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Numerical Simulation to Find Root Solution of Nonlinear Equation with Hybrid and Laguerre Method

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Abstract: Non-linear equations are a form of mathematical problem that is often found in various fields of science, such as engineering, physics, and computers. The solution cannot always be done analytically, so the numerical approach becomes a commonly used solution. This research discusses and compares two numerical methods, namely the Hybrid method and the Laguerre method, in numerically finding the roots of non-linear equations using Matlab software. The Hybrid method combines the advantages of two methods, such as Newton-Raphson and bisection, to improve convergence speed and stability. Meanwhile, the Laguerre method is known to be efficient in solving high degree polynomial equations. The assessment is based on three main aspects: convergence speed, stability, and computational efficiency. Simulation results on various types of functions - polynomial, exponential, trigonometric, and logarithmic-show that the Hybrid method tends to have shorter computation time in some cases, while the Laguerre method shows stable performance in most trials. This study concludes that both methods have their own advantages depending on the characteristics of the function being solved. Therefore, the results of this study can be an important reference for the selection of appropriate numerical methods in the application of solving non-linear equations effectively and accurately.

Keywords: Non-linear equation roots, Hybrid Method, Laguerre Method, Numerical Method Accuracy, Convergence Rate, Stability, Computational Speed.

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A. INTRODUCTION

Numerical methods are an approach to finding the solution of a mathematical problem through algorithms or procedures that involve stepwise calculations, instead of finding an exact solution (Mara Doli Nasution et al., 2017). In terms of solving the roots of non-linear equations, there are various numerical methods that can be used, two of which are the popular Newton Hybrid and Laguerre Methods (Purwatiet al, 2021). Numerical solution requires an iteration process (repeated calculation) of existing numerical data. The use of Scilab v.6.0.0 software will greatly reduce the iteration time (Wigati, 2020).

In general, numerical methods for finding the roots of functions are generally divided into two, namely the confinement method and the open method. In the confinement method, the roots are always enclosed in an interval (Frayudi et al., 2019). If these methods cannot be applied, then a frequently used approach is to use Al-Khwarizmi's formula, better known as the ABC formula. (S. M. Putri, 2019). Mathematical problems in engineering are often encountered with non-linear equations. Functions f (x) can be in the form of algebraic equations, polynomial equations, trigonometric equations, transcendental equations (Akbarita et al. 2019). In the field of mathematics, problems are often encountered related to finding solutions to the nonlinear equation $f(x) = 0$. The solution to the equation is not always in the form of simple roots, some are multiple roots with multiplicity m, with $m > 1$ (Nonlinear, n.d.). Not all equations can be solved simply using mathematical theory, but rather require a numerical approach or computational techniques (Purwati et al. 2021).

Hybrid numerical methods are methods that combine two or more numerical approaches to solve mathematical or scientific problems. In general, the hybrid recommender system approach is to combine more than one recommendation system approach with the aim of overcoming the shortcomings of each approach, resulting in a good recommendation (Utomo et al. 2015). In the field of architecture, the hybridization method can be applied in design. In this imaginative study, traditional market and mall functions were selected to be hybridized into a new system (Rompis et al. 2014). This approach aims to utilize the strengths of each method and reduce the weaknesses that exist in each method separately. This hybrid method is designed with the aim of obtaining the advantages possessed by both methods (Irfandi et al., 2015). This hybrid method needs to be applied because in addition to getting more accuracy than a stand-alone method. Thus, this hybrid method allows multimode with a flexible structure to produce a better and optimal system (Irfandi et al., 2015)

On the other hand, Laguerre Method is one of the numerical methods used to find the roots of polynomial equations. Laguerre functions are a set of orthonormal functions used to estimate the discrete-time impulse response H(k) of a dynamic system (Rehiara et al, 2020). This method is very effective for finding complex and real roots of polynomials of high degree (Suripah et al, 2017). In terms of usability, the Laguerre method is very effective for solving polynomials of high degree, especially for finding complex roots compared to the Hybrid method.

This research aims to compare the accuracy level of Hybrid and Laguerre methods in choosing the solution of non-linear equations using a systematic and comprehensive approach. The assessment criteria that will be used include convergence, stability, and computational speed of each method. The expected result of this research is to know the comparison of convergence in solving the system of Non-linear equations with Hybrid and Laguerre methods using Matlab applications.By conducting a comparative analysis of these two methods, it is hoped that the best solution can be found for practical applications in various fields of science that require numerical solving of the roots of non-linear equations.

A. METHOD

In this section, we will explain in detail about the two methods used to solve the roots of non-linear equations, namely the Hybrid Method and the Laguerre Method. Each method will be explained starting from its basic concept, iterative formula, to its numerical application to solve the roots of non-linear equations. In this study, researchers compared the solution procedures (Hybrid and Laguerre) in selecting the roots of non-linear equations. then for the implementation of each formulation, researchers used Matlab software. The description of the two methods used is explained below.

1. Hybrid Methods

Hybrid methods in the context of finding the roots of equations is a technique that combines two or more numerical methods to improve efficiency and accuracy in finding solutions. This method is often used to utilize the advantages of each method under certain conditions, such as accelerating convergence or overcoming the weaknesses of a single method. For example, in non-linear equation root finding, the Hybrid Method can combine the Newton-Raphson method that converges quickly near the root with the more stable bisection method to improve results when the convergence of the first method slows down.

In general, hybrid methods can be applied by first using a coarser method (such as bisection) to narrow the root interval and then using a faster method (such as Newton-Raphson) to find a more accurate solution within that interval. The following is an overview of the application of the hybrid method with a combination of the bisection method and the Newton-Raphson method:

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Step 1: Using the Bisection method The bisection method is used to narrow the interval containing the roots of the equation f(x)=0. For example, if f(a)·f(b)<0, then the roots are between a and b. This interval will be divided into two parts at the midpoint $c = \frac{a+b}{2}$

Step 2: Use of the Newton-Raphson method After getting a smaller interval, the Newton-Raphson method is used to find further roots with the formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1)$$

This method is used to obtain a more precise root value than the initial value already obtained.

Hybrid Procedure:

a. Start with the interval [a, b] known to contain roots.
b. Use the bisection method to narrow the interval.
c. Once the interval is small enough, use the Newton-Raphson method to calculate the roots more accurately.

2. Laguerre Methods

The Laguerre method is one of the iterative methods used to find the roots of polynomial equations. This method is very efficient, especially for high degree polynomials, because it has a fast convergence rate compared to the Newton Midpoint Halley method. The iteration formula for the Laguerre method is as follows:

$$x_{k+1} = x_k - \frac{n}{G \pm (\sqrt{n} - 1)(nG^2 - H)} \quad (2)$$

Where:

a. n: degree of the polynomial P(x),
b. $G = \frac{p'(x_k)}{P(x_k)}$: ratio of the first derivative to the polynomial value
c. $H = \frac{p''(x_k)}{P(x_k)}$: the ratio of the second derivative to the polynomial value,
d. The ± sign is chosen to maximize denomination, i.e. using the sign that produces the largest absolute value.

Alternatif:

a. Choose an initial guess x_0
b. Calculate the values of G, H, and use the Laguerre formula to determine x_{k+1}
c. Check convergence: if $| x_{k+1} - x_k |$ is smaller than a certain tolerance, the iteration is complete.
d. Otherwise, repeat the step until a solution is found or the iteration limit is reached.

Both methods are then used to solve the problem of non-linear equations. The non-linear equations used involve non-linear equations in the form of trigonometry, exponential, and logatihm:

a. $f(x) = 2x^2 \sin(3x + 2)$ (trigonometry)
b. $f(x) = 2x^3 + 5x - 2$ (polynomial)
c. $f(x) = 2ex^{-4x} + 2$(eksponetial)
d. $f(x) = 2x \log(2^{x+1} + 2)$ (logarithm)

From the above equations, experiments will be conducted using the Hybrid method and Laguerre Method with a maximum iteration of 100 and error <0.001.

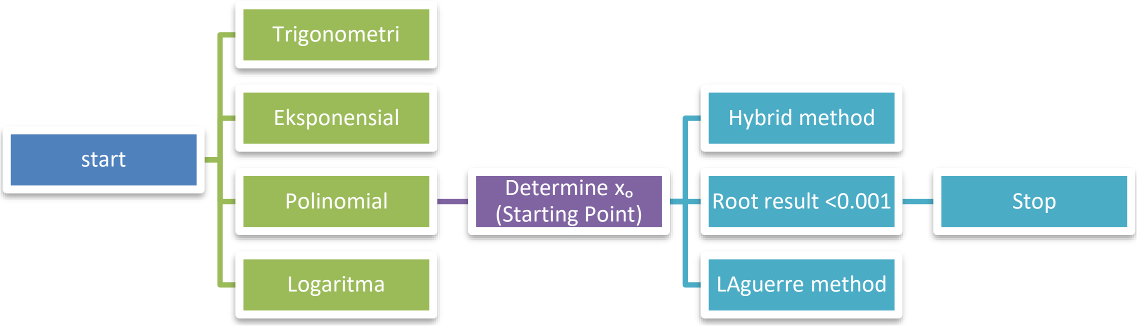
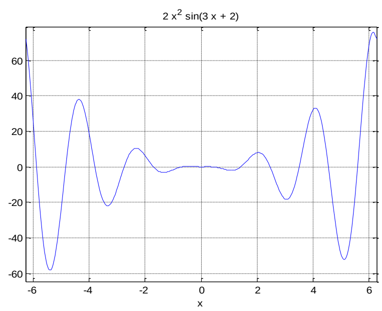


Figure 1. Flow Chart Research

The figure above illustrates the flow of the numerical simulation process in finding the root solution of non-linear equations using two methods, namely the Hybrid method and the Laguerre method. The process begins with the selection of the type of function to be solved, such as trigonometric, exponential, polynomial, or logarithmic functions. After that, the starting point \(x_0 \) is determined as the initial value to start the iteration process. Next, the user can choose one of the two solution methods, namely the Hybrid method or the Laguerre method, depending on the needs and characteristics of the function. Iterations are performed until a root result with an error smaller than 0.001 is obtained. If this criterion is met, the process is stopped. This diagram shows the systematic steps in the numerical approach to solve non-linear equations efficiently and accurately.

B. RESULTS AND DISCUSSION

Researchers used four non-linear equation problems consisting of trigonometric, polynomial, exponential, and logarithmic equations. According to the steps that have been carried out is a graph due to each synchronous equation Figure 1, Figure 2, Figure 3, and Figure 4.



1. Figure Trigonometry Graphics

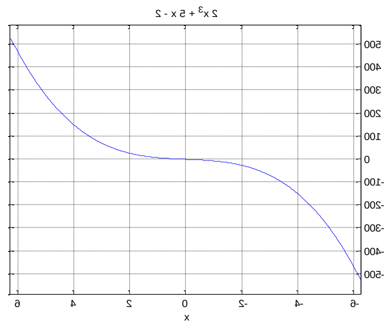


Figure 2. Polynomial Graphics

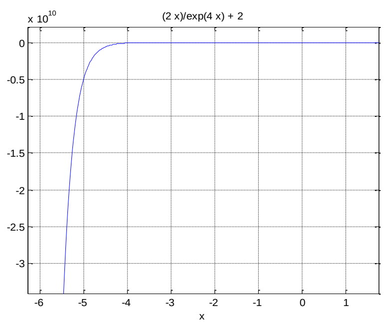


Figure 3. Eksponential Graphics

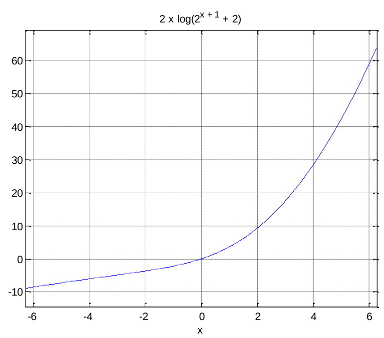


Figure 4. Logarithm Graphics

In Figure 2a above, it can be seen that the roots of the equation are in the interval [-6,6], in this case the crew point x0 =[-2] or x0 =[-1] is the starting point for finding the roots of trigonometric equations. Figure 2b above can be seen that the roots of the equation are in the interval [-6,6], in this case the starting point x0 =[-4] or x0 =[-2] is the starting point for finding the roots of the polynomial equation. In Figure 2c above, it can be seen that the roots of the equation are in the interval [-6,1], in this case the crew point x0= [-4] becomes the starting point for finding the roots of the exponential equation. In Figure 2d above, it can be observed that the root of the equation is in the interval [-6,6], in this case the crew point x0= [-2] becomes the starting point to find the root of the logarithm equation.

Table 1. Script Every Method

Method	script

Hybrid	clear;clc;
	disp('-------------------------------')
	disp('program : Metode Hybrid')
	disp('programer : Safaruddin')
	disp('-------------------------------')
	f=input ('f(x) = ');
	f_diff1=input ('df1(x) = ');
	f_diff2=input ('df2(x) = ');
	x1=input('x0 = ');
	imax=input('iterasi = ');
	galat1=input('Error = ');
	iter=0;
	fprintf('\n Iterasi Akar f(Akar) Galat\n')
	for k=1:imax
	    iter=iter+1;
	    %Rumus Hybrid
	    A=feval(f_diff2,x1);
	    B=6*feval(f_diff1,x1)-2*feval(f_diff2,x1)*x1;
	    C=6*feval(f,x1)-6*feval(f_diff1,x1)*x1+feval(f_diff2,x1)*(x1)^2;
	    x2=-B+(sqrt(B^2-4*A*C))/(2*A);
	    galat=abs((x2-x1)/x2);
	    x1=x2;
	    y=feval(f,x1);
	    fprintf('%10.0f %6.10f %6.10f %6.10f\n',[iter;x1;y;galat])
	    if (galat<galat1 || (iter>imax)),break,end
	end
	fprintf('Akarnya adalah = %6.10f\n',x1)

Laguerre

```
clear; clc;
disp('--------------------------------')
disp('Program   : Metode Laguerre')
disp('Programmer: Safaruddin')
disp('--------------------------------')
f=input ('f(x) = ');
n=input ('n = ');
f_diff1=input('df1(x) = ');
f_diff2=input('df2(x) = ');
x1=input('x0 = ');
imax=input('Iterasi = ');
galat1=input('Error = ');
iter=0;
fprintf('\n    Iterasi     Akar        f(Akar)         Galat\n');
for k=1:imax
   iter =iter+1;
   %Rumus Laguerre
   G=feval(f_diff1,x1)/feval(f,x1);
   H=G^2-(feval(f_diff2,x1)/feval(f,x1));
   a=n/(G+sqrt((n-1)*(n*H-G^2)));
   x2=x1-a;galat=abs((x2-x1)/x2);
   x1=x2;
   y=feval(f,x1);
   fprintf('%10.0f   %6.10f   %6.10f   %6.10f\n',[iter;x1;y;galat])
   if (galat<galat1 || (iter>imax)),break,end
end
fprintf('akarnya adalah = %6.10f\n',x1)
```

After determining the starting point of the graph above the next below is a script of Hybrid method and Laguerre method to find the roots of non-linear equations. Furthermore, from the script below, 8 trials will be conducted to find the iteration of the roots of the roots of the equations; Trigonometric, polynomial, exponential, and logarithmic.

Table 2. Iteration Result

No	Method	Non-linear equations	X Value	Iteration
1.	Hybrid Method	$2x^3 + 5x - 2$	2.8×10^{-13}	5
		$2x^2 \sin(3x + 2)$	0.0000000000	7
		$2xe^{-4x} + 2$	3.3×10^{-12}	2
		$2x \log(2^{x+1} + 2)$	0.0000000000	10
2.	Laguerre Method	$2x^3 + 5x - 2$	-0.1×10^{-9}	10
		$2x^2 \sin(3x + 2)$	-4.8×10^{-9}	4
		$2xe^{-4x} + 2$	9.1×10^{-9}	5
		$2x \log(2^{x+1} + 2)$	0.0000000000	10

Above is a table 2 of the results of iterated searches for the roots of non-linear trigonometric, polynomial, exponential, and logarithmic equations using the two scripts above.

Discussion of iteration results:

For case 1, it is known that the value of x such that f(x) < 0.001 is x =2.8x10^{-13}, the value was obtained after computing up to 5 iterations. By using the Laguerre method, for case 1 it is known that the value of x so that f(x) < 0.001 is x = 0.1x10^{-9}, the value was obtained after computing up to 10 iterations .Thus, for solving the root equation of f(x) = $2x^3 + 5x - 2$using the Hybrid method the rate of convergence is faster when compared to the Laguerre method. For case 2 is known by the hyrid method that the value of x as a result f(x) <0.001 meaning x =0, the value was obtained after computing up to 7 iterations. For case 2 using the Laguerre method it is known that the value of x such that f(x) <0.001 is x=-4.8x10^{-9}, the value was obtained after computing up to 4 iterations. after computing up to 4 iterations. Thus showing that for solving the root equation of f(x) = $2x^2 \sin(3x + 2)$ using the Hybrid method the rate of convergence is slower when compared to the Laguerre method.

For case 3 using the hybrid method it is known that the value of x as a result of f(x) <0.001 means x = 3.3x10^{-12}, the value was obtained after computing up to 2 iterations. For known problem 3 using the Laguerre method that the value of x as a result of f(x) <0.001 is x = 9.1x10^{-9} the value was obtained after computing up to 5 iterations. To solve the root equation of f(x) = $2xe^{-4x} + 2$ using the Hybrid method the convergence rate is faster when compared to the Laguerre method. For case 4, it is known that the value of x as a result of f(x)<0.0001 is x=0, the value was obtained after computing up to 10 iterations. For problem 4, it is known that the value of x as a result of f(x) <0.001 is x = 0, the value was obtained after computing up to 10 iterations in the Laguerre method. Thus, to solve the root equation of f(x) = $2x \log(2^{x+1} + 2)$using the Hybrid and Laguerre methods, the convergence rate is error or does not find the result.

Several studies have been conducted by several authors who simulate numerical methods. One of the studies conducted by (Rozi & Rarasati, 2022), examined the Comparison of Hybrid Methods and Laguerre Methods in solving the Roots of Non-Linear Equations. Furthermore, (Wigati, 2020), examined the Biseks Method using GUI Matlab: A Simulation and Solution of Non-Linear Equations. Furthermore, there is (Mandailina, 2020) examining the Wilkinson Polynomial: Accuracy Analysis according to the Numerical Method of Taylor Series Derivatives. And finally, there is (M. Putri & Syaharuddin, 2019), examining the Implementation of Open and Closed Methods Numerically: Convergence Test of Nonlinear Equation Solutions.

C. CONCLUSIONS AND SUGGESTIONS

The conclusion of this study shows that the Hybrid method and the Laguerre method have their respective advantages in solving the roots of non-linear equations numerically. Based on simulations carried out using the Matlab application, the Hybrid method is proven to be faster for several types of equations, especially with the number of iterations of 2, 5, 7, and 10 compared to the Laguerre method which requires 10, 4, 5, and 10 iterations. The experiment was carried out 8 times with an error limit of <0.001 and the number of iterations up to ±100, ensuring accurate results. Both methods were tested on equations with polynomial, exponential, trigonometric, and logarithmic characteristics, showing varying performance depending on the type of equation. These results indicate that the Hybrid method has an advantage in computational speed, although the Laguerre method remains competitive in stability and convergence. With this analysis, the study has succeeded in providing an in-depth evaluation of both methods for solving non-linear equations. This study is expected to be a reference and recommendation for practical applications in various fields of science that require effective and accurate solution of the roots of equations.

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