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Comparative Analysis of Fixetpoint and Halley Methods For Numerically Solving The Roots of Non-Linear Equations

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Abstract: This study aims to analyze the comparison of Fixetpoint and Halley methods in numerically solving the roots of non-linear equations. The assessment criteria to be used include convergence, stability, and computational speed of each method. The equations used include trigonometric, pilinomial, exponential and logarithmic. Experiments were conducted 8 times with an error of 0.001 and using a maximum of 100 iterations. Of the four cases of solving the tested equations, the fixetpoint method is faster than the halley method with 5 iterations on trigonometric equations. In polynomial equations the fixetpoint method is faster than newton rapshon with 100 iterations. in exponential equations the fixetpoint method is faster than hallley with 1 iteration. In the logarithmic equation the fixetpoint method is slower than the halley method with 14 iterations, therefore it can be said that the fixtpoint method has a faster convergence rate and higher accuracy than the Halley method in most cases. So it can be said that Fixetpoint is the best method in solving the roots of non-linear equations. These findings provide new insights in choosing the right method for numerical applications in solving the roots of non-linear equations, and contribute to the development of more efficient numerical algorithms.

Keywords: *Fixed-point* method, *Halley*'s method, roots of non-linear equations, numerical solution, convergence, and comparative analysis.

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A. INTRODUCTION

Non-linear equations often appear in various fields of science and technology, such as physics, economics, engineering, and computer science. Unlike linear equations that have simpler properties (Hutagalung, 2017). Non-linear equations have complex properties and cannot always be solved analytically. Therefore, to find the solution, the numerical approach becomes the most frequently used method because of its flexibility in solving these problems (Jumawanti et al., 2018). Numerical methods allow us to obtain approximation solutions of non-linear equations by using certain algorithms or iterative methods (Pandia & Sitepu, 2021).

One of the numerical methods often used in solving this problem is the fixetpoint method and the halley method (Vilinea et al., 2020). Both methods have their own characteristics and advantages in predicting the solution or roots of non-linear equations. Meanwhile, the Halley method is a method similar to the Newton-Raphson method but uses a faster iteration approach while maintaining higher accuracy in some cases (et al., 2024). This makes Halley's method often chosen to solve non-linear equations with faster convergence than Fixed Point methods, especially if the functionality used meets the convergence requirements (Azmi et al., 2019).

In its application, both methods have different characteristics in solving non-linear equations (Pandia & Sitepu, 2021). Therefore, it is important to compare the extent to which these two methods are effective and efficient in solving non-linear problems. By comparing the performance of the two methods, we can understand the advantages and disadvantages of each method (Apriano & Rizal, 2024). The problem that arises is how the Fixed Point and Halley methods work in solving non-linear equations and to what extent the differences in their convergence speed and accuracy affect the solution results (Mandailina et al., 2020).

The methodology used in this research is a numerical simulation approach. The steps include selecting a non-linear function as a case example, applying the Fixed Point and Halley methods to the equation, and analyzing the results of these methods in various aspects such as accuracy of results, speed of convergence, and ease of application. This approach is done so that we can understand the extent to which the two methods provide optimal solutions and compare the results obtained from each method (Ritonga & Suryana, 2019).

This research has a clear goal in order to provide an overview of the performance of the two methods in the context of solving non-linear equations (Wigati, 2020). The objectives of this study are to compare the Fixed Point and Halley methods in finding numerical solutions to various non-linear equations, analyze the speed of convergence of these methods, and identify the advantages and disadvantages of each method based on the results of the calculations carried out (Ritonga & Suryana, 2019). Thus, this research will focus on a comparative study conducted using several cases of non-linear equations.

B. METHODS

In this section, we will explain in detail about the two methods used to solve the roots of non-linear equations, namely the fixetpoint method and the halley method. Each method will be explained starting from the basic concept, iterative formula, to its numerical application to solve the roots of non-linear equations. In this study, researchers compared the solution procedures (fixetpoint and halley) in selecting the roots of non-linear equations. The equations used include trigonometric, pilinomial, exponential and logarithmic. Experiments were conducted 8 times with an error of 0.001 and using a maximum of 100 iterations. Of the four equations, the first step taken is to draw a graph of each equation with the aim of determining the starting point or x0. then the researcher performs a simulation using Matlab software. The description of the two methods used is explained below.

1. Fixetpoint method

The Fixed Point Method is a numerical technique to find the solution of a non-linear equation $f(x)=0$ by transforming it into a fixed point form $x=g(x)$. With this method, the solution of the equation is sought iteratively, starting with an initial guess and calculated repeatedly using the iteration formula (Karunia, 2021) . The process continues until the calculated value is close to the desired solution, which is when the difference between iterations becomes very small. The Fixed Point Method is often used in computer programming to solve equations that arise in mathematical simulations or numerical optimization.

Basic Formula of FixedPoint Method:

Suppose we have a non-linear equation f(x)=0, to find the root x=α of the equation, we can transform it into a fixed point form x=g(x), where g(x) is a function derived from the equation f(x). The iterative formula is:

$x_n + 1$ = g (x_n) (2)

Where

- x_n *adala nilai pendekatan solusi pada iterasi ke* $- n$.
- $g(x)$ *adalah fungsi yang di turunkan dari persamaan* $f(x) = 0$, *dimana solusi* x *berada pada titik tetap* $g(x) = x$.
- $x_n + 1$ *adalah nilai pendekatan solusi pada iterasi berikutnya.*

2. Halley Method

Halley's method is an iteration method that has a faster convergence speed than the Fixed Point method and has the basis of the Newton-Raphson method. The iteration formula for Halley's method is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}[1 - \frac{f(x_n).f''(x_n)}{2.(f'(x_n))^2}]$$

Where:

- f(x) is the function for which we want to find the root,
- f'(x) is the first derivative of f(x)
- f''(x) is the second derivative of f(x)
- x_n is the nth iteration guess
- x_{n+1} is the next iteration's guess.

Halley's method works by maintaining a higher convergence speed than the Fixed Point method in certain cases. Therefore, it is often chosen to solve non-linear equations that have complex properties and require high accuracy.

Both methods are then used to solve the problem of non-linear equations. The non-linear equations used involve trigonometric, exponential polynomial, and logarithmic non-linear equations. Furthermore, the problems used for the simulation consist of:

1.$2x^2\, sin(3x + 5)$ Trigonometry

2. $2x^3 + 5x - 5$ Polynomial

3. $2xe^{-4x} + 14$ Exponential

4. $2xlog(2^{x+1} + 5)$ Logarithms

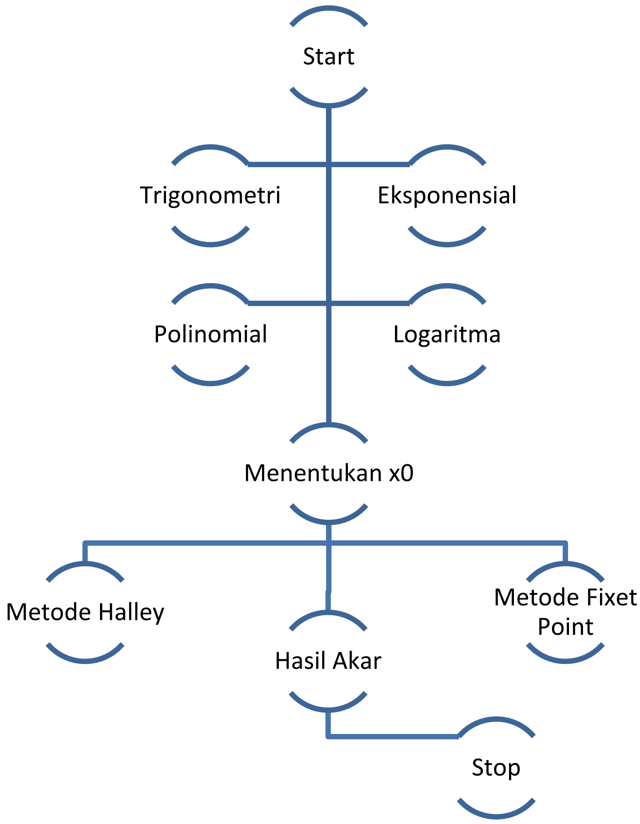

Figure 1 shows the flow of research conducted

C. RESULTS AND DISCUSSION

Researchers used four non-linear equation problems consisting of trigonometric, polynomial, exponential, and logarithmic equations. according to the steps that have been taken, the graphs resulting from each synchronous equation are Figure 1, Figure 2, Figure 3, and Figure 4 below.

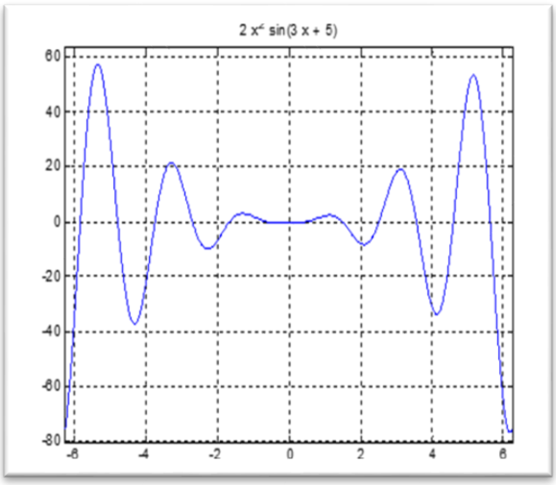

Figure 1. Graph Of Trigonometric Functions

Figure 1 above shows that the roots of the equation are in the interval [-6,6]. In this case, the starting point x0= [0.5] becomes the starting point for finding the roots of the trigonometric equation,

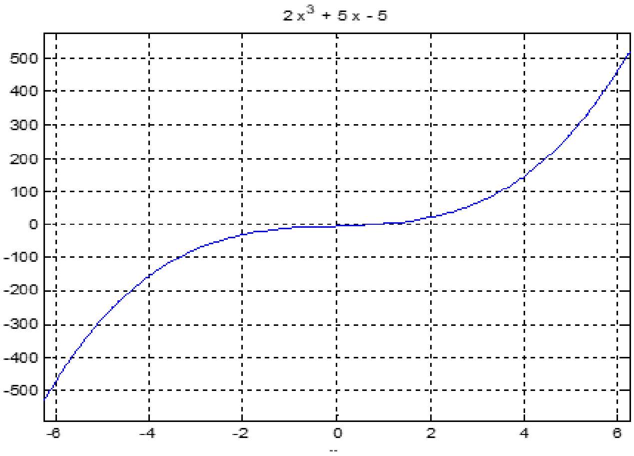

Figure 2. Polynomial Function Graph

Figure 2 above shows that the roots of the equation are in the interval [-6,6]. In this case, the starting point x0= [1] is used as the starting point to find the roots of the polynomial equation.

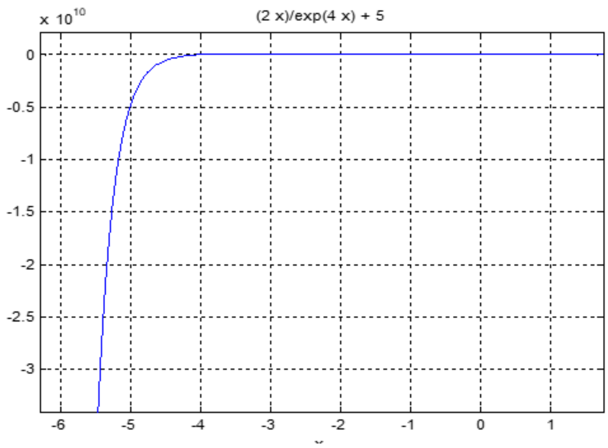

Figure 3. Graph of the exponential function

In Figure 3 above, it can be seen that the roots of the equation are in the interval [-6,1]. In this case, the crew point x0= [-5] becomes the starting point for finding the roots of the exponential equation.

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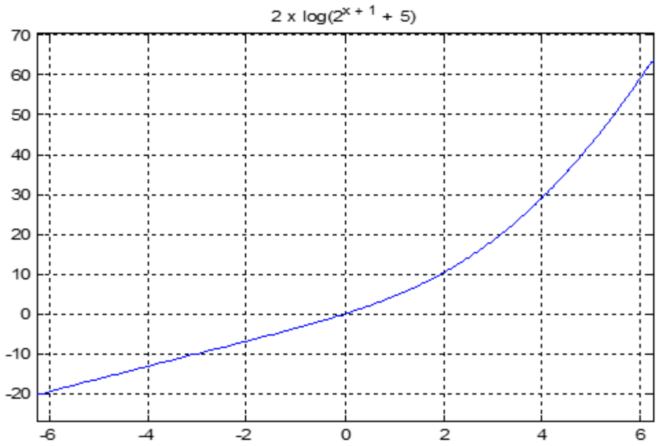

Figure 4: Graph of logarithmic function

In Figure 4 above, it can be observed that the root of the equation is in the interval [-6,6]. In this case, the starting point x0= [0] becomes the starting point find the root of the logarithmic equation.

Table 1. Simulation results

n	Case	Methods	Iterations	X	F(x)	Error
11	$2x^2 \sin(3x + 5)$	Fixet point	100	633.825	803.328	0.500
		Halley	5	0.427	0.0000000209	0.0000860
22	$2x^3 + 5x - 5$	Fixet point	2	19.910	15879.520	0.750
		Halley	4	0.797	0.0000000249	0.0000522
33	$2e^{-4x} + 5$	Fixet point	3	10.000	5.0000000000	0.000000000 8
		halley	1	4.999	4840662901.46 4	0.000107
44	$2x \log(2^{x+1} + 14)$	Fixet point	2	2.639	8.823	0.936
		halley	14	0.0000006259	0.00000243	2286.103

According to the results of the above computation using the fixetpoint method, for case 1 it is known that the value of x such that f(x) < 0.001 is 633825300114114700 x = 6 with f(x) = 803328896963385550O, the value was obtained after computing up to 100 iterations. Using halley method, for case 1 it is known that the value of x such that f(x) < 0.001 is x = 0.4277284547 with f(x) = 0.0000000209, the value was obtained after computing up to 5 iterations, Thus showing that for solving the root equation of f(x) =$2x^2 \sin(3x + 5)$ using fixetpoint method the rate of convergence is slower when compared to halley method.

According to the results obtained as a result of the above computation using the fixetpoint method, for case 2 it is known that the value of x such that f(x) < 0.001 is x = 19.9100000000 with f(x)=15879.5205420000, the value was obtained after computing up to 2 iterations. Using the halley method, for case 2 it is known that the value of x as a result f(x) < 0.001 is x = 19.9100000000 using f(x) = -0.0000000249, the value was obtained after computing up to 4 iterations of repetition, Thus For solving the root equation of f(x) = f(x) = $2x^3 + 5x - 5$ using the fixetpoint method the rate of convergence is slower when compared to the halley method.

According to the results obtained as a result of the above computation using the fixetpoint method, for case 3 it is known that the value of x such that f(x) < 0.001 is x = 10.0000000082 using f(x) = 5.0000000000, the value was obtained after computing up to 3 iterations. Using Halley's method, for case 3, it is known that the value of x such that f(x) < 0.001 is x = -4.9994601013 using f(x) = 4840662901.4644470000, the value is obtained after computing up to 1 iteration. Thus, for solving the root equation of f(x) =$2xe^{-4x} + 5$ using the fixetpoint method, the rate of convergence is slower than the Halley method.

According to the results obtained as a result of the above computation using the fixetpoint method, for case 4 it is known that the value of x as a result f(x) < 0.001 means x = -2.6391599469 using f(x) = -8.8236058705, the value was obtained after computing up to 2 iterations. . Using Halley's method, for case 4, it is known that the value of x as a result of f(x) < 0.001 is x = 0.0000006259 using f(x) = 2286.1039265839, the value was obtained after computing up to 14 iterations. Thus for solving the root equation of f(x) =$2x \log(2^{x+1} + 5$ using the fixetpoint method is slower than the halley method.

D. CONCLUSIONS AND SUGGESTIONS

Thus it shows that for solving the root equation of f(x) $= 2x^2 \sin(3x + 5)$ using the fixetpoint method the rate of convergence is slower when compared to the halley method. For solving the root equation of f(x) = f(x) = $2x^3 + 5x - 5$ using the fixetpoint method the rate of convergence is slower when compared to the halley method. For solving the root equation of f(x) =$2xe^{-4x} + 5$ using the fixetpoint method the rate of convergence is slower with the halley method. Then for solving the root equation of f(x) $= 2x \log(2^{x+1} + 5$ using the fixetpoint method is slower than the halley method.

From the four results of solving the equations (trigonometric, polynomial, exponential, and logarithmic), it can be concluded that Halley's method is more suitable for solving equations that have complex convergence rates or have roots that require more dynamic calculations. While the fixetpoint method can be faster and simpler but may be less accurate in some cases with high non-linearity.

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