Modeling and Analysis of the Dynamic Model of Bali Starling
(*Leucopsar Rothschildi*) Breeding in West Bali National Park

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ABSTRACT

Antara, the official news agency of the state, reported a record-breaking population of 303 Bali starlings in the West Bali National Park (WBNP) in June 2020, attributing this achievement to the park’s captive reproduction initiative. This paper presents a study on the dynamic equilibrium of Bali Starlings and proposes a mathematical model for analyzing this dynamic. The research also examines parameters ensuring the stability of the captive breeding model for Bali starlings in WBNP in a sustainable manner. The Bali starlings are categorized into two groups: those in the wild and those in captive breeding, with hatched eggs in captivity included in the latter. The dynamic model is analyzed for system stability around the endemic critical point using the Routh-Hurwitz stability criteria. As an illustrative example, a simulation is conducted to assess the model’s suitability under real field conditions. The model analysis reveals that the existence of an endemic critical point can be maintained if the percentage of stolen Bali starlings or eggs reintroduced to the wild is less than the difference between the percentage of Bali starlings laying eggs and the population growth rate in WBNP. Furthermore, the stability of the endemic critical point is confirmed as long as the percentage of Bali starlings laying eggs exceeds the population growth rate. This dynamic model offers a valuable tool for evaluating the sustainability of Bali starling breeding programs and optimizing the benefits associated with their conservation efforts.

Keywords: Bali Starling; Compartment of Dynamics; Endemic Critical Point; Stability of Dynamic Model.

A. INTRODUCTION

WBNP is located on the western tip of Bali. The area is home to a wide variety of protected and rare species of wildlife, including the Bali Starling (*Leucopsar rothschildi Stresemann*), which is classified as endemic to Bali. This bird is believed to be the original bird of WBNP. At the Curik Bali Animal Sanctuary, attempts were made to reproduce the bird for the purpose of replenishing their population. A strategy employed involved confining Bali Starlings at eight months old to facilitate the process of acclimatization before they are set free to roam in their natural habitat. A method to accomplish this is to introduce 8-month-old barrister chicks to acclimation cages, subject them to an acclimation procedure, and subsequently set them free into their natural habitat which is classified as endemic to Bali by (Ardhana & Rukmana, 2017).

Success stories as reported by Subrata et al. (2017) there was a very significant increase, namely in 2012 by 19.35%, in 2013 by 15.97%, in 2014 by 19.04%, in 2015 by 22.27% and in 2016 until April 30, 2016 by 23.38% and with these results, efforts to preserve the Bali Starling population are considered very successful. This success is also supported by Cheng & Ma (2023)
who states that artificial breeding is helpful in increasing the dwindling bird population. Artificial breeding has also managed to increase the mammal population as reported by (Khan et al. 2011). Bali starling in the wild is inseparable from the combined management of ex-situ (outside the habitat) and in-situ (inside the habitat), giving rights to the community, as well as supporting research findings that generate new management strategies for birds stolen from Bali in their habitat, as reported by Antara, the official news agency of the state. It is estimated that the population of Bali Starling by June 2020 is about 303 birds (Daniels J. M., 2020).

As the bird population increases, research concerning bird population dynamic need to be carried out. Callaghan et al. (2021) suggested a structure for avian species utilizing worldwide community-based scientific information from the Bird platform and a worldwide model for the danger of habitat transition. Şekercioğlu (2012) proposed management strategies should be developed to achieve the dual goals of biodiversity production and conservation that is, to use modeling to generate recognizable models of poultry dynamics covering different land use rates and to compare production sizes. Another proposal by Ouvrard et al. (2019) stated that the need for localized and enduring programs for observing biodiversity is imperative in comprehending and reducing the influence of worldwide transformations on tropical biodiversity while building capacity and educating the environment, school and community awareness, their production and ecology.

In order to investigate the worldwide decrease in biodiversity, accurate models of animal population dynamics are essential. Ouvrard et al. (2019) identified that the difficult problem of analyzing biodiversity loss is solved by modeling the dynamics of bird populations. More specifically, a new data-based model of bird population dynamics is proposed, using a variable-parameter partial differential equation (PDE) model. Mouysset et al. (2016) investigated various dynamic models linking bird abundance to agricultural land use. The paper emphasizes the importance of systemic, functional and mechanical relations between agricultural and bird populations in both described and intended context.

A model dynamic that supports the existence of predators is something model mathematics in which the describe dynamics cycle the growth population of bird with threat of predators (B. Ghosh et al., 2020; Rana et al., 2022). In this study, Bali starlings or common eggs were divided into two groups as Bali starlings or wild and captive eggs. Egg in captivity breeding which hatch entered in the group exposed. The built model will be analyzed for the stability of the system around its endemic critical point using Routh-Hurwitz criteria (Yang et al., 2022). As described from the model, perform a simulation to obtain a model that is compatible with the built description with the reality in the field. The research aims are to create a mathematical model that describes the population growth and dynamics of Bali Starlings within the WBNP and is to provide valuable insights into the population dynamics of Bali Starlings in a critically important conservation area and offer guidance for effective conservation and breeding efforts to ensure the survival of this endangered species.
B. METHODS

This study used data obtained from the West Bali National Park, Jembrana Regency, Bali. Data analysis refers to information and facts gathered from the agency based on structured interviews with trafficked persons informants as well as existing literature. The research method employed in this study utilizes mathematical modeling to investigate crucial interventions in the context of bird breeding and disease-related mortality (Khan et al., 2011) (Moudrý et al., 2017), (Abrahams & Geary, 2020), (S. Ghosh et al., 2021). Various mathematical analyses within the field of bird breeding have been conducted, with a focus on management strategies, host treatment control, and minimizing host-vector interactions while considering cost-effectiveness. (Cheng & Ma, 2023).

The study begins by considering a population function \( y = f(t) \) over time, where the time derivative \( dy/dt \) represents the rate of change of the population. This differential equation is characterized by a constant rate of growth \( (r > 0) \), with a solution given by \( y = y_0 \exp(rt) \), where \( y_0 \) is the initial population size and \( r \) represents a constant rate of growth (Ouvrard et al., 2019). The logistics model, which is widely used to depict population growth, is introduced, defined by \( dy/dt = r(1 - y/K)y \), where \( K = r/a \). The intrinsic growth rate \( (r) \) represents growth in the absence of limiting factors. Equilibrium solutions, where population size remains constant \((y = 0 \text{ or } y = K)\), are discussed, and these equilibria are termed critical points.

The study also delves into a system of nonlinear equations describing predator-prey interactions and then assesses the stability of critical points within this system. The method of linearization is applied, approximating the nonlinear differential equations with an equivalent system of linear differential equations. Stability analysis relies on eigenvalues obtained from matrices, with the Jacobi matrix evaluated at critical points (Markett, 2017); (Rana et al., 2022) (Grüne, 2020) (Iglesias & Ingalls, 2010).

The Routh-Hurwitz criterion is introduced as a necessary and sufficient criterion for determining the stability of linear systems. It involves the assessment of coefficients within the characteristic equation and the calculation of the Hurwitz determinant. A tabular form of this criterion is presented to simplify stability assessment (Yang et al., 2022). The entire research process involves calculating carrying capacity, constructing dynamic differential equations for breeding birds (specifically, Bali starlings), identifying endemic equilibrium conditions, and conducting stability analysis. Finally, data-driven simulations are used to assess parameters in the captive breeding model within West Bali National Park (WBNP).

C. RESULT AND DISCUSSION

1. Result

Data analysis refers to information and facts gathered from the agency based on structured interviews with trafficked persons informants as well as existing literature. All information needed in the analysis has been adjusted to the needs of calculating the dynamic model parameters of the Bali starling captive system. The population expansion pattern of the Bali starling \((M)\) in the WBNP habitat, coupled with the danger posed by predators \((P)\). The development of free-living Bali starlings in WB NP follows a logistical rule, in which the varieties produced by Bali starlings are bred in captivity (on-site) and spawned outdoors (ex-situ), introduced into captivity with proposal \((\alpha)\). The percentage of Bali starlings laying eggs
is \((\theta)\) so the number of eggs in captivity is \((\alpha \theta M)\). Bali starlings hatched from eggs hatched in captivity \((T_p)\) with a percentage \((\beta)\), were not released directly into WBNP, but were pre-adjusted and classified as exposed \((E)\) during spawning. When ready, the Bali starling was released into the wild \((T_b)\) of WBNP with percentage of \((\gamma)\). The more the value of \(((1 - \alpha) \theta M)\), the greater the growth rate \(\rho\). In addition, we denoted \(\mu_1\) be the natural mortality rate of the Bali starling.

The WBNP’s local records did not contain thorough information on predators, hence the impact of these animals on the fluctuating captive breeding cycle of Bali starlings is assumed to be incorporated in the rate of development of Bali starlings. Therefore, the parameter is understood as the growth rate of the Bali flute in the wild. In addition, the capacity of the Bali starling in the WBNP is very large compared to the existing Bali starling population, so the \(M/k\) (correction coefficient) is too small to ‘may’ omitted in the model. The flow diagram of the Bali Starling and the interaction is depicted in the diagram as shown in Figure 1 and Table 1.

![Diagram Compartment of Dynamics Model Captivity Breeding of Bali Starling](image)

**Figure 1.** Diagram Compartment of Dynamics Model Captivity Breeding of Bali Starling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>The rate of Bali starling population growth</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Percentage egg sneaky which hatch becomes population exposed</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>Natural death rate of Bali starling</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Percentage of Bali starling moved and added from region conservation</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Coefficient displacement population exposed from captivity to area WBNP</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Bali starling growth rate in the natural wild</td>
</tr>
<tr>
<td>((1 - \alpha))</td>
<td>Percentage of eggs or Bali starling which permanently is at in natural wild region conservation</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Percentage Bali starling lay eggs</td>
</tr>
<tr>
<td>(k)</td>
<td>Power accommodates (carrying capacity)</td>
</tr>
</tbody>
</table>
The dynamic model of the captivity breeding of Bali starling can be written in the form of differential equation systems (DES) as follows (Amini Tehrani et al., 2021; Coppée et al., 2022):

\[
\begin{align*}
\frac{dM}{dt} &= M(r - \theta) + \gamma E + \rho Tb \\
\frac{dTp}{dt} &= \alpha \theta M - \beta Tp \\
\frac{dE}{dt} &= \beta Tp - (\gamma + \mu_1)E \\
\frac{dTb}{dt} &= \theta (1 - \alpha)M - (\rho + \mu_1)Tb
\end{align*}
\]

The critical point derived from the DES algorithm can be expressed through the comprehensive equations presented below:

\[
\begin{align*}
\frac{dM}{dt} &= 0, & \frac{dTp}{dt} &= 0, & \frac{dE}{dt} &= 0, & \frac{dTb}{dt} &= 0,
\end{align*}
\]

So that it is obtained the critical point in terms of endemic condition, i.e.

\[
T = (M^*, Tp^*, E^*, Tb^*)
\]

where

\[
\begin{align*}
M^* &= \frac{E(\gamma + \mu_1)}{\alpha \theta}, & Tp^* &= \frac{E(\gamma + \mu_1)}{\beta}, & Tb^* &= \frac{E(\gamma + \mu_1)(1 - \alpha)}{\alpha(\rho + \mu_1)}
\end{align*}
\]

Combining equation (1)-(7) gives

\[
-\gamma E = \frac{[(\rho + \mu_1)(r - \theta) + \rho \theta (1 - \alpha)](\gamma + \mu_1)E}{(\rho + \mu_1)\alpha \theta}
\]

Since the value of \((1 - \alpha) > 0, M^* > 0, Tp^* > 0, dan Tb^* > 0, then the value of \(E^* > 0.\) This gives a condition \([\rho + \mu_1](r - \theta) + \rho \theta (1 - \alpha) < 0\) or \((1 - \alpha) < (\theta - r).\) Thus, the critical survival point \(T\) can be achieved if the percentage of eggs being stolen/returned to the wild \((1 - \alpha)\) is lower than the difference between the proportion of Bali starlings with could allow spawning with the growth rate of Bali starling populations in the WBNP area.

The determination of the stability of the critical point \(T\) relies on the eigenvalues and coefficients obtained from the equality characteristics \(det(\lambda I - A) = 0,\) where \(A\) represents the matrix coefficients of the DES model. The process of identifying the characteristics of the roots, which are assessed at the critical point \(T = (M^*, Tp^*, E^*, Tb^*)\), involves utilizing the Routh-Hurwitz criteria derived from the characteristic equation. The purpose of this analysis is to examine the characteristic of roots with positive values in a dynamic model, which may lead to instability at a critical point represented by a 4th degree polynomial,

\[
a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0
\]
The coefficients of the fourth-degree polynomial for $T$ in terms of $s$, as determined by the captive breeding model DES, are listed below:

$$
\begin{align*}
    a_0 &= 1 > 0 \\
    a_1 &= \rho + \gamma + 2\mu_1 + \theta + \beta - r > 0 \\
    a_2 &= (\rho + \mu_1)(\gamma + \mu_1) + \beta(\theta - r) \\
         &\quad + (\theta + \beta - r)(\rho + \gamma + 2\mu_1) > 0 \\
    a_3 &= (\rho + \mu_1)(\gamma + \mu_1)(\theta + \beta - r) + \beta(\theta - r)(\rho + \gamma + 2\mu_1) > 0 \\
    a_4 &= \beta(\theta - r)(\rho + \mu_1)(\gamma + \mu_1) > 0, \text{ if } (\theta - r) > 0
\end{align*}
$$

The Routh-Hurwitz method is utilized for determining stability, with a fixed model declaration for DES provided in the Table 2.

**Table 2.** Table Routh-Hurwitz.

<table>
<thead>
<tr>
<th>$s^4$</th>
<th>1.00</th>
<th>2.5199</th>
<th>0.1510</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^3$</td>
<td>2.4775</td>
<td>1.0263</td>
<td>0</td>
</tr>
<tr>
<td>$s^2$</td>
<td>2.1052</td>
<td>0.1510</td>
<td>0</td>
</tr>
<tr>
<td>$s^1$</td>
<td>0.8488</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$s^0$</td>
<td>0.1510</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Data are analyzed by authors

By applying the Routh-Hurwitz criteria, it has been determined that every figure in the initial column of the Routh-Hurwitz matrix is affirmative and there are no alterations in signs, provided that the condition $(\theta - r) > 0$ is fulfilled. This has resulted in the inference that there exists a crucial point ($T$) within the captive breeding scheme of the Bali starling, which remains stable so long as the rate of egg-laying among the starlings surpasses the rate of population expansion.

To present the growth dynamics of Bali starling populations in their natural habitat, simulations were conducted. These simulations utilized a dynamic model of Bali starling from DES and were carried out using Matlab. The simulation produces a curve describing the growth of each captive population the ecosystem using estimated parameters $r = 0.315, \mu_1 = 0.032, \rho = 0.398, \alpha = 0.70, \theta = 0.80, \gamma = 0.878, \beta = 0.850, k = 750000$. Those parameters were estimated during the period of 2016–2021. Time interval observation or simulation are made in year.

It is possible to attain the critical point $T$ when the prevalence of the endemic situation is such that the proportion of eggs stolen/returned to their natural habitat $(1 - \alpha) = 0.30$, which is lower than the disparity between the proportion of Bali starling that can lay eggs and the rate at which the starling population is expanding $(\theta - r) = 0.485$. Similarly, the endemic critical point is satisfied because the value of $(\theta - r) = 0.485$ is positive. The simulation model in endemic conditions was performed starting at early 2015 with the population of $M = 23$, eggs in captivity to breed ($T_p = 175$), the exposure period $E = 112$, eggs in the natural wild area ($T_b = 60$). The observations or simulations in each population with the interval of $0 \leq t \leq 6$, and unit time in year are shown in Figure 2.
Based on Figure 2, (top-right), it is seen that the proportions of total population ($M$), captive subpopulation ($T_p$), and wild subpopulation ($T_b$) show increasing conditions throughout the year. This result is in contrast to the report published by Ardhana & Rukmana (2017), a survey in WBNP discovered that there were only 10 specimens present, whereas Birdlife International reports that the remaining natural habitat houses a mere 49 individuals. This observation pertains to the Bali starling population in the West Bali National Park. The bottom right phase diagram shows that the exposed subpopulation ($E$) and $T_b$ shows that $T_b$ and $E$ decrease over time, but $T_p$ increases. Because of this situation, there is a need to increase the number of starlings released into the wild and to improve the care of young starlings in special cages after they are weaned from their mothers. The relationship of $M$, $E$ and $T_b$ as seen in the bottom-left curve, it decreases uniformly under all conditions, and decreasing $T_b$ and $E$ subpopulations clearly decrease the total population $M$. The relationship between $M$, $T_p$ and $E$, as shown in the top-left diagram, $E$ and $M$ decrease, but $T_p$ increases. Based on this condition, improvements in the care process for hatching eggs should be adjusted so that more chicks (exposure) grow, as shown in Figure 3.
As seen in Figure 3, subpopulations within captive breeding range will grow fast enough over the next few years. This shows the success of the team that bred starlings in captivity. As a result, it can be affirmed that the endeavors to conserve the Bali starling population at the WBNP Bali starling population development center have been successful. The captive breeding from eggs was well managed and hatched successfully, so dead thieves of starlings are extremely rare. Efforts to raise the birds are carried out at WBNP. This is one of the ways to secretly acclimatize 8-month-old starling back to their cages, before releasing them into their natural habitat. One method that can be applied before returning the starling into wild is by carrying out an adaptation in a cage with similar condition as in their habitat. Even though, there are still wild predators in the WBNP, but our purpose is to release the Bali starling into the natural habitat.

2. Discussion

Based on population size estimations, Ardana (2017) suggests that the actual population size of the Bali Starling remains uncertain, with estimates ranging from as low as 10 individuals in specific locations to around 49 individuals in the natural habitat. This is added by Pramatana et al. (2022a) who mentioned that Bali Starlings are shifting their habitat away from soft release sites, making accurate population assessments challenging. However, in terms of habitat shift and suitability, Pramatana et al. (2022a) highlights the shifting habitat preferences of Bali
Starlings within WBNP, which may be influenced by artificial treatments imposed by park authorities. This is supported by Ardana's results and Birdlife International's data implying that the Bali Starling population is struggling to thrive in its natural habitat. Moreover, in terms of human impact, our results provide specific recommendations for Bali Starling conservation. We emphasize the importance of maintaining a certain percentage of Bali Starlings with higher egg-laying growth rates to ensure population growth and stability. We also suggest developing more comprehensive dynamic models that consider predation effects and the predation level relative to Bali Starlings' flying activity per unit time. Pramatana et al. (2022b) emphasizes the negative impact of Bali Starlings residing near human activities, as they become dependent on resources provided by the community. This dependence hampers their ability to adapt to a natural habitat.

D. CONCLUSION AND SUGGESTIONS

Based on the discussion, it can be summarized the following results: (a) a dynamic model formulation system for captive-breeding Bali starlings describes the dynamics between subpopulations that reflect the growth of community-stage Bali starlings into the ecosystem of the WBNP; (b) the critical (equilibrium) points from captive breeding of stable dynamic model systems satisfies the Routh-Hurwitz stability criterion; and (c) the model parameters that satisfy the existence condition and a stable (unique) critical point (equilibrium point) are the percentage of Bali Starlings that lay eggs must be greater than the growth rate of the Bali Starling population.

Based on the results, it can be suggested the following important points: (1) to ensure the continued growth of the Bali starling populations, the existence and the stability of the endemic system, that is percentage Bali starlings that lays eggs bigger from the rate growth population Bali starling should be maintained; and (2) to get more complete dynamic models, to capture the required predation effect and balance the predation level for flying Bali per unit time should be proposed. The future research in this topic is to develop more advanced and dynamic mathematical models involving optimal strategies to simulate and predict Bali Starling population dynamics, incorporating factors like climate change and habitat fragmentation.

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