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Comparison of the Accuracy of Solution of Nonlinear Equations by Chebyshev and Euler Methods through Numerical Simulation

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Abstract: Non-linear equations have an important role in various fields of science such as physics, biology, and especially mathematics. The complexity of the mathematical form of non-linear equations often makes it difficult to solve analytically, so numerical approaches become a widely used alternative. Numerical methods offer approximate solutions to analytical solutions with certain errors, but are still reliable for various practical purposes. This study compares the accuracy of two numerical methods, namely the Chebyshev method and the Euler method, in solving non-linear equations through numerical simulations. The Chebyshev method is used to analyze the proportion of values within a certain standard deviation from the mean, while the Euler method is known as a simple one-step method with low accuracy, but faster computation process. Simulations were performed on four types of functions: trigonometric, exponential, logarithmic, and polynomial. The simulation results show that the Euler method gives a faster convergence rate than the Chebyshev method in all cases of the functions tested, so it is considered more accurate in solving the roots of non-linear equations in the context of this study. This finding is expected to be a reference in the selection of efficient and appropriate numerical methods in solving non-linear mathematical problems.

Keywords: Non-Linear Equations, Numerical Methods, Chebyshev Method, Euler Method, Numerical Simulation, Approximate Solution, Convergence Rate.

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A. INTRODUCTION

Nonlinear equations have a very important role in all scientific fields such as physics and biology, especially in the field of mathematics (Pandia et al. 2021). Most nonlinear equations appear in the form of mathematical equations that are quite complicated and complex, so analytical solutions are often unable to find solutions to these nonlinear equations (Yahya et al. 2018). If the equation is linear, then analytical methods can be used to solve it. However, if the equation is in nonlinear form, then not all analytical methods can solve it (Estuningsih et al. 2019).

Analytic solution is a solution that produces two forms of solutions, namely explicit and implicit forms, while numerical is an approximate solution. The result of numerical solution is an approximate or approximate value of the analytical solution. The difference between the two is called the error (Salwa et al., 2022). The main difference between numerical methods and analytical methods is twofold. First, solutions using numerical methods are always numerical. When compared to analytical methods that usually produce solutions in the form of mathematical functions, the mathematical functions can then be evaluated to produce values in the form of numbers. Second, with numerical methods, it can only obtain solutions that approach or approach the true solution so that the solution of the numerical calculation results is called an approximate solution (approximation) and can be made as precise as desired (Pandia et al., 2021).

The method that can be used to solve nonlinear equations is a numerical method that produces iterated approximate solutions (Mathematics et al., 2021). Numerical method is a technique used to formulate mathematical models so that they can be solved by ordinary calculation/arithmetic operations. Numerical methods can always solve problems, but the value obtained is an approximate value (Estuningsih & Rosita, 2019). Numerical solution requires an iteration process (repeated calculation) of existing numerical data (Wigati & Mathematics, 2017). To note that the best or most efficient numerical method is the method that has the smallest average iteration in determining the approach to variable values (Akmala et al., 2022). Determining the roots of a nonlinear equation of one variable, $f(x)=0$ is a topic that is always discussed in numerical methods courses, because this problem arises from various scientific problems that require mathematical solutions (Pandia et al. 2021).

The Chebyshev method is an improved method of the Newton-Raphson method. This method uses a more stable approach and has a better convergence speed in some cases (Deswanta et al. 2019). In principle, the Chebyshev method utilizes a series of iterations to produce a more accurate solution in a few iterations compared to the Newton-Raphson method (Farida, 2016). The main advantage of this method is more effective numerical error reduction, although in some specific cases, it can be more complex than the Newton-Raphson method (Raenagus, 2021).

On the other hand, the Euler Method can be applied in the educational aspect, namely in learning mathematics on maximum and minimum values (Dharmawan et al., 2014). There are special methods to calculate nonlinear methods, including numerical methods, if the objective function cannot be solved analytically. Jacobian method and Euler method are examples of numerical methods (Aadam, 2017). The Jacobian method is a method of solving equations through an iteration process using equations. While the Euler method is used to solve differential equations numerically when the value of the function in the initial state is known (Dwi Rahayu Septiani et al., 2022). The Euler method is also a method taken from the first two terms of the Taylor series (Azis et al. 2021). Nevertheless, if the step size is chosen carefully, this method can still provide a fairly good solution (Wahyuni et al., 2023).

In the local context, research on numerical solutions for non-linear equations has also been conducted in Indonesia, with many journals examining the application of numerical methods to engineering and science problems (Akmala et al., 2022). Therefore, this simulation aims to find out which method is more accurate when performing numerical solutions for non-linear equations.

B. METHOD

The researcher compared the algorithms (Chebyshev, and Euler) in determining the roots of non-linear equations. The explanation of the two methods used is explained below.

1. Chebyshev's method

Chebyshev's theorem is used to find the minimum proportion of data that occurs within a certain number of standard deviations from the mean. Chebyshev's theorem is more general and can be applied to many different distributions. This theorem states that at the smallest number, a value falls within the standard deviation of the mean regardless of its shape. Here is the formula for the Chebyshev method.

$$x_{i+1} = x_i\left(1 + \frac{1}{2}L(x_n)\right) \cdot \frac{f(x_n)}{f'(x_n)} \qquad (1)$$

2. Euler's Method

Euler's method is one of the simplest one-step methods. Compared to other methods, it is the least rigorous. Here is the iterative formula of Euler's method:

$$x_{i+1} = x_i \frac{f(x_i)}{[f'(x_i)} \cdot \frac{2}{\sqrt{1-2L(x_n)}} \qquad (2)$$

Both methods are then used to solve non-linear equation problems involving polynomial, trigonometric, exponential and logarithmic non-linear equations. Furthermore, the problems used for simulation consist of:

1) Trigonometry Problem: $f(x) = 2x^2 \sin(3x + 6)$
2) Exponential Problem: $f(x) = 2xe^{-4x} + 6$
3) Logarithm Problem: $f(x) = 2x \log(2^{x+1} + 6)$
4) Polynomial Problem: $f(x) = 2x^3 + 5x - 2$

Simulation parameters when solving non-linear equations (trigonometric, exponential and logarithmic) in Chebyshev's method and Euler's method are set to an error of 0.001.

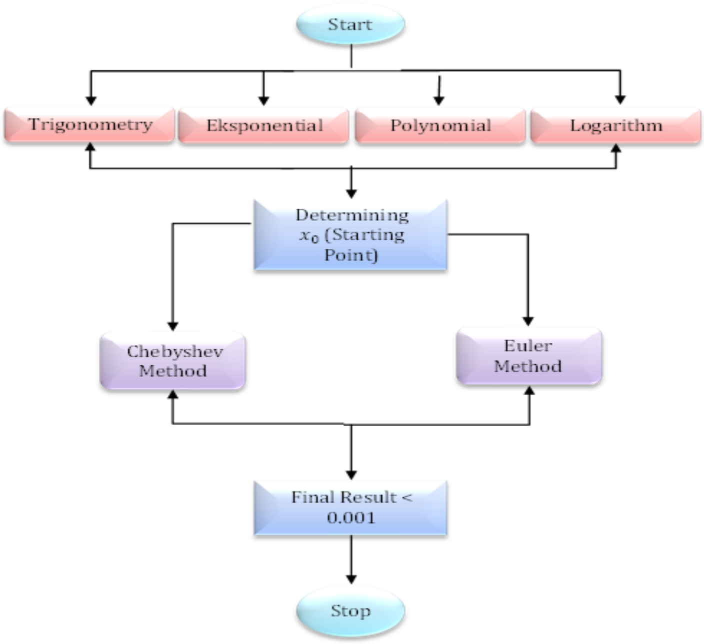


Figure 1. Shows The Flow Of Research Conducted

The figure is a flowchart of the numerical simulation process in comparing the accuracy of the Chebyshev method and the Euler method in solving non-linear equations. The process starts from the Start stage, followed by the selection of the type of function to be simulated, namely trigonometric, exponential, polynomial, or logarithmic functions. After the type of function is determined, the next step is to determine the starting point x0x0 for the iteration process. Then, numerical solutions are sought using two compared methods, namely the Chebyshev method and the Euler method. The results of each method are then evaluated until they meet the convergence condition, which is the final value smaller than 0.001. If the condition is met, then the process is stopped. This diagram shows the systematic flow in determining the most accurate method based on the convergence rate of the two methods tested.

C. RESULTS AND DISCUSSION

The simulation process is done using Matlab software by running the Chebyshev and Euler method algorithms. Furthermore, the results of the two methods are compared in determining the roots of the Non-Linear equation as follows:

1. **Trigonometry graphics**

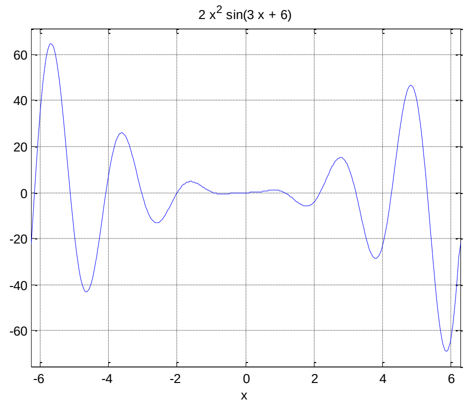


Figure 2.a

2. **Polynomial graphics**

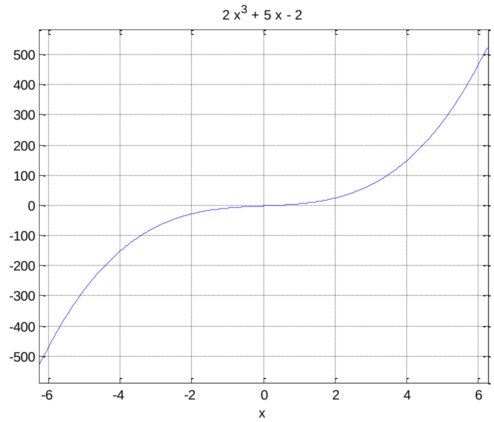
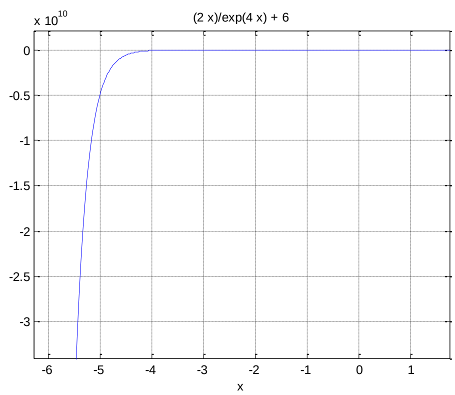


Figure 2.b

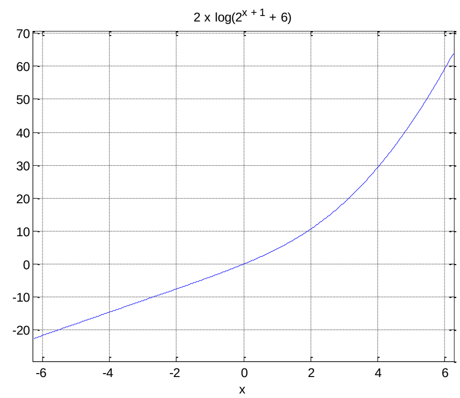
3. **Eksponential graphics**

Figure 2.c



4. **Logarithm graphics**

Figure 2.d



In Figure 1 above, it can be seen that the roots of the equation $f(x) = 2x^2 \sin(3x + 6)$ are in the interval [-6,6]. In this case, the interval [-3,-2] is chosen as the starting point to find the roots of the equation. In Figure 2 above, it can be seen that the root of the equation $f(x) = 2x^3 + 5x - 2$ is in the interval [-6,5]. In this case, the starting point x0= [3] is obtained as the starting point to find the roots of the polynomial equation. In Figure 3 above, it can be seen that the root of the equation $f(x) = 2xe^{-4x} + 6$ is in the interval [-6,6]. In this case, the interval [-0.5,1] is chosen as the starting point to find the roots of the equation. In Figure 4 above, it can be seen that the root of the equation $f(x) = 2x \log(2^{x+1} + 6)$ is in the interval [- 6,6]. In this case, the interval [-1,0] is chosen as the starting point to find the root of the equation. In the simulation process in matlab with the algorithms of the two methods, namely the Chebyshev method and the Euler method, the following results were obtained:

Table 1. Script

Chebyshev

```
for k=1:imax
   iter=iter+1;
   %Rumus Chebyshev
   L=feval(f_diff2,x1)*feval(f,x1)/feval(f_diff1,x1)^2;
   x2=x1-(1+0.5*L)*(feval(f,x1)/feval(f_diff1,x1));
   galat=abs((x2-x1)/x2);
   x1=x2;
   y=feval(f,x2);
   fprintf('%10.0f     %6.10f     %6.10f     %6.10f\n',[iter;x1;y;galat])
   if (galat<galat1 || (iter<imax)),break,end
end
fprintf('Akarnya adalah = %6.10f\n',x1)
```

Euler

```
for k=1:imax
   iter=iter+1;
   %Rumus Euler
   L=feval(f_diff2,x1)*feval(f,x1)/feval(f_diff1,x1)^2;
   x2=x1-((feval(f,x1))/(feval(f_diff1,x1))*(2/(1+sqrt(1-2*L))));
   galat=abs((x2-x1)/x2);
   x1=x2;
   y=feval(f,x1);
   fprintf('%10.0f     %6.10f     %6.10f     %6.10f\n',[iter;x1;y;galat])
   if (galat<galat1 || (iter>imax)),break,end
end
fprintf('Akarnya adalah = %6.10f\n',x1)
```

The scripts in Table 1 above, namely the Euler and Chebysev method scripts, are iterative implementations of the Chebyshev method and the Euler method to find the roots of non-linear equations by utilizing the first and second derivatives of functions until they reach an error below the tolerance limit.

Table 2. Simulation results

No	Kasus	Metode	Iterasi	X	F(x)	Galat
1	$2x^2 \sin(3x + 6)$	Chebyshev	2	-3.0471975511	-0.0000000035	0.0000997506
		Euler	2	-3.0471975512	-0.0000000002	0.0000426531
2	$2xe^{-4x} + 6$	Chebyshev	2	-0.4657042165	0.0000000154	0.0007443605
		Euler	2	-0.4657042162	-0.0000000016	0.0004526771
3	$2x \log(2^{x+1} + 6)$	Chebyshev	3	0.0000000000	0.0000000000	Inf
		Euler	3	-0.0000000000	-0.0000000000	Inf
4	$2x^3 + 5x - 2$	Chebyshev	5	0.3783379433	0.0000000000	0.0000000007
		Euler	4	0.3783379433	0.0000000000	0.0000000352

Discussion

Based on the computational results above using the Chebyshev method, for case 1 it is known that the value of x so that $f(x) < 0.001$ is $x = -3.0471975511$ with $f(x) = -0.0000000035$, and error 0.0000997506 the value is obtained after computing up to 2 iterations. For case 2, it is known that the value of x so that $f(x) < 0.001$ is $x = -0.4657042165$ with $f(x) = 0.0000000154$, and error 0.0007443605 the value is obtained after computing up to 2 iterations. For case 3, it is known that the value of x so that $f(x) < 0.001$ is $x = 0.0000000000$ with $f(x) = 0.0000000000$ the value is obtained after computing up to 3 iterations. For case 4, it is known that the value of x so that $f(x) < 0.001$ is $x = 0.3783379433$ with $f(x) = 0.0000000000$ the value is obtained after computing up to 5 iterations. However the table above shows that the simulation results of the roots of non-linear equations in the Chebyshev Method have not been able to find the roots of the equation with the expected accuracy. The simulation results of the application of the Chebyshev Method in solving difficult logarithm cases show an error result or infinity value. This shows that the effectiveness of the Chebyshev Method cannot solve the non-linear equation problem with the expected accuracy.

By using the Euler method, for case 1 it is known that the value of x so that $f(x) < 0.001$ is $x = -3.0471975512$ with $f(x) = -0.0000000002$ error 0.0000426531 the value is obtained after computing up to 2 iterations. For case 2, it is known that the value of x so that $f(x) < 0.001$ is $x = -0.4657042162$ with $f(x) = -0.0000000016$ error 0.0004526771 the value is obtained after computing up to 2 iterations. For case 3, it is known that the value of x so that $f(x) < 0.001$ is $x = -0.0000000000$ with $f(x) = -0.0000000000$ the value is obtained after computing up to 3 iterations. For case 4, it is known that the value of x so that $f(x) < 0.001$ is $x = 0.3783379433$ with $f(x) = 0.0000000000$ the value is obtained after computing up to 4 iterations. However, the table above shows that the simulation results of the roots of non-linear equations in the Euler Method have not been able to find the roots with the expected accuracy. The simulation results of the application of the Euler Method in solving difficult logarithm cases show an error result or infinity value. This shows that the effectiveness of the Euler Method cannot solve the non-linear equation problem with the expected accuracy. Thus, this result shows that the Euler Method is not effective in solving complex non-linear equations.

Thus it shows that for solving the root equation of $f(x) = 2x^2 \sin(3x + 6)$ using Euler's method the rate of convergence is faster when compared to Chebyshev's method. For solving the root equation of $f(x) = 2xe^{-4x} + 6$ using Euler's method the rate of convergence is faster than Chebyshev's method. For solving the root equation $f(x) = 2x \log(2^{x+1} + 6)$ using Chebyshev and Euler methods, the results show that both methods do not converge in solving complex non-linear equations. It can be concluded that in the case of 1) trigonometric function problem, 2) exponential function problem, 3) logarithm function problem and 4) polynomial function problem, the results show that the best method in solving the roots of non-linear equations is the Euler method with the fastest convergence rate compared to the Chebyshev method.

D. CONCLUSIONS AND SUGGESTIONS

From the four results of solving equations (trigonometric, exponential, logarithmic and polynomial), it can be concluded that the fastest rate of convergence in solving equations (trigonometric, exponential, logarithmic and polynomial) is with Euler's method So the best method in solving non-linear equations is Euler's method.

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