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Comparative Analysis of Newton Midpoint Halley and Olver Methods for Solving the Roots of Nonlinear Equations

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Abstract: This study compares the Newton Midpoint Halley (NMH) method and the Olver Method in numerically solving the roots of non-linear equations. The main objective of this study is to evaluate the efficiency and convergence speed of both methods. Results show that the Newton Midpoint Halley method is superior in convergence, with a faster quadratic convergence rate and fewer iterations to achieve a certain accuracy than the Olver method. Although Olver's method is effective, NMH is able to achieve solutions with small errors in fewer iterations. In addition, the NMH method has a simpler and more intuitive algorithm, making it easier to implement in math teaching, especially for math education students. Although the Olver method offers higher accuracy, in terms of efficiency and ease of implementation, the Newton Midpoint Halley method is more recommended. Knowledge of both methods is important for students and can be applied in teaching and further applied mathematics research.

Keywords: Newton Midpoint Halley, Olver Method, Solving Roots Of Non-Linear Equations, Convergence, Efficiency, Numerical, Math Education, Error, Iteration, Algorithm.

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A. INTRODUCTION

In various fields of science, especially in exact sciences such as mathematics, physics, and engineering, we are often faced with problems involving non-linear equations (Rangkuti, 2019). These non-linear equations are often difficult or even impossible to solve analytically, namely by using standardized mathematical formulas. Therefore, numerical methods become a very important tool in finding solutions to these equations (Tukan et al., 2024). A non-linear equation is a mathematical equation where the variables do not have a linear relationship with each other. In other words, it cannot be expressed in the general form y = mx + c, where m is the gradient and c is a constant. Non-linear equations often involve variable powers higher than one, trigonometric functions, exponentials, polynomials or logarithms (Syata et al., 2023).

In this study, a comparative analysis between the Newton Midpoint Halley Method (NMH) and the Olver Method in numerically solving the roots of non-linear equations is conducted. The Newton Midpoint Halley method is a combination of three approaches designed to improve the accuracy and speed of convergence in finding roots. Through simulations on various non-linear functions, it is evident that the Newton Midpoint Halley method gives better results in terms of convergence speed compared to the Olver method, although Olver shows good robustness under less than optimal initial conditions. This makes the Newton Midpoint Halley method an attractive option for applications in the context of mathematics education, where a deep understanding of numerical methods is necessary.

The analysis shows that the Newton Midpoint Halley Method, by utilizing a combination of the advantages of each method, is able to overcome some of the problems often encountered in conventional methods. Tests conducted on various functions with different levels of complexity, such as polynomial functions, show that the Newton Midpoint Halley method's consistency in convergence makes it a more efficient option for handling non-linear equations. While Olver's method provides reliable solutions under certain conditions, the speed and accuracy of Halley's Newton Midpoint method make it more attractive for application in a broader context. This study shows that the selection of an appropriate numerical method is highly dependent on the characteristics of the function being analyzed (Charismana et al., 2022). The Newton Midpoint Halley method results in innovation in numerical approaches that can support students to understand more deeply the issues of convergence and accuracy in solving non-linear equations (Pratamasyari et al., 2017).

This research lies in the lack of a comprehensive study comparing the two numerical methods Newton Midpoint Halley, and Olver in solving the roots of non-linear equations simultaneously. Although each method has been widely used in various contexts, a direct comparison between the two methods in terms of efficiency, convergence speed, and solution accuracy has not been widely discussed, particularly in the context of applications to more complex non-linear equations (Bintang et al., 2024). In addition, variations in the characteristics of different non-linear equations can affect the performance of each method, so this research seeks to fill the gap in the literature by providing a deeper insight into the practical and theoretical comparison between the methods.

This study aims to compare which Newton Midpoint Halley method and Olver method is faster in terms of converging by identifying the advantages and disadvantages of each method in terms of convergence, accuracy, and efficiency of computing time in finding the root solution of non-linear equations is expected to provide deeper insight into which methods are more effectively used in the context of solving non-linear equations, both in the aspect of convergence speed and accuracy of the results obtained (Mandailina et al., 2020).

B. RESEARCH METHOD

In this study, we compared the procedure of solving non-linear equations using Newton Midpoint Halley and Olver methods to find the roots of these equations. Each method was tested and implemented using Matlab software. This implementation aims to evaluate the performance of both methods in terms of accuracy, convergence speed, and computational efficiency. The explanation of the two methods used (Newton Midpoint Halley and Olver) will be discussed in detail in the following section.

1. Newton Midpoint Halley Numerical Method

The Newton Midpoint Halley Method (NMH) is a numerical technique developed to solve nonlinear systems of equations by combining three approaches: Newton Method, Newton Midpoint Method, and Halley Method. The solution process begins by using the Newton Method to find the initial solution of the system of nonlinear equations (Yudhi, 2020) . After obtaining the initial solution, the value is then substituted into the Midpoint Newton Method to improve the result. The final step involves using the solution from the Midpoint Method in the Halley Method to reach the final solution. Iteration will stop when the error of the last iteration is smaller than the specified tolerance limit (Estri et al., 2018) . Research shows that the Newton Midpoint Halley method has a faster convergence rate than other methods, so it can reduce the number of iterations and computation time in finding the roots of nonlinear equations.

The formula for the Newton Midpoint Halley (NMH) method in solving systems of nonlinear equations can be expressed as follows:

$$xi + 1 = xi - \frac{2f(xi)}{f'(xi)} \cdot \frac{f'(xi)^2}{2f^{"}(xi)+f(xi)f"(xi)} \qquad (1)$$

Where:

- $xi + 1$ is the next iteration value,
- xi is the current iteration value,
- $f(x)$ is the function that we want to find the root of,
- $f'(x)$ is the first derivative of the function $.f(x)$
- $f"(x)$ is the second derivative of the function $.f(x)$

The method starts by finding the initial solution using Newton's method, then the result is improved by Newton's Midpoint method, and finally substituted into Halley's method to get the final solution. The iteration process continues until the error of the last iteration is smaller than the predetermined tolerance limit.

Table 1. Script Table

Methods	Script

Newton Midpoint Halley	for k=1:imax
	iter=iter+1;
	%NMH formula
	xnn=x1-(feval(f,x1)/feval(f_diff1,x1));
	xnb=x1-(feval(f,x1)/feval(f_diff1,0.5*(xnn+x1)));
	x2=xnb-((2*feval(f,xnb)*feval(f_diff1,xnb))/(2*feval(f_diff1,xnb)^2
	feval(f,xnb)*feval(f_diff2,xnb)));
	error=abs((x2-x1)/x2);
	x1=x2;
	y=feval(f,x1);
	fprintf('%10.0f %6.10f %6.10f %6.10f\n',[iter;x1;y;error]);
	if (error < error1 || (iter>imax)),break,end
	end
	fprintf('The root is = %6.10f\n',x1);

Using the script, the researcher conducted 4 experiments using polynomial, trigonometric, exponential, and logarithmic problems with a maximum iteration of 100 and an error of 0.001.

2. Olver Numerical Method

The Olver method is a numerical approach developed to solve complex mathematical problems, especially in the context of high-oscillation integrals and Riemann-Hilbert problems (Khaidir, 2019). The method focuses on developing analytical and numerical techniques that can improve accuracy and efficiency in numerical calculations. In this context, Olver and his colleagues have shown that the method can be effectively applied to various types of mathematical problems, including differential and integral equations that are difficult to solve analytically (Murtafiah, 2017).

One of the main applications of Olver's method is in the solution of Riemann-Hilbert problems, where this approach allows the computation of solutions by taking into account the behavior of solutions in distant regions. This is particularly important in the context of complex analysis, where solutions often have complicated and unpredictable behavior (Juldial & Haryadi, 2024). The method exhibits high stability and good approximation properties, which make it an attractive option for handling problems with noisy or unstable initial conditions.

In addition, Olver's method is also used in the context of high-oscillation integrals. Research shows that this method can calculate high-oscillation integrals with better efficiency compared to traditional methods (Dewi et al., 2024). In this regard, Olver showed that by utilizing the asymptotic behavior of the integral, this method can provide more accurate results Sari et al. (2018), even when the integral is highly oscillatory. This shows that the more oscillatory an integral is, the better the numerical approximation can be achieved, which is a finding contrary to the common view that highly oscillatory problems are difficult to solve.

Olver's method also has wide applications in solving differential equations. In this context, the method is used to solve inverse scattering problems, which allows the calculation of solutions without the need to perform spatial discretization or time steps. This demonstrates the versatility of Olver's method in a wide range of mathematical and physical applications, as well as its ability to provide accurate and efficient solutions in complex contexts (Zahri, 2023). Overall, the Olver method in a numerical context is a powerful and efficient approach to solving a wide range of mathematical problems, focusing on stability, accuracy, and speed of calculation. The method has proven effective in a variety of applications, including high-oscillation integrals and Riemann-Hilbert problems, as well as in the context of complex differential equations. The formula for the Olver method in solving systems of nonlinear equations can be expressed as follows:

$$xi + 1 = xi\,\frac{f(xi)}{f'(xi)} - \frac{1}{2}\left(\frac{[f(xi)]^2}{f'(xi)[f''(xi)]}\right) \qquad (2)$$

This formula shows that Olver's method requires the calculation of the second derivative, so although the results converge faster, the complexity of the calculation is higher. This makes Olver's method more efficient in situations where high accuracy is required.

Although Olver's method offers better speed and efficiency, its use is not as popular as Halley's Newton Midpoint method. This is due to the need for more complicated calculations and a general lack of understanding of the method among practitioners. However, research shows that in many cases, especially for polynomials of high degree, Olver's method can give very accurate results with very small errors in fewer iterations compared to other methods.

Table 2. Script Table

Methods	Script

Olver	for k=1:imax
	iter=iter+1;
	%Over formula
	x2=x1-(feval(f,x1)/(feval(f_diff1,x1))-
	0.5*(((feval(f,x1))^2*feval(f_diff2,x1))/(feval(f_diff1,x1))^3);
	error=abs((x2-x1)/x2);
	x1=x2;
	y=feval(f,x1);
	fprintf('%10.0f %6.10f %6.10f %6.10f\n',[iter;x1;y;error])
	if (error < error1 || (iter>imax)),break,end
	end
	fprintf('The root is = %6.10f\n',x1)

Using the script, the researcher conducted 4 experiments using polynomial, trigonometric, exponential, and logarithmic problems with a maximum iteration of 100 and an error of 0.001.

Solution 4 questions

- Polynomials
  Specify $f(x) = x^3 - 6x + 3$
- Trigonometry
  Specify $f(x) = \sin(x) - \frac{1}{2}$
- Exponential
  Specify $f(x) = e^x - 3x$
- Logarithms
  Specify $f(x) = \log(x) + x^2 - 2$

The following is the flow of research conducted. In the initial stage, the researcher compiled a script to simulate each method, then in the second stage, the simulation results were compared by looking at the percentage of error and the number of iterations produced. The research procedure is described in Figure 1 below.

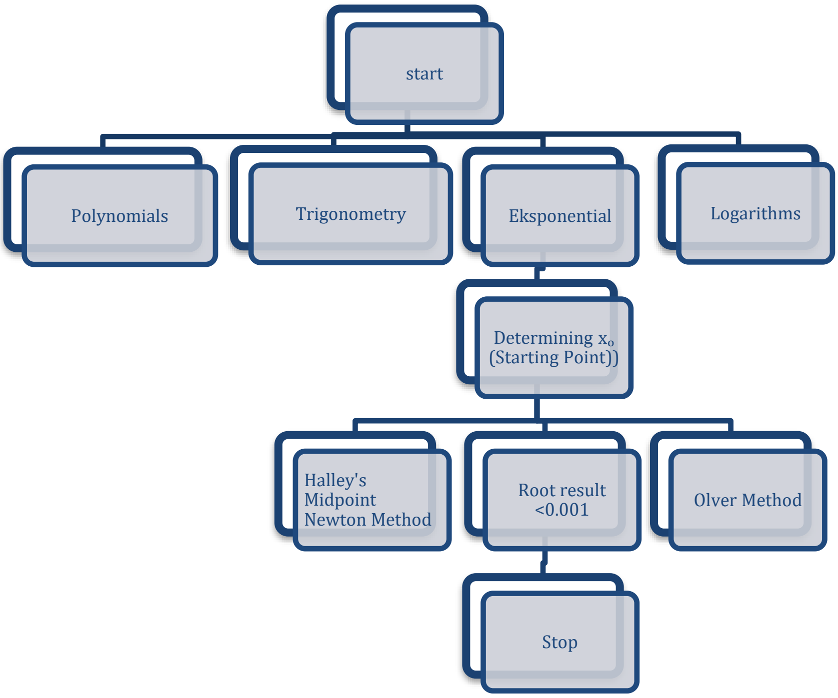
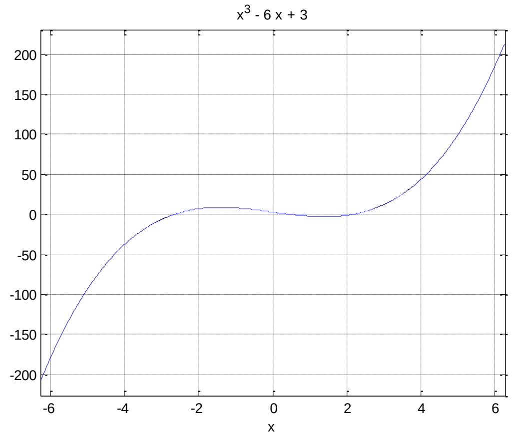


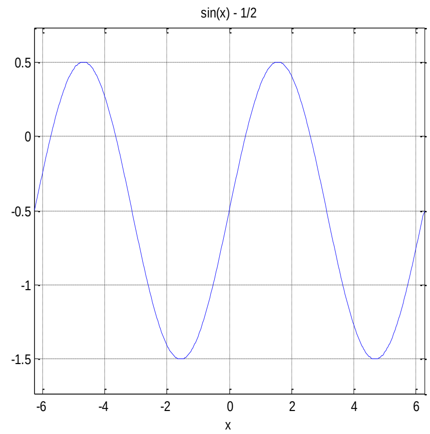
Figure 1. Shows The Flow Of Research Conducted

C. RESULTS AND DISCUSSION

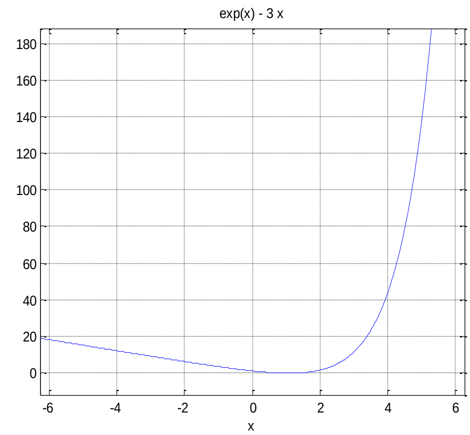
Researchers used four non-linear equation problems consisting of polynomial, trigonometric, exponential, and logarithmic equations. In accordance with the steps that have been done is a graph due to each equation. Synchronize Figure 1, Figure 2, Figure 3 and Figure 4 below:



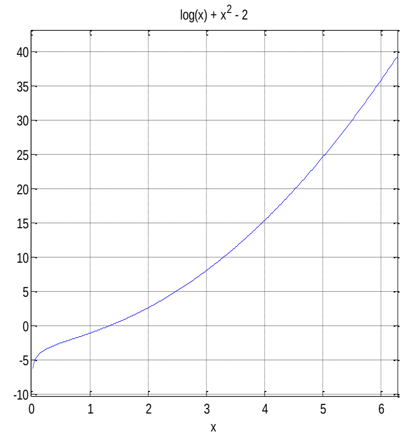
a. Figure Polynomial Function Graph



b.Figure Graph of Functions



c.Figure Exponential Function Graph



d.Figure Logarithmic Function Graph

In the polynomial function graph image above, point 0.5 is selected as the starting point for testing using the newton midpoint halley method and the olver method, in the trigonometric function graph image point 1 is selected as the starting point for testing using the newton midpoint halley method and the olver method, in the exponential function graph image point 0.6 is selected as the starting point for testing using the newton midpoint halley method and the olver method and in the logarithm function graph point 1 is selected as the starting point for testing using the newton midpoint halley method and the olver method.

Table 1. Simulation Results

No.	About	Methods	Iterations	X	F(x)	Error
1	$x^3 - 6x + 3$	Newton Midpoint Halley	3	0.5239758378	0.0000028949	0.0000363102
		Olver	4	0.5239568241	0.0001013171	0.0002584222
2	$\sin(x) - \frac{1}{2}$	Newton MidpointHalley	2	0.5235987756	0	0.0000228944
		Olver	4	0.5235987756	0	0.0000000039

3	$e^x - 3x$	Newton Midpoint Halley	2	0.6190612867	0	0
		Olver	2	0.6190612867	0	0.0000162694
4	$\log(x) + x^2 - 2$	Newton Midpoint Halley	2	1.3140968043	0	0
		Olver	2	1.3140968044	0.0000000001	0.0005463908

According to the results of the above computation using the Newton Midpoint Halley method, for problem 1 it is known that the value of x so that f(x) < 0.001 is x = 0.5239758378 with f(x) = 0.0000028949 and error = 0.0000363102, the value was obtained after computing up to 3 iterations. Using the Olver Method, for problem 1, the value of x such that f(x) < 0.001 is x = 0.5239568241 with f(x) = 0.0001013171 and error = 0.0002584222, the value was obtained after computing up to 4 iterations. Thus it shows that for solving the root equation of$f(x) = x^3 - 6x + 3$ using the Newton Midpoint Halley method the rate of convergence is faster when compared to the Olver method. For problem 2 using Halley's Neton Midpoint method, the known value of x such that f(x) < 0.001 is x = 0.5235987756 with f(x) = 0 and error = 0.0000228944, the value was obtained after computing up to 2 iterations. Using Olver's method, for problem 1, the known value of x such that f(x) < 0.001 is x = 0.5235987756 using f(x) = 0 and error = 0.0000000039, the value was obtained after computing up to 4 iterations. Thus showing that for solving the roots of$f(x) = \sin(x) - \frac{1}{2}$ using the Newton Midpoint Halley method the rate of convergence is faster when compared to the Olver method.

For problem 3, using the Newton Midpoint Halley method, it is known that the value of x such that f(x) < 0.001 is x = 0.6190612867 with f(x) = 0 and error = 0, the value was obtained after computing 2 iterations. Using Olver's method, for problem 1, the known value of x such that f(x) < 0.001 is x = 0.6190612867 with f(x) = 0 and error = 0.0000162694, the value was obtained after computing up to 2 iterations. Thus it shows that for solving the roots of$f(x) = e^x - 3x$ using the Newton Midpoint Halley method and the Olver method the rate of convergence is the same but the error is different, faster using the Newton Midpoint Halley method than the Olver method. For case 4, using the Newton Midpoint Halley method, the value of x such that f(x) < 0.001 is x = 1.3140968043 with f(x) = 0 and error = 0, the value was obtained after computing 2 iterations. Using Olver's method, for problem 1, the known value of x such that f(x) < 0.001 is x = 1.3140968044 with f(x) = 0.0000000001 and error = 0.0005463908, the value was obtained after computing 2 iterations. Thus it shows that for solving the roots of$f(x) = \log(x) + x^2 - 2$ using the Newton Midpoint Halley method and the Olver method the rate of convergence is the same but the root value,$f(x)$ and error are different, faster using the Newton Midpoint Halley method than the Olver method.

Several studies have been conducted by various researchers that discuss the application of numerical methods in solving non-linear equations. One of them is a study by Comparing the effectiveness of Newton Midpoint Halley method and Olver method in finding the roots of non-linear equations. The study showed that although both methods are effective, they have different characteristics in terms of convergence speed and stability. In addition, another study also examined the use of the Newton Midpoint Halley method with a Matlab graphical interface (GUI) to facilitate visualization and implementation in solving non-linear equations.

D. CONCLUSIONS AND SUGGESTIONS

In this report, a comparative analysis between the Newton Midpoint Halley method (NHM) and the Olver Method in solving the roots of non-linear equations has been conducted. Both methods have their own characteristics and advantages, but the analysis shows that the Newton Midpoint Halley method is superior in terms of convergence and efficiency. By utilizing the combination of Newton and Halley approaches, Newton Midpoint Halley method is able to reach the solution faster than Olver method.

One important aspect analyzed is the convergence rate of the two methods. The Newton Midpoint Halley method shows faster quadratic convergence, which means that the number of iterations required to achieve a certain level of accuracy is much less compared to the Olver method. In the simulations performed, the Newton Midpoint Halley method only requires a few iterations to achieve very small errors, while the Olver method, while still effective, requires more iterations to achieve similar results.

In addition, this report also highlights the ease of implementation of both methods. Halley's Newton Midpoint method, with its relatively simple and intuitive algorithm, makes it easy for students and practitioners to apply it to various non-linear problems. On the other hand, Olver's method, while also easy to understand, is not as efficient as Halley's Newton Midpoint method in terms of convergence speed. This makes the Newton Midpoint Halley method a more attractive option to use in the context of mathematics education.

REFERENCES

Charismana, D. S., Retnawati, H., & Dhewantoro, H. N. S. (2022). Learning Motivation and Learning Achievement in Ppkn Subjects in Indonesia: A Meta-analysis Study. *Bhineka Tunggal Ika: Review of Theory and Practice of Civics Education*,*9* (2), 99-113. https://doi.org/10.36706/jbti.v9i2.18333

Dewi, A. K., Dyah Lestari, & Muhammad Afnan Habibi. (2024). Design and Analysis of PID Controller Using Genetic Algorithm Method on CCM Type Boost Converter. *Emitter: Journal of Electrical Engineering*, 50-56. https://doi.org/10.23917/emitor.v24i1.2817

Estri, M. N., Nurshiami, S. R., Reorita, R., & Ibrohim, M. O. (2018). Determination of Iteration Termination Criteria in the Strawberry Algorithm. *Scientific Journal of Mathematics and Mathematics Education*,*10* (1), 27. https://doi.org/10.20884/1.jmp.2018.10.1.2834

Juldial, T. U. H., & Haryadi, R. (2024). Analysis of Computational Thinking Skills in the Learning Process. *Basicedu Journal*,*8* (1), 136-144. https://doi.org/10.31004/basicedu.v8i1.6992

Kasnanda Bintang, Y., Imaduddin, H., Kasnanda, Y., & Corresponding Author, B. (2024). *Development of Deep Learning Model for Diabetic Retinopathy Detection Using Transfer Learning Method*. *9*(3), 1442-1455. https://doi.org/10.29100/jipi.v9i3.5588

Khaidir, C. (2019). The Effectiveness of Textbooks for Numerical Methods Based on Constructivism at Iain Batusangkar. *Unpam Saintika Journal: Unpam Journal of Science and Mathematics*,*2* (2), 109. https://doi.org/10.32493/jsmu.v2i2.3321

Mandailina, V., Pramita, D., Ibrahim, M., Ratu Perwira Negara, H., & History, A. (2020). http://ejournal.radenintan.ac.id/index.php/desimal/index Wilkinson Polynomials: Accuracy Analysis Based on Numerical Methods of the Taylor Series Derivative ARTICLE INFO ABSTRACT. *Decimal: Journal of Mathematics*,*3* (2), 155-160. https://doi.org/10.24042/djm

Murtafiah, W. (2017). Profile of Students' Creative Thinking Ability in Posing Differential Equation Problems. *JIPM (Scientific Journal of Mathematics Education)*,*5* (2), 73. https://doi.org/10.25273/jipm.v5i2.1170

Pratamasyari, D. A., Silalahi, B. P., & GURITMAN, S. (2017). Combination of Variants of Newton's Method and Halley's Method for Solving Nonlinear Equations. *Journal of Mathematics and Its Applications*,*16* (2), 1-12. https://doi.org/10.29244/jmap.16.2.1-12

Rangkuti, A. N. (2019). *EDUCATIONAL RESEARCH METHODS Quantitative, Qualitative, PTK, and Development Research Approaches*.

Sari, W. M., Darnius, O., & Sembiring, P. (2018). Comparison of the Accuracy of the Grouped Frequency Distribution Table Model Between the Sturges Method and the Scott Method. *Talenta Conference Series: Science and Technology (ST)*,*1* (1), 001-009. https://doi.org/10.32734/st.v1i1.182

Syata, I., Suriadi, P. N., & Halim, S. N. H. (2023). Concentration Value of Paracetamol and Caffeine Elements Forming a System of Nonlinear Equations with Broyden's Method. *Journal of MSA (Mathematics and Statistics and Its Applications)*,*11* (1), 103-112. https://doi.org/10.24252/msa.v11i1.37744

Tukan, H., Maure, O. P., & Ina, K. T. D. (2024). Analysis of the Accuracy of Numerical Methods in Solving Derivatives of Nonlinear Equations. *Leibniz: Journal of Mathematics*,*4* (2), 10-22. https://doi.org/10.59632/leibniz.v4i02.446

Yudhi, Y. R. E. N. (2020). Modification of Newton's Method with Third Order Convergence to Solve Nonlinear Equation Systems. *Bimaster: Scientific Bulletin of Mathematics, Statistics and Applied*,*9* (4), 533-540. https://doi.org/10.26418/bbimst.v9i4.43316

Zahri, I. N. (2023). Optimization Strategy in Big Data Analysis: An Integrative Approach Using Semantic and Blockchain Technologies. *Researchgate.Net*, *December*, 0-5. https://doi.org/10.13140/RG.2.2.26750.56645