking errors on the subject. This logical thinking error creates a scheme for constructing incomplete troubleshooting. This is in line with the Subanji & Subanji (2021) statement that construction holes occur because of an incomplete problem-solving scheme. S1 can confirm the answer correctly after confirmation. This is because S1 experienced an incomplete algebraic thinking process. While S2 experienced a complete algebraic thinking process, every algebraic operation that S2 performs is correct and well confirmed. Hence, forming a complete structure requires cognitive skills and systematic thinking processes. This is in line with the research of Murtianto et al. (2019), which states, "The definition of thinking is about: Thinking is a systematic transformation of mental representations of knowledge". Therefore, students with a systematic way of thinking can download the correct information in sequence, transforming their representational rights of knowledge so
that they can form mental descriptions that can be manipulated to form other mental descriptions (Mainali, 2021).

During the pattern-seeking stage, it was observed that the subjects focused on data patterns while transferring information about the problem situation. Therefore, pattern checking on math tasks is crucial during pattern-seeking (du Plessis, 2018). Furthermore, determining variables and mathematical modelling are important aspects that must be analyzed during the pattern-seeking stage. This is similar to the research of Yeh et al. (2019); when viewed from its potential to make students acquire this strategy, it is related to the importance of writing formulas to obtain mathematical patterns.

During the pattern recognition stage, the subject has successfully applied mathematical modelling in problem-solving and algebraic manipulation. This is research by Hartono (2020), that the success of mathematics students depends on the learner’s ability to choose the right variables and build relationships between variables through a good understanding of the problems to be solved. Since the model is a mathematical representation of the problem, errors in setting the model will result in an incorrect solution (Duong et al., 2017). Producing a model is the same as carrying out a plan, while model interpretation and validation is an activity of looking back (Fukushima, 2021).

Due to the similarities between the modelling and problem-solving processes, it can conclude that modelling is part of the problem-solving process. In contrast, according to Noh (2019), mathematical modelling provides a new perspective for problem-solving, namely the process of interpreting a situation mathematically, which often involves repeated cycles of expressing, testing, and revising mathematical interpretations and activities of selecting, sorting, integrating, revising, and refining. Grouping of mathematical concepts for topics within and outside mathematics.

In the generalizing stage, subjects can articulate the general rules of their pattern in the way they feel most comfortable using words, diagrams, symbols they made, or equations and explain them by relating them to the original situation and drawing logical arguments in the form of a conclusion. This is in line with the research of Dani et al. (2017) that generalizing needs to appear in activities related to situations of rich experience. Many mathematical generalizations that students learn come from thinking about how physical quantities change or remain invariant due to actions and operations.

D. CONCLUSION AND SUGGESTIONS

Students’ algebraic thinking ability from online and offline learning fulfils the process of investigating algebraic thinking of pattern-seeking, pattern recognition, and generalizing. In the pattern-seeking stage, students found out what was asked, determined variables in mathematical problems, and made mathematical models. In the pattern recognition stage, students have already solved mathematical modelling problems and performed algebraic manipulation at the pattern recognition stage. While at the generalizing stage, students can articulate the general rules of their pattern in the way they feel most comfortable using words, diagrams, symbols they made, or equations and explain them by relating them to the original
situation, as well as drawing logical arguments in the form of conclusions. The algebraic thinking ability of students in online learning subjects is said to be incomplete because they experience construction holes at the pattern-seeking and generalizing stages. In contrast, students who are subjects in offline learning have complete algebraic thinking according to the process of investigating Herbert and Brown's algebraic thinking. This research is limited to algebra material for junior high school students to discover the differences in algebraic thinking in online and offline learning. However, research can be developed on a larger sample and other materials to discover students' thinking processes and their relationship to the psychology of students' cognitive development.

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REFERENCES


