Exploring Students Learning Difficulties in Linear Function: A Diagnosis of Grade 9

Sarah Inayah1, Al Jupri2, Darhim3, Sufyani Prabawanto4

1,2,3,4 Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

inayahsarah@upi.edu1, aljupri@upi.edu2, darhim@upi.edu3, sufyani@upi.edu4

ABSTRACT

The aim of this research is to determine students’ learning difficulties in completing diagnostic tests on linear function material. In managing data, quantitative procedures are used with the aim of reducing data. After that the data is analyzed using inductive data analysis and the processed data will be presented in narrative form. So this type of research is qualitative research. The subjects in this research were class IX students at a junior high school in Cianjur. The instruments used in this research were documentation, tests and interviews. The conclusions of the research results obtained include the types of student difficulties in straight line equation material are (1) difficulties in algorithmic abilities including a lack of planning abilities (strategy knowledge) and in solving abilities (algorithmic knowledge) which are shown from incomplete answers or lack of steps, the lack of accuracy of students in working; (2) difficulties in using the principle of linear functions, lack of mastery of the basics of algebra and lack of understanding (schematic knowledge) as indicated by difficulties in recognizing linear functions in contextual problems, errors in algebraic computations, difficulty in determining the point through which the line passes, and difficulty in apply the principle of parallel or perpendicular gradients; and (3) difficulties in using the concept including the inability to remember the concept, the inability to deduce useful information from a concept and the lack of understanding skills (schematic knowledge) which is shown by incompleteness in writing formulas. This research will be useful as a preliminary study in making learning designs to overcome student learning difficulties in linear function material based on empirical findings.

Keywords: Diagnosis; Learning Difficulties; Linear Function.

A. INTRODUCTION

Understanding linear functions is key for students to develop a solid foundation for understanding algebraic topics (Pierce et al., 2010; She et al., 2014). Meanwhile, algebra is a topic that students must master because algebraic knowledge and skills, both implicitly and explicitly, can be used directly in daily life activities or as a prerequisite for further mathematics learning (Jupri et al., 2014; Tunstall, 2018). Previous studies have found errors or limitations in students’ understanding of linear functions and their graphs (Acuña, 2007; Zaslavsky et al., 2002). Errors made by students in learning often reflect the difficulties experienced by students in understanding or mastering the subject matter.

Early identification and intervention are very important in overcoming students’ learning difficulties in mathematics (Gersten et al., 2005; Hanley, 2005). This is because in Mathematics, each new learning skill is built from previous learning and is related to each other.
Learning difficulties experienced by students on the topic of linear functions will have an impact on the achievement of learning skills on the following topic. To recognize and explore learning difficulties experienced by students, teachers can make a diagnosis through a diagnostic test.

The mathematics diagnostic test is a good predictor of showing students' initial knowledge of mathematics (Simion, 2022). Through diagnostic tests, teachers can diagnose student errors in answering questions, teachers can identify patterns and causes of errors, which in turn can help identify potential student learning difficulties. The next step is to take appropriate action, such as providing additional support, asking questions to dig deeper, providing constructive feedback, or providing additional material to help students overcome these learning difficulties.

Linear function material is mathematical content chosen to develop various mathematical abilities, including reasoning abilities as researched by Ellis (2007) and Troup et al. (2017). The focus of Ellis's (2007) research is junior high school students' generalized and justified while exploring linear functions, while in Troup et al. (2017) undergraduate students' geometric reasoning on the subtopic of line slope using the help of the Geometer's Sketchpad (GSP). Apart from that, research on linear function content to develop mathematical representation skills was carried out by Adu-Gyamfi & Bosse (2014). The purpose of the study was to investigate students' interpretations and specific ways of working within table, graph, and the algebraic on notions fundamental to a conceptualization of linear functions.

Another mathematical ability that can be developed through linear function material is functional thinking. Research has shown that the use of linear function tables can enhance students' functional thinking, particularly in relation to covariation and generalization (Tanışlı, 2011). This is further supported by the successful introduction of linear functions through a functional approach, which significantly influenced students' algebraic expectation (Pierce, 2005). The development of learning materials based on mathematical understanding and representation, such as those for linear algebra, can also contribute to the enhancement of students' algebraic thinking skills (Rosita et al., 2019). Additionally, the use of ICT and a realistic mathematics education approach has been found to be effective in improving learning functions and increasing algebraic thinking skills (Hajizah & Wijayanti, 2023). So, it is important for students to master linear function material because it is mathematical content that can be chosen to develop various mathematical abilities. In the Indonesian curriculum, linear functions are given to junior high school students.

Research has identified several key difficulties that students face in learning about linear functions. Ansaldo-Leyva (2019) found that students struggle with the graphic representation and conversion between graphic and algebraic. Wijayanti (2018) drew attention to the ways that textbooks for lower secondary schools overlook possibilities to help pupils comprehend the theoretical concept of linear functions. Further, Fatio (2020) noted that a considerable proportion of pupils had trouble with the idea of finding the solution to linear equations in two variables. All of these research point to the necessity of implementing focused teaching methods to overcome these challenges in student learning.

Understanding the cognitive nature of different mathematical domains is crucial in addressing learning difficulties in mathematics (Karagiannakis et al., 2014). This understanding can guide the creation of classification models for these challenges.
(2019) and Fuchs (2008) highlight the significance of cognitive processes in addressing difficulties in mathematics learning. They propose that integrating instruction on these processes into direct skills intervention or tailoring interventions based on cognitive profiles can yield positive outcomes. Duval (2006) emphasizes the significance of semiotic representation and cognitive processes in understanding mathematics, and the necessity of taking these into account when addressing issues.

There are several previous studies that explored the learning difficulties of junior high school students in learning mathematics. Ali (2011) in his research describes how teachers try to use pedagogical solutions to help their students overcome learning difficulties that hinder depth in mathematics classes. Furthermore, Ramli (2013) describes teachers’ perspectives on pedagogical improvements that can help their students overcome difficulties that hinder deep learning in mathematics classes such as fun learning, effective communication, problem-based learning, constructivism approaches, real-life applications, and technology-integrated learning. Both of these studies did not provide a detailed explanation of the learning difficulties experienced by students. Various learning difficulties will certainly require different efforts to overcome them. This study aims to determine students’ learning difficulties in completing a diagnostic test on linear function material. This research will be useful as a preliminary study in making learning designs to overcome student learning difficulties in linear function material based on empirical findings.

B. METHODS

This research is descriptive research with a qualitative approach. The subjects of this research were class IX students at a junior high school in Cianjur, where these students had studied linear function material. The focus of the problem in this study is the results of diagnostic tests and interviews. This research began by formulating a diagnostic test on linear function material which was validated by mathematics learning experts, language experts, mathematicians and assessment experts. All experts stated that the test design was suitable for use with revision. After that, revisions were made to the test questions, Table 1 describes the grid of the diagnostic tests given to students.

<table>
<thead>
<tr>
<th>No. Ques</th>
<th>Material</th>
<th>Question Indicator</th>
<th>Cognitive Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A linear function is a relationship between a number/quantity pair, both of which can change at a constant rate of change</td>
<td>Given a function of inverse comparison, students determine whether it is a linear function or not with reasons</td>
<td>C2 (Understand)</td>
</tr>
<tr>
<td>2</td>
<td>Rate of change in a linear function</td>
<td>Students can determine the rate of change in a linear function from a given equation</td>
<td>C1 (Remember)</td>
</tr>
<tr>
<td>3</td>
<td>The relationship between the rate of change and the graph of a linear function</td>
<td>Students can determine the rate of change in values in a linear function of a given graph</td>
<td>C3 (Apply)</td>
</tr>
<tr>
<td>4</td>
<td>Line slope</td>
<td>Students can determine the slope of a line from a known equation</td>
<td>C4 (Analyze)</td>
</tr>
</tbody>
</table>
Table 1 illustrates that the test instrument contains various material which is a subtopic of linear functions. In Table 1 it is also written about the question indicators. Question indicators are a benchmark for competency achievement that can be proven measurably through knowledge attainment. Apart from that, the diagnostic tests given to students consist of various cognitive levels from the lowest level, namely remembering, to the highest level, namely creating. It is hoped that the results of the analysis of the tests will also describe the level at which students’ thinking skills are low, medium or high. After obtaining a suitable instrument, several stages are carried out until the reporting the finding and conclusion. The research stages are explained through the flowchart in Figure 1 below.

The data collection referred to in Figure 1 was by carrying out diagnostic tests on 34 class IX students at one of the junior high schools in Cianjur. The test results are analyzed and data is obtained on which questions students have difficulty with. Apart from that, data analysis also produced three groups of students based on total scores, namely high, medium and low groups. From each group, three people are selected who have the most mistakes or are interesting from their group. In other words, data reduction is carried out on questions and students who will go through the next stage, namely interviews. Interviews are conducted to confirm students’ answers to diagnostic tests in order to obtain more in-depth information regarding students’ learning difficulties. The questions asked in the interview are semi-structured interviews. Common questions asked include “is this question difficult?”; “Do you understand what is asked in the question?”; then “how do you get results like this?”. The interview results were then analysed using inductive data analysis and then interpreted as research findings. The last is reporting the findings and conclusions.
C. RESULT AND DISCUSSION

1. Student learning difficulties based on indicators on a diagnostic test

The diagnostic test used consists of seven questions with seven different indicators. Based on the results of the analysis found results as in Table 2.

<table>
<thead>
<tr>
<th>No. Ques</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The student's score on this question is 91%, so it can be said that students understand the attributes of a linear function. When given a function of inverse ratio, 91% of students can determine that it is not a linear function with reasons.</td>
</tr>
<tr>
<td>2</td>
<td>Achievement of student scores on this problem is 94%, so that it can be said 94% of students can determine the level of change in the linear function of the given equation.</td>
</tr>
<tr>
<td>3</td>
<td>The score for this question is 84%, so it can be said that 84% of students can determine the rate of change in the value of the linear function of the given graph.</td>
</tr>
<tr>
<td>4</td>
<td>The score for this question is 58%, because the achievement does not reach the minimum completeness of 75% so it can be said that students have difficulty determining the slope of a line from a known equation.</td>
</tr>
<tr>
<td>5</td>
<td>The score for this problem is 81%, so it can be said that 81% of students can draw a graph of a linear function on Cartesian coordinates.</td>
</tr>
<tr>
<td>6</td>
<td>The score for this question is 0%, none of the students answered this question correctly. Because the achievement did not reach the minimum completeness of 75%, it can be said that students have difficulty making linear equations based on graphs that are known to use parallel or perpendicular relationships by creating the necessary points</td>
</tr>
<tr>
<td>7</td>
<td>The score for this question is 2%, because the achievement does not reach the minimum completeness of 75% so it can be said that students have difficulty solving contextual problems using linear functions</td>
</tr>
</tbody>
</table>

The material in question Number 4 regarding the slope of the line. The results of the analysis show that there are errors and limitations in students' understanding of linear functions in the discussion of the slope of the line. This is in line with research Acuña (2007); Zaslavsky et al. (2002) which found that there was a lot of confusion among students regarding the relationship between the algebraic and geometric aspects of slope, scale, and angles. In question Number 6 ask students to make a linear line by first determining the required point based on a graph that is known to use parallel or perpendicular relationships. In this question, none of the student’s answers were correct, the students only redraw the graphs of the questions and rewrote the information on the questions. Similar results were found in the study (NoprianiLubis et al., 2017) with the results showing that the average percentage of students in solving problems in determining linear equations from graphs reached 49.25%, which is in the less good category.

In question Number 7 it was found that students had difficulty solving contextual problems using linear functions. This is in line with findings Adu et al. (2015); Bardini et al. (2004) which found that students had difficulty solving word problems on linear functions. The use of contextual problems offers some potential to engage and motivate students in learning mathematics as well as presenting some challenges for students in the classroom (Widjaja, 2013). This is in line with the opinion of Salavera et al. (2019) which states that in order for students' knowledge to be effectively constructed from students' daily lives, the problems presented during learning must start from contextual problems.
2. **Students’ learning difficulties based on cognitive level**

The diagnostic test used consisted of seven items with six Bloom’s taxonomy on the cognitive domain revised by Anderson and Krathwohl (Anderson & Krathwohl, 2021). The following details of the cognitive level in each question are shown in Table 3.

<table>
<thead>
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<th>No. Ques</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student completeness in answering problems on linear function material as a relationship between pairs of numbers/quantities both of which can change with a constant level of change at the cognitive level of understanding is 91%</td>
</tr>
<tr>
<td>2</td>
<td>Mastery of students in answering problems in the matter of the level of change in linear functions at the cognitive level remembering is 94%</td>
</tr>
<tr>
<td>3</td>
<td>Student completeness in answering problems in the matter of the relationship between the level of change and the graph of a linear function at the applying cognitive level is 84%</td>
</tr>
<tr>
<td>4</td>
<td>Student completeness in answering problems on the material of the slope of the line with the cognitive level of analyzing is 58%</td>
</tr>
<tr>
<td>5</td>
<td>Student completeness in answering problems on linear function graph material at the applying cognitive level is 81%</td>
</tr>
<tr>
<td>6</td>
<td>None of the students were able to answer problems in the matter of linear equations/lines from graphs that were known to be at the cognitive level of creating</td>
</tr>
<tr>
<td>7</td>
<td>Student mastery in answering contextual problems using linear functions with a cognitive level evaluating is 2%</td>
</tr>
</tbody>
</table>

Based on the cognitive level students can remember, understand and apply the concept of linear function but still experience learning difficulties at high levels namely applying, evaluating and creating. The creating level, which is the highest level in the cognitive taxonomy, is closely related to the previous level. When students face difficulties in implementing, analyzing and evaluating a problem, indirectly, they also have challenges in making or creating solutions (Yee et al., 2010). In questions with a cognitive level creating open-ended problems are used which makes students confused and unfamiliar with the problems given. Learning with open-ended problems can develop students’ mathematical creative thinking skills (Damayanti & Sumardi, 2018; Rahayuningsih et al., 2021).

3. **Students’ learning difficulties in completing diagnostic tests on linear function material**

Based on the results of the analysis on the results of the written test, questions Number 4, 6 and 7 indicated that students had learning difficulties in completing the diagnostic test on linear function material. After the written test was carried out to obtain more in-depth data an interview was conducted. The following are questions on diagnostic test Number 4.

\[
\text{is known that the line } g \text{ is perpendicular to the line through the points (3,-1) and (2,5), determine the slope of the line } g!
\]

**Exploration results of student answers on question Number 4:**
Figure 2. Exploration Results of Student Answers to Question Number 4

Figure 2(a) shows students’ unfinished answers. Students have been able to determine the slope of the line through points (3, -1) and (2, 5), but have not answered questions that ask for the slope of a line perpendicular to that line. Figure 2(b) shows that there was no accuracy in the students’ answers to the questions. Students know the strategy for determining the slope of a line, namely using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ but found inaccuracies in substituting points in the formula. Using a dot with one of the negative signs will show the student’s thoroughness.

Based on the analysis of students’ answers then supported by interview results, it was found that in question Number 4 it was found difficulties in algorithmic abilities including a lack of planning ability (strategy knowledge) and solving ability (algorithmic knowledge). In doing, basically students know how the slope of two lines that are parallel or perpendicular but admit that they forget to use it to solve problems. This is due to a lack of planning ability. In addition, students are also able to perform operations on integers but are not careful in inputting values causing errors in operating numbers. The following are questions on diagnostic test Number 6:

Here are the lines $k, l, m, n$ and $g$ with $k$ parallel to $l$, $m$ parallel to $n$ and $g$ perpendicular to $k$

Make an equation of the unknown line on the graph by first determining the required point.
Exploration results of student answers on question Number 6, as shown in Figure 3.

![Figure 3](image)

**Figure 3. Exploration Results of Student Answers to Question Number 6**

Based on the analysis of students' answers and then supported by interview results, it was found that in question Number 6 it was found difficulties in using the principle of linear function, and a lack of understanding ability (schematic knowledge) as indicated by the difficulty in determining the point through which the line passed, and the difficulty in applying the principle of parallel gradients or perpendicular but also difficulties in using the concept including the inability to remember the concept, the inability to deduce useful information from a concept. None of the student's answers led to a solution. Students only copy the pictures on the answer sheet.

The following is a question on diagnostic test Number 7:

Car rental fees are offered in two different methods. In the first method, tenants can pay IDR 400,000 per day and in the second method, tenants pay IDR 250,000 per day plus IDR 1,200 per kilometer of mileage.

What advice do you give to people who rent a car for a day, about a payment method based on the number of kilometers traveled to make it more economical?

Exploration results of student answers on question Number 7, as shown in Figure 4.

![Figure 4](image)

**Figure 4. Exploration Results of Student Answers to Question Number 7**
Among the many students who took the diagnostic test, only two students whose answers led to the correct solution. In Figure 4(a) it was found that students were able to evaluate the situation even though they did not provide clear boundaries regarding near and far at the distance traveled. Whereas in Figure 4(b) the student’s answers have led to the correct evaluation results. However, between the two answers, there were no students who changed the conditions in the contextual questions into a mathematical model in the form of a linear function. The other answer is in the form of a conclusion without a clear basis, more accurately called an intuitive guess.

Based on the analysis of students’ answers then supported by interview results, it was found that in question Number 7 it was found difficulties in using the principle of linear functions, lack of mastery of the basics of algebra and lack of ability to understand (schematic knowledge) which is indicated by difficulty recognizing linear functions in contextual problems, errors in algebraic computing, difficulties in using the concept include the inability to remember the concept, the inability to deduce useful information from a concept and the lack of ability to understand (schematic knowledge) which is indicated by incomplete writing of formulas. In general, students’ learning difficulties in completing diagnostic tests on linear function material are presented in Table 4.

<table>
<thead>
<tr>
<th>No. Ques</th>
<th>Analysis of Learning Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Difficulties in algorithmic abilities include a lack of planning abilities (strategy knowledge) and problem-solving abilities (algorithmic knowledge).</td>
</tr>
</tbody>
</table>
| 6        | Difficulties in using the principle of linear function, and lack of ability to understand (schematic knowledge) which is indicated by the difficulty in determining the point through which the line passes, and the difficulty in applying the principle of parallel or perpendicular gradients.  
|          | difficulties in using the concept including the inability to remember the concept, the inability to deduce useful information from a concept and the lack of ability to understand (schematic knowledge) which is indicated by incomplete writing of formulas. |
| 7        | Difficulties in using the principle of linear functions, lack of mastery of the basics of algebra and lack of ability to understand (schematic knowledge) which is indicated by the difficulty of recognizing linear functions in contextual problems, errors in algebraic computations,  
|          | difficulties in using the concept including the inability to remember the concept, the inability to deduce useful information from a concept and the lack of ability to understand (schematic knowledge) which is indicated by incomplete writing of formulas. |

D. CONCLUSION AND SUGGESTIONS

Based on the analysis of student errors, the types of student difficulties in understanding and using concepts/principles in linear function material were obtained, including: difficulties in algorithmic abilities, difficulties in using linear function principles, and difficulties in using concepts. The limitation of this research is the limited subject. The number of subjects studied is still minimal, so it is possible that there are things that might happen but are not found in the subjects studied. This research can be continued by creating appropriate learning designs to overcome all learning difficulties found.
REFERENCES


