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Comparative Analysis of Halley and Hybrid Methods for Numerically Solving the Roots of Non-Linear Equations

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Abstract: Finding the roots of nonlinear equations is a fundamental problem in numerical analysis with wide applications in engineering, science, and applied mathematics. The purpose of this study is to conduct a comparative analysis between Halley's method and a selected hybrid method for numerically solving the roots of nonlinear equations. Halley's method is a third-order iterative technique known for its fast convergence when provided with a good initial guess. On the other hand, hybrid methods are designed to combine the strengths of multiple numerical algorithms to enhance accuracy, stability, and robustness against different function characteristics. This study employs four test functions—polynomial, trigonometric, exponential, and logarithmic—to evaluate the performance of both methods in terms of convergence speed, computational efficiency, and sensitivity to initial guesses. The results indicate that Halley's method performs better in terms of speed under ideal conditions, while the hybrid method is more reliable in handling diverse nonlinear behaviors. Therefore, the appropriate method selection should consider both the nature of the function and the need for speed or stability in the computation.

Keywords: Non-linear equation roots, Hybrid Method, HalleyMethod, Numerical Methods.

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A. INTRODUCTION

Mathematics is a science that is applied in everyday life and plays an important role in solving various problems. Problems are often encountered such as those related to finding solutions to non-linear equations (Amri et al., 2021). Mathematical problems in engineering often encounter non-linear equations. Functions f (x) can be in the form of algebraic equations, polynomial equations, trigonometric equations, transcendental equations. Finding the roots of these equations means making the equation zero or $f(x)=0$. Not all equations can be solved easily using mathematical theory, but require numerical or computational techniques (Salwa et al., 2022).

Solving the roots of non-linear equations is a major challenge in various disciplines, such as physics, engineering, and economics (Arruda, 2024). Many physics and engineering problems require numerical solutions to find the roots of non-linear equations that cannot be solved analytically. Numerical solutions require an iteration process (repeated calculations) of existing numerical data. The use of Scilab v.6.0.0 software will greatly reduce the iteration time. However, since numerical solution is also an approximation, errors are inevitable. Error is closely related to accuracy or states how far the solution is from the exact value (Wigati, 2020). Several numerical methods are used to solve these equations, including the Newton-Raphson method, Halley's method, and hybrid methods that combine several numerical techniques to obtain more accurate and efficient results. One of the popular and powerful numerical methods in determining the roots of the solution is the Newon-Raphson (NR) method, NR method is widely applied in several disciplines such as estimation, informatics and electronics. The concept of the NR method is to use derivatives to accelerate convergence, but the NR method can also diverge (away from the roots) (Darmawan & Zazilah, 2019).

According to Sanchez (2009), the Newton-Raphson method is also one of the best methods to determine the root solution of nonlinear equations. In its development this method has undergone many advances, not only finding the root of a function, but this method is also used to find the optimal point of an equation in nonlinear optimization. In recent years, the development of iteration methods to determine the roots of nonlinear equations has been carried out by modifying existing methods or combining their use together (Raenagus, 2021). Alternatively, Halley's method was developed to improve the convergence rate. This method has the advantage of cubic convergence, which is faster than the Newton-Raphson method in some cases. However, more complex calculations, such as the need to calculate the second and third derivatives of the function, can be a drawback of this method.

On the other hand, hybrid methods are approaches that combine the advantages of different methods to overcome the weaknesses of a single method (Kong & Tsai, 2014). The most common hybrid method is a combination of the Newton-Raphson method and other methods, such as the secant method or the Halley method itself. In many cases, hybrid methods can provide more stable and faster convergence, especially when the functions used have complex or ill-defined properties at some points Ana Sutisna suggests that Hybrid learning is a learning method that combines two or more methods and approaches in learning to achieve the goals of the learning process (Fauzan & Arifin, 2017).

While the Halley and Hybrid methods are claimed to have a faster convergence rate than other methods and both apply the concept of derivatives to the iteration calculation process (Darmawan & Zazilah, 2019). This study aims to analyze the comparison between Halley's method and the hybrid method in numerically solving the roots of non-linear equations. The analysis involves testing several non-linear equations with different properties, to evaluate the convergence speed, solution accuracy, and stability of the methods under varying conditions. It is hoped that the results of this study can provide insight into the selection of more efficient and reliable methods in solving non-linear problems.

B. METHOD

In this section, we will explain in detail about the two methods used to solve the roots of non-linear equations, namely the Hybrid Method and the Halley Method. Each method will be explained starting from its basic concept, iterative formula, to its numerical application to solve the roots of non-linear equations. In this study, the researchers compared the solution procedures (Hybrid and Halley) in selecting the roots of non-linear equations. then for the implementation of each formulation, the researchers used Matlab software. The description of the two methods used is explained below.

1. Method Halley

Halley's method is a numerical method developed from the Newton-Raphson method, which uses a second-order Taylor series approach. Unlike the Newton method which only uses the first derivative, the Halley method involves the first and second derivatives of the function to accelerate convergence. The Halley method iteration formula is as follows:

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'(x_n)^2 - f(x_n)f''(x_n)} \quad (1)$$

The main advantage of Halley's method is its faster convergence speed compared to Newton's method for some cases, as the approach uses more information.

2. Method Hybrid

The Hybrid method is to combine several methods contained in the recommendation system to produce recommendation items that match the user's wishes. The following is one of the formulas of the Hybrid method, namely Hybrid Bisection and Modified Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (2)$$

$$x_{n+1} = \frac{x_{n+}x_{n-1}}{2} \qquad (3)$$

Both methods are then used to solve the problem of non-linear equations in the form of functions. Furthermore, the problems used for simulation consist of:

a. F(x) = $2x^3 + 4x^2 + 5x + 6$ (Polinomial)
b. F(x) = $2x^2 \sin(3x + 6)$ (Trigonometri)
c. F(x) = $2xe^{-4x} + 6$ (Eksponensial)
d. F(x) =$2x \log(2^{x+1} + 6)$ (Logaritma)
e.

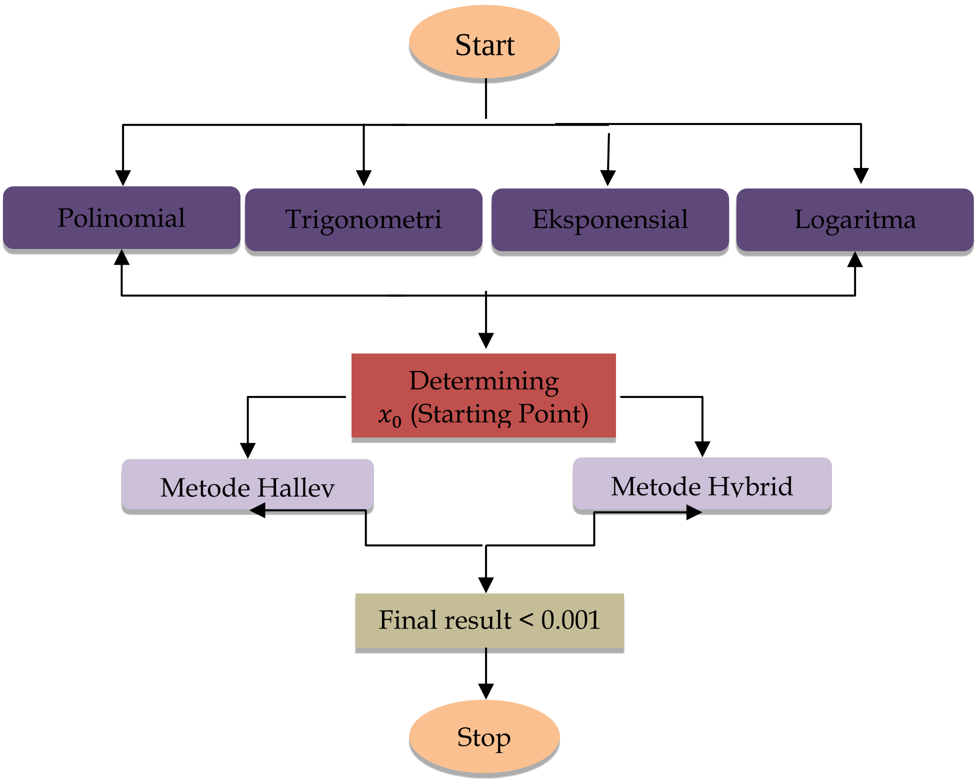

Figure 1. shows the flow of research conducted

The flowchart above illustrates a systematic process for solving the roots of a mathematical function using numerical methods. The process begins with the Start step, followed by the identification of the type of function to be analyzed. The diagram categorizes functions into four main types: Polynomial, Trigonometric, Exponential, and Algebraic. Identifying the type of function is essential, as each has unique characteristics that influence the selection of an appropriate numerical method. Once the function type is determined, the

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next step is to define the initial value , which serves as the starting point for the iteration process. The choice of this initial value is critical, as it significantly affects the convergence behavior of the selected numerical method. The initial value is then used as input for one of the two available iterative methods: the Halley Method or the Hybrid Method.

The Halley Method is an iterative technique that utilizes the first and second derivatives of the function and is known for its faster convergence compared to the Newton-Raphson Method. Meanwhile, the Hybrid Method combines two or more numerical techniques to enhance accuracy and stability, especially when a single method is insufficient. The choice between these methods depends on the characteristics of the function and the desired outcome. The iteration process continues until the final result meets a specified error tolerance, defined as less than 0.001. Once this criterion is achieved, the process stops, indicated by the Stop step. This signifies that the function's root has been found with an acceptable level of accuracy. Overall, the flowchart provides a comprehensive and structured overview of the steps involved in numerically solving root-finding problems. It aids in the proper selection of methods and the efficient organization of the iteration process.

C. RESULTS AND DISCUSSION

Researchers used 4 non-linear equation problems consisting of trigonometric, polynomial, exponential, and logarithmic equations. Based on the steps that have been taken, the following are the graphical results of each equation according to Figure 1, Figure 2, Figure 3, and Figure 4 below:

$$\mathbf{2x^3 + 4x^2 + 5x + 6}$$

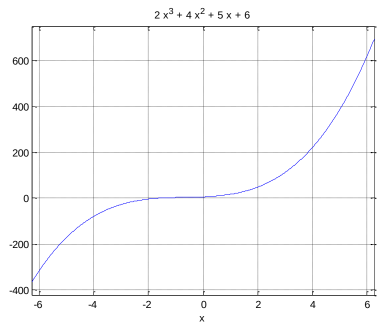

Figure 1. Graph of a polynomial

$$\mathbf{2x^2 sin(3x + 6)}$$

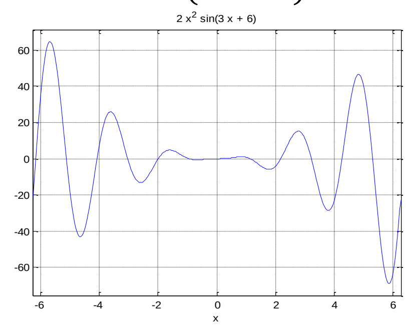

Figure 2. Graph of atrigonometric

$$\mathbf{2xe^{-4x} + 6}$$

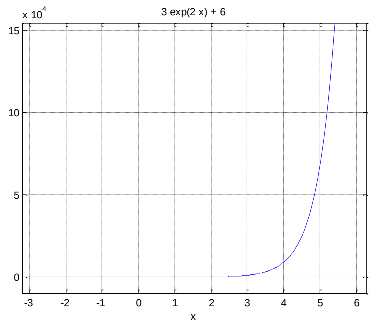

Figure 3. Graph of a exponential

$$\mathbf{2xlog(2^{x+1} + 6}$$

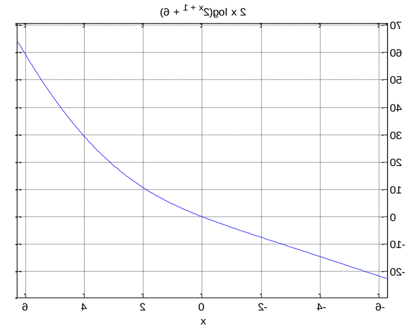

Figure 4. Graph of a logarithmic

In Figure 1 above, it can be seen that the roots of the equation are in the interval [-6,6]. In this case, the starting point x * 0 = [3] is chosen as the starting point to find the roots of the polynomial equation. In the simulation process in Matlab with the algorithm of two methods, namely the Halley method and the Hybrid method, the following results are obtained. In Figure 2 above, it can be seen that the equation f (x) = 2x ^ 2 * sin (3x + 6) is in the interval [-6,6]. In this case, the interval [-3,-2] is chosen as the starting point to find the roots of the equation. In Figure 3 above, it can be seen that the roots of the equation f (x) = 3e ^ (2x) + 6 are in the interval [-3,6]. In this case, the interval [-0.3,5] is chosen as the starting point to find the roots of the equation. In Figure 4 above, it can be seen that the roots of the equation f (x) = 2x log (2 ^ (x + 1) + 6) are in the interval [-6,6]. In this case, the interval [-1,0] is chosen as the starting point for finding the roots of the equation.

The following is the script used in each method. This script is used for simulation in the matlab application. Simulation on matlab application.

Tabel 1. SCRIPT Halley end Hybrid

Method Halley	for k=1:imax
	iter=iter+1;
	%Rumus Halley
	L=feval(f_diff2,x1)*feval(f,x1)/feval(f_diff1,x1)^2;
	x2=x1-((2/2-L)*(feval(f,x1)/feval(f_diff1,x1)));
	galat=abs((x2-x1)/x2);
	x1=x2;
	y=feval(f,x1);
	fprintf('%10.0f %6.10f %6.10f %6.10f\n',[iter;x1;y;galat])
	if (galat<galat1 || (iter>imax)),break,end
	end
	fprintf ('Akarnya adalah = %6.10f\n',x1)

Method Hybrid	for k=1:imax
	iter=iter+1;
	%Rumus Hybrid
	A=feval (f_diff2,x1);
	B=6*feval(f_diff1,x1)-2*feval(f_diff2,x1)*x1;
	C=6*feval(f,x1)-6*feval(f_diff1,x1)*x1+feval(f_diff2,x1)*(x1)^2;
	x2=-B+(sqrt(B^2-4*A*C))/(2*A);
	galat=abs((x2-x1)/x2);
	x1=x2;
	y=feval(f,x1);
	fprintf('%10.0f
	%6.10f %6.10f %6.10f\n',[iter;x1;y;galat])
	if (galat<galat1 || (iter>imax)),break,end
	end
	fprintf ('Akarnya adalah = %6.10f\n',x1)

Using the script, the researchers then simulated 8 times with the matlab script. The simulation was carried out with the aim of calculating the iteration and the root of the equation, then obtained the simulation results in table 1 below

Table 2. Simulation results

NO	Kasus	Metode	Iterasi	X	f(x)	Galat
1	2x^3 + 4x^2 + 5x + 6	Halley	11	-1.6063195738	-0.0000001301	0.0000545979
		Hybrid	1	-31.6300000000	-59290.7001720219	1.0015795165
2	2x^2 sin (3x + 6)	Halley	3	-0.9528046880	0.0000121972	0.0006249058
		Hybrid	4	0.0101435000	-0.0000514602	0.0007750661
3	3*exp(2*x) + 6	Halley	2	4.1053241937	11045.7831339531	0.0000662652
		Hybrid	3	31212.1869778478	6.0000000000	0.9999350130
4	2*x*log (2^(x+1)) + 6)	Halley	3	1.9154169347	9.9830662356	0.4469538922
		Hybrid	2	-2043.7553595427	-7323.8360364925	0.9959015237

According to the results obtained as a result of the above computation using Halley's method, for case 1 it is known that the value of x such that f(x) < 0.001 is x =-1.6063195738 with f(x) = -0.0000001301, the value was obtained after computing up to 11 iterations. for case 2 it is known that the value of x such that f(x) < 0. 001 is x = -0.9528046880 with f(x) =0.0000121972, the value was obtained after computing up to 3 iterations. for case three it is known that the value of x as a result f(x) <0.001 means x = 4.1053241937 using f(x) =11045.7831339531, the value was obtained after computing up to 2 iterations. . for case four, it is known that the value of x as a result of f(x) < 0.001 means x = 1.9154169347 using f(x) = 9.9830662356, the value was obtained after computing up to 3 iterations.

By using the Hybrid method, for case 1 it is known that the value so that f(x) < 0.001 is x = -31.6300000000 with f(x) = -59290.7001720219, the value is obtained after computing up to 1 iteration. for case 2 it is known that the value of x so that f(x) < 0.001 is x = 0.0101435000 with f(x) =-0.0000514602, the value is obtained after computing up to 4 iterations. for case 3 it is known that the value of x so that f(x) < 0.001 is x = -31212.1869778478 with f(x)=6.0000000000, the value is obtained after computing up to 3 iterations. for case 4 it is known that the value of x so that f(x) < 0.001 is x = -2043.7553595427 with f(x)= -7323.8360364925, the value is obtained after computing up to 2 iterations.

Thus it shows that for solving the root equation of f(x) = 2x sin (3x+6) using the Halley method the convergence rate is slower when compared to the Hybrid. For solving the root equation of f(x) = 2xe + 6 using the Halley method the convergence rate is faster when compared to the Hybrid method. And for solving the root equation of f(x) = log (2 + 6) using the Halley and hybrid methods.

Several studies have been conducted by several authors who simulate numerical methods. One of the studies conducted by (Salwa, 2022), examined the Comparison of the Newton Midpoint Halley Method, the Olver Method and the Chabysave Method in solving the Roots of Non-Linear Equations. Furthermore, (Qisty, 2022), examined the Biseks Method using GUI Matlab: A Simulation and Solution of Non-Linear Equations. Furthermore, there is (Mandailina, 2020) examining the Wilkinson Polynomial: Accuracy Analysis according to the Numerical Method of Taylor Series Derivatives. And finally, there is (Putri & Syaharuddin, 2019), examining the Implementation of Open and Closed Methods Numerically: Convergence Test of Equation Solutions.

D. CONCLUSIONS AND SUGGESTIONS

From the four cases of equation solving (trigonometric, polynomial, exponential, and logarithmic) tested, it can be concluded that the hybrid method has a faster convergence rate and higher accuracy than the Halley method in most cases, although the iteration calculation is more complex. Therefore, it can be concluded that the hybrid method is the best method in solving the roots of non-linear equations. Based on these findings, it is recommended that future research should further explore the application of the hybrid method on a wider variety of non-linear equations, especially those with higher degrees of complexity or those arising in real-world engineering problems. Additionally, the efficiency and stability of the hybrid method under different initial conditions and computational constraints should also be investigated. This could help in developing more robust numerical tools for practical implementation in scientific and engineering fields.

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