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Comparison Of The Accuracy Of The Solution Of Non-Linear Equations By Euler And Fixet Point Methods Through Numerical Simulation

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Abstract: This study aims to analyze the comparison of Euler method and FixedPoint method in solving the roots of non-linear equations numerically. The assessment criteria to be used include convergence, stability, and computational speed of each method. The equations used include trigonometric, pilinomial, exponential and logarithmic. Experiments were conducted 8 times with an error of 0.001 and using a maximum of 100 iterations. From the four cases of solving the tested equations, the results show that the FixedPoint method is more accurate in calculating the solution of non-linear equations compared to the Euler method, with 1 iteration on trigonometric equations. on polynomial equations the Euler method is faster than the Fixed point with 1 iteration. on exponential equations the Euler method is faster than the Fixed point with 1 iteration. In the second logarithmic equation Euler method is faster than Fixed point with 1 iteration. Therefore it can be said that the Euler method has a faster convergence rate and higher accuracy than the Fixed point method in all cases. So it can be said that Euler is the best method in solving the roots of non-linear equations. These findings provide new insights in choosing the right method for numerical applications in solving the roots of non-linear equations, as well as contributing to the development of more efficient numerical algorithms.

Keywords: Non-Linear Equation, Euler Method, Fixed Point Method, Numerical Simulation.

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A. INTRODUCTION

Non-linear equations are equations that contain variables in non-linear operations, such as quadratic, exponential, or trigonometric functions (Vilinea et al., 2020) . Common examples include$x^2 - 2 = 0$ and sin(x)=0, which cannot be solved analytically. In many cases, exact solutions are difficult to find, so numerical techniques are used to approximate the solution. One of the methods used is Euler and Fixed Point, which allows us to find solutions iteratively. Non-linear equations often appear in various fields such as engineering, physics, and economics (Rohmatulloh, 2024). Therefore, numerical approaches are very important in solving these problems. This method calculates a solution that is close to the correct value through repeated calculations (Fanani et al., 2024).

The Euler method is one of the explicit numerical techniques used to calculate the solution of differential equations or non-linear equations (Nuryadi, 2023) . In this method, the solution at the next step is calculated based on the value at the previous step, using an iterative formula involving a time step size h Although simple, the Euler method has limitations in terms of accuracy, especially for problems with high variability (Syaharuddin et al., 2019) . On the other hand, the fixedpoint method is a numerical technique used to find the solution of an equation that can be expressed in the form x=g(x), where g(x) is a continuous function and satisfies certain convergence conditions. In this method, the solution is sought by repeated iterations, namely by calculating new values based on previous values until it converges on the desired solution (Karunia, 2021). These two methods have advantages and disadvantages in terms of convergence speed and accuracy, and their comparison is often analyzed in numerical research to choose the most effective method (Sihotang, 2023).

An accuracy comparison between the solution of non-linear equations using Euler and Fixed Point methods reveals important insights into the performance and applicability of both methods (Wigati, 2020). Various studies have explored the convergence rate, accuracy, and computational efficiency of these methods, providing a comprehensive understanding of their respective advantages and disadvantages. For example, studies on the Euler and FixedPoint methods show that the Euler method can face instability problems if not carefully regulated. On the other hand, the Fixed Point method requires proper selection of initial values to ensure good convergence (Rozi & Rarasati, 2022).

Research related to numerical method convergence and stability in solving initial value problems (IVP) shows that classical numerical methods such as Euler and Fixed Point methods often face difficulties in dealing with complicated non-linear differential equations (Hoyali et al., 2024) . In many cases, more sophisticated methods tailored to the problem conditions can provide more accurate and stable results (Gofur, 2023). However, the application of these methods requires extra attention to the proper selection of parameters and numerical techniques to avoid potential computational errors that can occur, especially in simulations with complex dynamics (Gde Agung et al., 2021).

In addition, research comparing the Newton-Raphson and Regula-Falsi methods shows that Newton-Raphson has a better convergence rate and is more effective in solving algebraic and transcendental equations (Batarius, 2018). This method provides higher accuracy in solution approximation. On the other hand, combining ordinary numerical methods with multiplication methods, as found in research Darwin Damanik et al. (2022), can also produce more accurate results for non-linear equations involving exponential and logarithmic functions (Azmi et al., 2019).

Although many studies have discussed the Euler and Fixed Point methods, there are still some aspects that have not been widely explored, especially in the context of applications to more complex non-linear equations (Diningsih & Yulia, 2023). Most studies only test both methods on relatively simple cases and do not take into account the influence of parameter selection, such as step size in the Euler method or initial value in the FixedPoint method, on the results obtained. In addition, there are still few studies that discuss the stability of these two methods under extreme conditions or when the equation parameters are very large or small (Hoyali et al., 2024). Therefore, further research is needed to find out how these two methods work on more complicated problems and how to choose the right parameters to improve the accuracy and efficiency of the solution (Luthfiana, 2020) .

This study aims to compare the accuracy and efficiency of Euler and Fixed Point methods on various non-linear equations (Salimi et al., 2017). The main focus is to explore the effect of parameter selection such as Euler step and FixedPoint initial value on convergence and accuracy. This study will also examine the stability of both methods under various numerical conditions, both in the case of simple and complex equations. It is hoped that this research will provide better insight in choosing the right numerical method, as well as contribute to practical applications in mathematics education and scientific research in the fields of science and engineering (Nugraha & Nurullaeli, 2023).

B. RESEARCH METHOD

In this section, we will explain in detail about the two methods used to solve the roots of non-linear equations, namely the Euler Method and the Fixed Point Method. Each method will be explained starting from the basic concept, iterative formula, to its numerical application to solve the roots of non-linear equations. In this study, the researcher compares the procedure of solving non-linear equations using Euler and Fixed Point methods to find the roots of these equations. The non-linear equations used involve non-linear equations in the form of trigonometric, logarithmic, polynomial and exponential. Each method was tested and implemented using Matlab software. This implementation aims to evaluate the performance of both methods in terms of accuracy, convergence speed, and computational efficiency. The explanation related to the two methods used (Euler and Fixed Point) will be discussed in detail in the following section.

1. Euler Numerical Method

The Euler method is one of the numerical methods used to solve ordinary differential equations (PDB) that cannot be solved analytically. This method is Pandia & Sitepu (2021) a simple explicit method and is often used in computer programming to solve dynamics problems or phenomena described by differential equations (Yudhi, 2020). Euler's method is used to solve ordinary differential equations that do not have analytic solutions, with a simple and fast numerical approach

The Euler method is an explicit method used to approximate the solution of a first differential equation that cannot be solved analytically. This method works by using initial values and approximating the solution of the differential equation by dividing the time interval into steps (Sitompul & Siahaan, 2023) . This method is very useful for solving the first differential equation that cannot be solved analytically, by using a given initial value (Sihombing & Dahlia, 2018). Euler Method Formula. Interactive Formula of Euler's Method that can be used to solve ordinary differential equations (PDB) by Euler's method. Suppose we have a differential equation:

$$\frac{dy}{dt} = \mathrm{f}(t\,, y) \tag{1}$$

Where:

- $y\ adalah\ fungsi\ dari\ waktu\ (t)$
- $f(t\,, y)\ adalah\ fungsi\ yang\ menentukan\ perubahan\ y\ terhadap\ t$

2. Fixet Point Numerical Method

The The fixed point method is one of the numerical methods used to find the solution of non-linear equations, especially in finding the roots of a function (Yahya & Nur, 2018). In simple terms, we find the value of x that satisfies the equation f(x)=0 by transforming it into a fixedpoint equation x = g(x). This method is very useful when non-linear equations are difficult to solve analytically. In practice, this method is widely used in various fields such as physics, economics, and engineering (Anuraga et al., 2021). The main advantage of the FixedPoint method is its ability to handle non-linear equations that cannot be solved directly.

The FixedPoint Method is a numerical technique to find the solution of a non-linear equation f(x)=0 by transforming it into a fixedpoint form x=g(x). With this method, the solution of the equation is sought iteratively, starting with an initial guess and calculated repeatedly using the iteration formula (Karunia, 2021) . The process continues until the calculated value is close to the desired solution, which is when the difference between iterations becomes very small. The FixedPoint Method is often used in computer programming to solve equations that arise in mathematical simulations or numerical optimization.

Basic Formula of FixedPoint Method

Suppose we have a non-linear equation f(x)=0, to find the root x=α of the equation, we can transform it into a fixedpoint form x=g(x), where g(x) is a function derived from the equation f(x). The iterative formula is:

$$x_n + 1 = \mathrm{g}\ (x_n) \qquad (2)$$

Where:

- x_n *adala nilai pendekatan solusi pada iterasi ke* $- n$.
- $g(x)$ *adalah fungsi yang di turunkan dari persamaan* $f(x) = 0$, *dimana solusi* x *berada pada titik tetap* $g(x) = x$.
- $x_n + 1$ *adalah nilai pendekatan solusi pada iterasi berikutnya*.

Both methods are then used to solve the problem of non-linear equations. The non-linear equations used involve trigonometric, logarithmic, polynomial and exponential non-linear equations. Furthermore, the problem used to perform the simulation consists of:

Solution 4 Problem

- Trigonometr
  Find f(x) = sin (x) - $\frac{x}{2}$
- Logarithms
  Find f(x) = log(x) +x^2 - 4 = 0
- Polynomials
  Find f(x)=x^3 - 4x^2 + x - 1= 0
- Exponential
  Determine f(x)=e^x -x^2 = 0

In order to clarify the stages of solution in this research, an algorithm is presented below that illustrates the logical flow from the process of identifying the type of function to determining the starting point that is the basis for iterating the numerical method. This algorithm represents the systematic steps used in the research as a guideline in obtaining the solution of nonlinear equations.

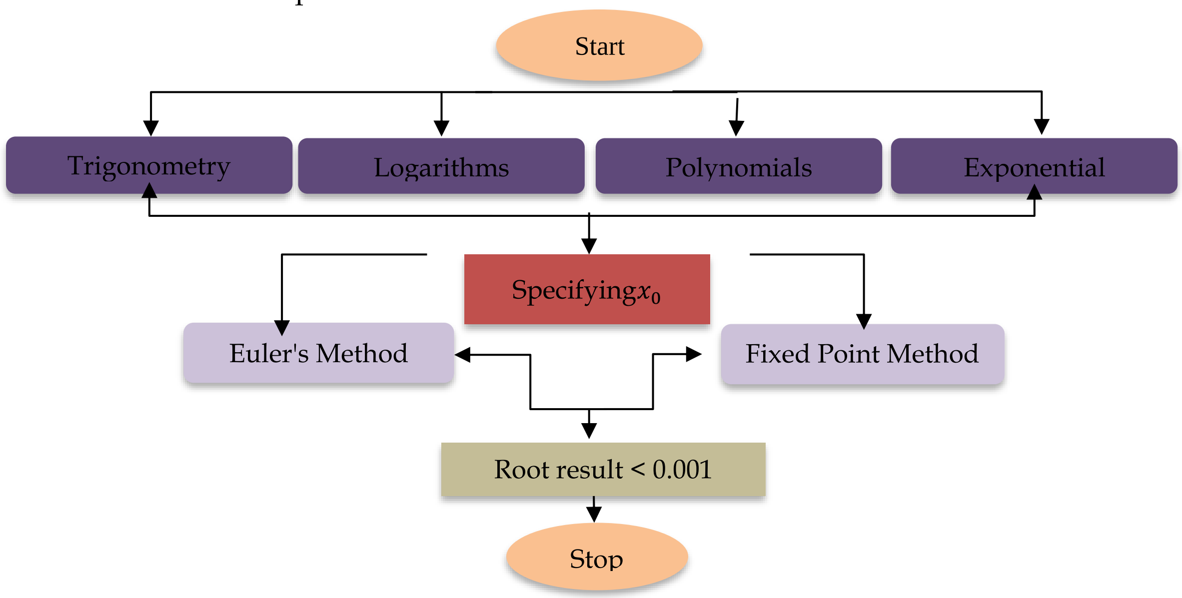

Figure 1. Shows The Flow of Research Conducted

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C. RESULTS AND DISCUSSION

Researchers used 4 non-linear equation problems consisting of trigonometric, polynomial, exponential, and logarithmic equations. Based on the steps that have been taken, the following are the graphical results of each equation according to Figure 2, Figure 3, Figure 4, and Figure 5 below.

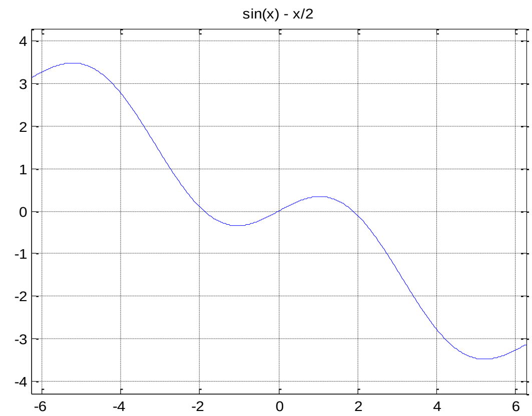

Figure 2. shows the graph of solving trigonometry problems

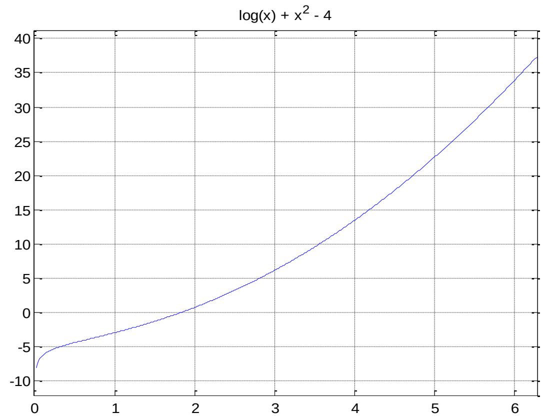

Figure 3. shows the graph of logarithm problem solving

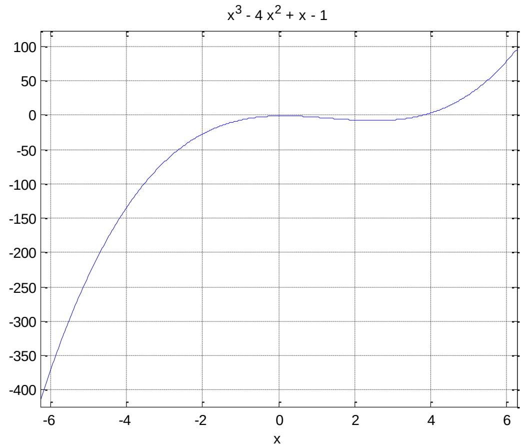

Figure 4. shows the graph of polynomial problem solving

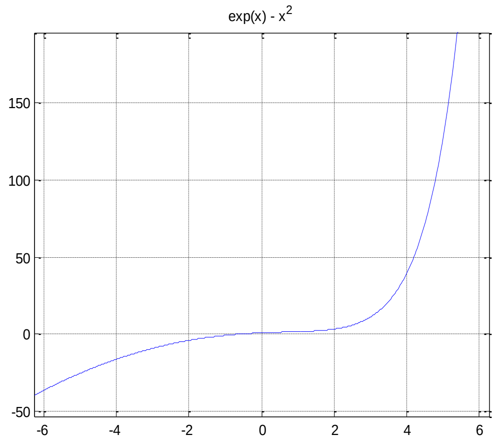

Figure 5. shows the graph of exponential problem solving

Based on Figure 2 above, it can be seen that the root of the equation is in the interval [-6, 6]. So in this case, point 2 is chosen as the starting point for testing using the euler method and the foxed point method in the matlab application cript. Based on Figure 3 above, it can be seen that the root of the equation is in the interval [0, 6]. So in this case, point 2 is chosen as the starting point for testing using the Euler method and the foxed point method in the Matlab application script. Based on Figure 4 above, it can be seen that the root of the equation is in the interval [-6, 6]. So in this case, point 4 is chosen as the starting point for testing using the Euler method and the foxed point method in the Matlab application cript. Based on Figure 5 above, it can be seen that the root of the equation is in the interval [-6, 5]. So in this case, point -1 is chosen as the starting point for testing using the Euler method and the foxed point method in the Matlab application script.

As a follow-up to the graphical analysis in Figures 2,3,4 and 4, a numerical approach was used to test the solution through a predetermined starting point. The implementation of the numerical method is done by utilizing Matlab scripts, which are designed for two different methods, namely the Euler method and the fixedpoint method. The details of the script for each method are presented in Table 1 below.

Table 1. Script Of Each Method

<table>
<tr><th>Methods</th><th>script</th></tr>
<tr><td>Euler</td><td>for k=1:imax
   iter=iter+1;
   % Chebyshev formula
 L=feval(f_diff2,x1)*feval(f,x1)/feval(f_diff1,x1)^2;
   x2=x1-(1+0.5*L)*(feval(f,x1)/feval(f_diff1,x1));
   error=abs((x2-x1)/x2);
   x1=x2;
   y=feval(f,x2);
   fprintf('%10.0f %6.10f %6.10f %6.10f\n',iter,x1,y,error);

   if (error< error1 || (iter>=imax)), break, end
end
fprintf('The end is = %6.10f\n',x1);</td></tr>
<tr><td>Fixed Point</td><td>for k=1:imax
   iter=iter+1;
   x2 = feval(g, x1); % Calculate the next iteration
   error = abs((x2 - x1) / x2); % Relative error
   y = feval(f, x2); % Function value at current root
   fprintf('%10.0f %6.10f %6.10f %6.10f\n',iter,x2,y,error);

   if error < error1 || iter >= imax
      break;
   end
   x1 = x2; % Update x1 value
end

fprintf('The end is = %6.10f\n', x2);
figure;
plot(x, y, '-o', 'LineWidth', 1.5);
xlabel('x');
ylabel('y');
title('Numerical Solution with Euler's Method');
grid on;</td></tr>
</table>

By using the eular method script and the fixedpoint method above, the researcher conducted 8 trials using polynomial, trigonometric, exponential, and logarithmic problems with an error value = 0.001 and a maximum iteration value = 100. Then the simulation results are shown in Table 2 below

Table 2. Simulation Results

No.	About	Methods	Iterations	X	F(x)	Error
1	x^3 - $4x^2$ + x - 1= 0	Euler	1	3.8088744148	0.0361153419	0.0501790199
		Fixed point	9	3.8119935828	0.0800168586	0.0007113892
2	$sin(x)$ - $\frac{x}{2}$	Euler	1	1.8961312990	-0.0005219358	0.0547792767
		Fixed point	12	1.8959954872	-0.00046296	0.0006791657
3	$e^x - x^2$ =0	Euler	1	-0.7005661860	0.0055112428	0.2862915596
		Fixed point	100	-3.1237878037	-9.7140600167	1.0671423553
4	$log(x)$+x^2 - 4 =0	Euler	1	1.8413538898	0.0010852581	0.086157316
		Fixed point	4	1.8411699997	0.0003082068	0.0003081593

In accordance with the above simulation results explain that:

According to the results of the above computations using the Euler method, for case 1 it is known that the value of x so that f(x) < 0.001 is x = 1.8961312990 with f(x) = -0.0005219358, the value was obtained after computing only 1 loop. for case 2 it is known that the value of x so that f(x) < 0.001 is x = 1.8413538898 with f(x) = 0.0010852581, the value was obtained after computing up to 1 loop. for case 3 it is known that the value of x as f(x) < 0.001 means x = 3.8088744148 using f(x) = 0.0361153419, the value was obtained after computing only 1 loop. And for case 4, it is known that the value of x as f(x) < 0.001 means x = -0.7005661860 using f(x) = 0.0055112428, the value was obtained after computing only 1 iteration.

According to the results of the above computation using the FixedPoint method, for case 1 it is known that the value of x so that f(x) < 0.001 is x = 1.8959954872 with f(x) = -0.0004106296, the value was obtained after computing up to 12 iterations. for case 2 it is known that the value of x so that f(x) < 0.001 is x = 1.8411699997 with f(x) = 0.0003082068, the value was obtained after computing up to 4 iterations. for case 3 it is known that the value of x as f(x) < 0.001 means x = 3.8119935828 using f(x) = 0.0800168586, the value was obtained after computing up to 9 iterations. And for case 4, it is known that the value of x as f(x) < 0.001 means x = -3.1237878037 using f(x) -9.7140600167, the value was obtained after computing up to 100 iterations.

Several studies have been conducted by various researchers who discuss the application of numerical methods in solving non-linear equations. One of them, research by (Hutagalung, 2017) .Compares the effectiveness of the Euler method and the Fixed Point method in finding the roots of non-linear equations. The study showed that although both methods are effective, they have different characteristics in terms of convergence speed and stability. In addition, another study also examined the use of the FixedPoint method with a Matlab graphical interface (GUI) to facilitate visualization and implementation in solving non-linear equations.

D. CONCLUSIONS AND SUGGESTIONS

Based on numerical simulation results, the FixedPoint method is proven to be more accurate in calculating the solution of non-linear equations compared to the Euler method. The FixedPoint method gives results that are closer to the correct analytical solution, although it requires more iterations to converge. In contrast, the Euler method is faster in calculation, but has larger errors, especially for non-linear equations with sensitive initial conditions.

In terms of accuracy, Fixed Point is superior because it is able to produce more precise solutions despite its longer computation time. However, Euler's method remains useful in problems that require fast calculations with a lower level of accuracy, especially in applications that do not require high precision. This research shows that the selection of an appropriate method depends on the specific needs of the problem at hand, such as computational speed and the desired level of accuracy.

Therefore, it is recommended to conduct further research by expanding the variety of functions and initial conditions to compare in more detail the performance of the Euler method, and the Fixedpoint method in solving root equations, as well as exploring the possibility of parameter optimization or modification to the Euler method, and the Fixedpoint method to improve the success in finding solutions for various types of functions. In addition, future researchers can conduct a more in-depth analysis of the factors that affect the convergence speed of each method, such as numerical stability and computational complexity. Research the possibility of combining or integrating different methods to create a new, more efficient approach to solving root equations.

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