Control Chart for Correcting the ARIMA Time Series Model of GDP Growth Cases

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ABSTRACT

The essential prerequisite for attending the G20 conference is a country's GDP because G20 members can significantly boost the economy and preserve the nation's financial stability. Time series data can be thought of as a country's Gross Domestic Product (GDP) at a particular point in time. In this research, the GDP numbers from five Southeast Asian nations that are attending the G20 conference were used. The total was 47 observations made yearly, which extended from 1975 to 2001. A time series analysis was performed on the data gathered. The correctness of time series models is also evaluated using control charts based on this research. The control chart is constructed using the time series model's residuals as observations. After applying the IMR control chart for verification, the results revealed that the residuals, specifically the models for GDP in Malaysia, Singapore, and Thailand, are out of control. The white noise assumption is fulfilled by the time series model obtained for Brunei and Indonesia's GDP, but the residuals are out of control. Whether controlled residuals are used depends on the accuracy with which the time series model predicts the future. If the amount of residuals is under control, then the time series model produced is accurate and good enough for prediction. After using the IMR control chart to verify the residuals, the results indicate that the residuals, namely the models for GDP in Malaysia, Singapore, and Thailand, are not under control. The assumption of white noise is proved correct by the time series model obtained for the GDP of Brunei Darussalam and Indonesia. With that being said, the residuals are entirely out of control. The model must improve its ability to forecast various future periods. It is a consequence of the unmanageable residuals that the model contains. Even if the best available model has been obtained based on the criteria that have been defined, it is anticipated that the research findings will improve the theories that have previously been developed and raise knowledge regarding the usefulness of testing the time series model. In addition to all of that, it is intended that the research will produce a summary of cases of an increase in GDP from five Southeast Asian countries participating in the G20 conference.

A. INTRODUCTION

As one of the largest economic powers, the influence of economic development is reflected in the development of the Gross Domestic Product (GDP). GDP is the value of goods and services in a country produced by the factors of production owned by citizens of these countries and foreign countries. GDP is also defined as a method of calculating national income. GDP is
considered gross or gross, so income derived from the production of goods and services produced by residents of a country who are abroad is not considered (Hasanov et al., 2021).

The GDP value of a country is the main requirement for becoming a member of the G20 forum, because G20 members are considered capable of making a significant contribution to the economy and maintaining the country's financial stability (Terra dos Santos et al., 2023). The GDP determines the ranking or order of the G20 countries.

The Group of Twenty, also known as the G20, is an important forum for international economic cooperation comprising the countries with the largest economies in the world. There are now 19 countries and one institution from the European Union members of the G20. The main goal of the G-20 is to bring together the leaders of the world's primary developed and developing economies to address global economic challenges. Indonesia is considered a member of the G20 because it is experienced in overcoming economic crises, seen as a developing economy with huge potential in terms of both size and economic impact in the Asian region (Purwati et al., 2023). Apart from Indonesia, countries in Southeast Asia that are also members of the G20 are Brunei Darussalam, Malaysia, Singapore, and Thailand. The five countries each have GDP characteristics.

The value of a country’s GDP as it stood during a specific era is time series data. One of the approaches in time series analysis is called the Autoregressive Integrated Moving Average (ARIMA) model, and it is one of the ways that may capture the required information on the value of GDP over a particular period. A significant amount of study has been conducted on applying the ARIMA model to GDP instances, some of which are (Abdelghafar et al., 2023), (Hằng & Dũng, 2022), and (Muma & Karoki, 2022). ARIMA is a method for forecasting time series that is useful for reliably predicting many different variables quickly, efficiently, and affordably. The ARIMA method sometimes goes by the term Box-Jenkins time series because it was developed further in 1970 by George Box and Gwilym Jenkins. Order identification, parameter estimation, and diagnostic checking are the three iterative processes of the Box-Jenkins technique that can be utilized when modeling time series data (Farimani et al., 2022).

The diagnostic test step is when time series analysis yields the best model, and the residual must be white noise for this to be the case. It is essential to determine how accurate the model is compared to the best model to make accurate projections for the following periods to predict some different periods in the future.

The findings of the research that Imro’ah et al. (2023) and Imro’ah & Huda (2022) carried out indicate that the residuals are a good indicator of how accurate the time series model is for the time series. The residuals can be used to determine how accurate a time series model is. An accurate time series model will have residuals that are in a state where they are statistically controlled (Herdiani et al., 2018). Furthermore, it is evident on the control chart. Numerous scholars have contributed to the extant information on control charts by using time series data in their inquiries. Furthermore, this study investigates the resistance towards integrated quality, maintenance, and production models, specifically focusing on the issue of delayed monitoring as observed in the ARMA control chart.

In this study, the correctness of a time series model is investigated by employing a control chart to collect data. Data on the gross domestic product (GDP) of five countries Indonesia, Brunei Darussalam, Malaysia, Singapore, and Thailand have been incorporated into the case
study. The Individual Moving Range (IMR) control chart displays the residuals derived using the best models. The time series model will produce reliable results for various forecasts provided that the residuals can be controlled. Even though predetermined criteria have acquired the best available model, it is anticipated that the research findings will enhance the theories produced and expand knowledge regarding the use of time series model verification. In addition to that, the results of this research are anticipated to provide a summary of the combined GDP of the five Southeast Asian countries that are participants in the G20.

B. METHODS

The GDP numbers of five Southeast Asian countries participating in the G20 fulfillment are used in this analysis. The period utilized is on an annual basis, extending from 1975 to 2001, and there were 47 observations. After that, the data were analyzed using the ARIMA method, which included three iterative steps of Box-Jenkins analysis. An IMR control chart is utilized to perform validation checks on the model. We should be using the model with the Akaike Information Criterion (AIC) value, the lowest possible score.

To begin the process of data analysis for this study, utilize ARIMA to model the GDP data for each of the five countries. Testing for stationarity, determining order by examining plots of the autocorrelation function (ACF) and the partial autocorrelation function (PACF), estimating parameters, and verifying diagnostics are all components of this modeling process. After selecting the best model, the model will be put through a verification process. How successfully the verification is carried out can be determined by building an IMR control chart with residuals collected from the most potent ARIMA model. It is possible to conclude that the ARIMA model makes correct predictions regarding the subsequent period if the plot demonstrates that everything is under control. On the contrary, the model can be deemed as inaccurate when the plot demonstrates that conditions are out of control.

This research has two primary stages: the first involves modeling GDP data using ARIMA, and the second involves evaluating the model created in the first stage. To begin modeling data using ARIMA, the first step is to check the stationarity of the data. When the mean and variance of the data are not steady, differentiation and data transformation are utilized. Order identification, parameter estimation, and diagnostic testing are the three iterative stages followed by the succeeding process, which involves the Box-Jenkins algorithm. To achieve the utmost precision, it is necessary to have the most exact model. The next step is to use the control chart, which is the IMR control chart, to validate the most accurate ARIMA model created during the modeling stage. The process starts with the calculation of residuals, followed by the construction of an IMR control chart. In the event that the residual is under control, the generated model is regarded to be accurate to make predictions that stretch into several potential future periods. Every step involved in the research process is described in further detail in Figure 1.
If the ARIMA process is followed by a random variable called $Z_t$, $Z_t$ can be defined as in Equation 1 (Bhattacharyya et al., 2022). It has $\phi_p(B) = (1 - \phi_1B - \cdots - \phi_pB^p)$, $\theta_q(B) = (1 - \theta_1B - \cdots - \theta_qB^q)$, and $a_t$ is an error at time $t$. Equation (1) is named $ARIMA(p,d,q)$ model with $p$; $d$; $q$ are the auto-regressive; differentiation; and moving average orders. If $p = 0$, the model is named $IMA(d,q)$. Similarly, if $q = 0$, the model is named $ARI(p,d)$ model.

$$\phi_p(B)(1-B)^dZ_t = \mu + \theta_q(B)a_t$$  \hspace{1cm} (1)

According to Auger-Méthé et al. (2021) state that time series modeling involves three primary processes. The initial stage involves model identification, wherein the orders of $(p,d,q)$ in the ARIMA model are selected based on the examination of the patterns observed in the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The subsequent stage involves the estimate of parameters by widely employed techniques such as Maximum Likelihood estimate (MLE) and Ordinary Least Squares (OLS). The last stage involves diagnostic testing to assess the model’s residual, encompassing tests for residual white noise and residual normalcy. Following three primary stages, the optimal model is selected and employed as a point of reference for forecasting the future, as documented by Bae et al. (2021).

**Figure 1.** Flowchart of the Control Chart Implementation in Time Series Model
A control chart is a method that can be used to examine whether a process is statistically in control, fix problems, and enhance quality. The control chart comprises three lines: the Center Line, also known as CL, which represents the average value of quality attributes; the Upper Control Limit, also known as UCL; and the Lower Control Limit, also known as LCL. The UCL and LCL are the upper and lower control limits, respectively (Zeng et al., 2023). Figure 2 is an example of a control chart for some conditions. The process is called to be out of control if Imro’ah et al. (2023), Lindgren et al. (2020): some points lie outside the control limits (Figure 2a), seven straight points above or below CL (Figure 2b), there exists a sequence of six or seven consecutive points that exhibit an ascending or descending trend (Figure 2c).

![Figure 2](image.png)

**Figure 2.** The instances of a control chart illustrating a procedure that is not inside control limits

If a process is out of control, the cause should be investigated and analyzed (Alevizakos et al., 2021). In the meantime, the other criterion that determines whether or not a process is considered under control is whether or not the sample points fall inside the control limits. This circumstance is represented in Figure 2(d). Suppose \( w \) is a statistical sample of the quality and \( \mu_w \) as its mean and \( \sigma_w \) as its standard deviation. The control limits and CL are expressed as follows (Hossain & Riaz, 2021).

\[
\begin{align*}
UCL &= \mu_w + k \sigma_w \\
CL &= \mu_w \\
LCL &= \mu_w - k \sigma_w
\end{align*}
\]  

The distance between the control limit and the CL is \( k \) in standard deviations (Zwetsloot et al., 2023). Control charts are created using a set of principles called Shewhart control charts. Variable and attribute charts are the two primary categories of Shewhart charts. These categories are depending on the qualities being measured (Ahsan et al., 2022). If the sample's quality parameters can be measured and stated in numbers, then a variable control chart
should be utilized. There are three distinct types of variable control charts, which are referred to as $\bar{X} - R$ (for sample sizes of ten or fewer), $\bar{X} - s$ (for sample sizes of ten or more), and IMR (for sample sizes of one exclusively) (Fan et al., 2021).

According to (Padmarajan & Selvaraj, 2021), an IMR chart is a type of variable control chart that utilizes individual observations, where each observation consists of a single value. Because the range is determined by shifting data from one test to the next, this method of determining the range is referred to as the moving range (Leonov et al., 2020). This control chart determines if a process is stable when the data being gathered are variable, and every element being collected is an independent piece of data. In most cases, it is utilized in businesses that operate nonstop, such as the cement and chemical fertilizer manufacturing industries. Because the outcomes are consistent across all the data sets, a single representative sample is all that is required. When only a modest amount of manufacturing is available or when only a small amount of testing is allowed to be undertaken owing to cost restrictions, it is also possible to use it.

A basic assumption of control charts is about independence, and there is no correlation between observations (Sales et al., 2020). In a time series model, the assumption of independence and lack of correlation among residuals is necessary to construct a time series control chart, the observations that are used only one variable observed, called the residuals (Haddaji et al., 2022). Therefore, any model can use a time series control chart if the residuals fulfill the white noise's assumption (Qiu et al., 2020). A control chart to be used in this condition is the IMR chart, and its calculation is shown in Table 1 (Wang et al., 2022).

<table>
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<tr>
<th>Table 1. The Formula of IMR Control Chart</th>
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<tr>
<td><strong>Individual plot</strong></td>
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<td>$UCL = 3 \times \frac{MR}{d_2}$</td>
</tr>
<tr>
<td>$CL = 0$</td>
</tr>
<tr>
<td>$LCL = -3 \times \frac{MR}{d_2}$</td>
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Where $\overline{MR}$ is the average of $MR_i$ with $MR_i = |x_i - x_{i-1}|$, $x_i$ is $i$-th observation, $d_2$ and $D_4$ are constants. Let the residuals ($e_i$) be independent and identical to normal distribution so it has zero means, and $\sigma_e^2$ is used for the time series control chart, so the steps to construct a time series control chart is (Imro’ah & Huda, 2022): compute the moving range ($MR_i$) and mean of moving range ($\overline{MR}$); calculate the control limits and CL (see Table 1); construct the IMR control chart. Residuals are plotted in the individual plot, while the moving ranges that have been calculated are plotted in the moving range plot. If the residuals are in control, the time series models are accurate and predictive (Kim & Ha, 2022). If the residuals are out of control, low precision in the time series model (Lai et al., 2022). So the model needs to be evaluated and re-identified (Rasheed et al., 2023).
C. RESULT AND DISCUSSION

This study uses data on the GDP values of Indonesia, Brunei Darussalam, Malaysia, Singapore, and Thailand from 1975 - 2021, totalling 47 observations. The data was obtained from the website of The World Bank. The GDP values of the five countries tend to have the same pattern (see Figure 3). Apart from Brunei Darussalam, in 1998, all countries had the lowest GDP values. It is because, at that time, there was a massive financial crisis. The lowest GDP value in 1998 occurred in Indonesia. Indonesia has faced three recessions, namely in 1963, 1998, and 2020/2021. The three crises were triggered by different causes and with different impacts. The recession in 1963 was triggered by hyperinflation. Indonesia’s economy improved after the dark period of 1965 and boomed in the 1970s and 1980s. In the early 1990s, the Indonesian economy was in a high growth period of around 6%. Indonesia's inflation was also only at 5.1%. After going through high growth, Indonesia experienced a severe recession in 1998. The economy contracted by 13.13%, while Indonesia’s inflation soared by 77.63% in 1998. The domestic economy contracted by 6.4% in the first quarter. The contraction grew to 16.8% in the second quarter and 17.4% in the fourth quarter. The Asian Financial Crisis triggered the 1998 recession. The crisis started when Thailand abandoned its fixed exchange rate policy against the US dollar in July 1997. This policy made many companies default due to the weakening currency. The crisis spread to Southeast Asian countries, including Indonesia. The crisis dropped the rupiah exchange rate from Rp 2,500 to Rp 16,900 per US dollar. As a result of the recession, large and medium industries decreased drastically from 22,997 companies in 1996 to 20,422 in 1998. The number of workers during that period dropped to 18.5% or 3.53 million people. The third recession that Indonesia is experiencing is in 2020/2021. The national GDP contracted for four quarters, namely from the second quarter of 2020 to the first quarter of 2021, caused by the health crisis, as shown in Figure 3.

![Figure 3. Time Series Plot of GDP values](image)

Brunei Darussalam had the lowest GDP value during the observation period, namely 19,827, which occurred in 1981. It occurred due to the decline in the price of crude oil, which is the primary source of state revenue. The policy of reducing oil production to extend the life of oil reserves and increase the recovery rate is also the cause. After implementing the production cut policy, Brunei’s average oil production fell to 154,805 barrels per day from 1960 to 2021. Increased awareness in Brunei Darussalam of the country’s depleted natural resources
has prompted the government to diversify its economy from excess dependence on oil and gas. Figure 4 presents the analysis results of this study based on the steps presented in Figure 1.
Based on Figure 4, the best time series model (marked with AIC values printed in red) for each country is Indonesia, Brunei Darussalam, Malaysia, Singapore, and Thailand respectively are ARIMA (0,0,1), ARIMA (0,0,2), ARIMA (0,0,1), ARIMA (0,1,1), ARIMA (1,1,1). For Malaysia, Singapore, and Thailand, the models obtained did not pass the diagnostic test; the residuals were not independent (seen in the ACF residual plot in step 5). The IMR control chart (in step 6) shows that the residuals are out of control. The residual value significantly differs from other residuals if it is outside the control limits. Consequently, there is a significant disparity between the estimation results and the observations. It means the model obtained is not accurately used to predict future periods. While the best model obtained for Indonesia and Brunei Darussalam passed the diagnostic test (residuals were mutually independent and normally distributed), the residuals were also out of control. It indicates that other models may not be identified in the ACF and PACF plots. In addition, the existence of outliers in the data is one of the causes of this. Except for Brunei Darussalam, all countries’ IMR control charts (step 6) reveal that the residual out-of-control happened during the 24th observation or in 1998. It was brought on at the time by a severe financial crisis. The decline in asset values, the difficulties that firms and consumers are having paying off their loans, and the lack of liquidity in the financial system are the causes. Investors panicked at the possibility of their assets losing
value, selling assets, or withdrawing funds from savings accounts during the financial crisis. Additionally, the bursting of speculative financial bubbles, stock market collapses, government defaults, and currency crises all impacted the monetary crisis.

D. CONCLUSION

It was discovered that time series models could not fulfill the white noise assumption followed by time series data modeling steps using GDP data from five nations. The residuals, namely the models for GDP in Malaysia, Singapore, and Thailand, are out of control after verification using the IMR control chart. The time series model obtained for the GDP of Indonesia and Brunei Darussalam fulfill the white noise assumption. Nevertheless, the residuals are out of control. Consequently, the model must be better at predicting many future periods. It is a result of the model's uncontrollable residuals. As a result, even the most incredible model developed during the time series modeling step may only sometimes be a reliable forecasting tool. Making a control chart of the best residual model is one method of confirming the best model.

According to the research findings, selecting the best model for a time series analysis is not enough if the individual does not also pass the diagnostic test stage (for example, the residuals must fulfill the white noise assumption). It was one of the key takeaways from the study. To assess whether the residuals from the model are statistically controlled (in control) or not (out of control), it is required to develop a control chart utilizing the best model. The time series model's predictive accuracy is evaluated using controlled residuals or not. If the residual is within control, the time series model that was created will have high accuracy and will work well for predictive purposes. However, if the residual is out of control, the time series model that was developed needs to be investigated.

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