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Comparative Analysis of Newton Rapshon and Chebyshev Methods for Numerical Solving the Roots of Nonlinear Equations

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Abstract: This study aims to analyze the comparison of Newton-Raphson and Chebyshev methods in numerically solving the roots of non-linear equations. The assessment criteria to be used include convergence, stability, and computational speed of each method. The equations used include trigonometric, pilinomial, exponential and logarithmic. Experiments were conducted 8 times with an error of 0.001 and using a maximum of 100 iterations. Of the four cases of solving the tested equations, the newton rapshon method is faster than the cchebyshev method with 2 iterations on trigonometric equations. on polynomial equations the chebyshev method is faster than newton rapshon with 3 iterations. on exponential equations the chebyshev method is faster than newton rapshon with 14 iterations. In the logarithmic equation both methods do not find results or errors, therefore it can be said that the chebyshev method has a faster convergence rate and higher accuracy than the Newton-Raphson method in most cases. So it can be said that Chebyshev is the best method in solving the roots of non- linear equations. These findings provide new insights in choosing the right method for numerical applications in solving the roots of non-linear equations, and contribute to the development of more efficient numerical algorithms.

Keywords: Newton Raoshon, Chebyshev, Nonlinear Equations, Numerical Methods.

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A. INTRODUCTION

Solving the roots of non-linear equations is one of the problems often encountered in various fields of science, such as physics, engineering, economics, and computer science (Wigati, 2020). The roots of non-linear equations are values that satisfy an equation that cannot be solved analytically easily, especially when the functions involved in the equation are not linear (Ratu Perwira Negara et al., 2017). Therefore, numerical methods are needed to find solutions that are close to the roots of the equation (Pandia & Sitepu, 2021). to solve problems in non-linear equations there are many methods and algorithms that can be used, but each method and algorithm has its own advantages and disadvantages (Negara et al., 2019).

Numerical methods are an approach to finding the solution of a mathematical problem through algorithms or procedures that involve stepwise calculations, instead of finding an exact solution (Rozi & Rarasati, 2022). In terms of solving the roots of non-linear equations, there are various numerical methods that can be used, two of the most popular of which are

Newton- Raphson Method and Chebyshev Method (Sharma et al., 2023). Numerical solutions require a process of iteration (repeated calculations) based on existing numerical data. The use of Scilab v.6.0.0 software can greatly reduce the time required to perform these iterations (Wigati, 2020).

One of the numerical methods is used when analytical calculations cannot be performed. Numerical methods are techniques used to formulate mathematical problems so that they can be solved through arithmetic operations and logical operations. These operations are usually carried out sequentially (iteratively) or repeatedly to obtain an approximation value that is close to the true value (M. Putri & Syaharuddin, 2019)

In general, what comes to every reader's mind related to determining the roots of an equation f(x)=0 is the graph method, factoring, completing the perfect square, or if these methods cannot be used, then using Al-Kharizmi's formula which is better known as the ABC formula (Darmawan & Zazilah, 2019). In engineering, non-linear equations are often found. These functions $f(x)$ can be in the form of algebraic, polynomial, trigonometric, or transcendental equations (Putri & Hasbiyati, 2016). Finding the roots of these equations means making the equation zero, or$f(x)$ = 0. However, not all equations can be solved by simple methods using basic mathematical theory. Some equations require numerical or computational techniques to obtain a solution (Batarius et al., 2018).

The Newton-Raphson method is one of the well know methods in solving the roots of non- linear equations, which works by iterating based on the first derivative of the function in question (Utami et al., 2024). The advantage of this method is its fast convergence, especially if the initial guess is close enough to the root. However, this method also has disadvantages, namely its dependence on good initial conditions and the possibility of failing to converge if the initial guess is too far from the root (So Ate, 2022). In some studies, it is stated that the Newton- Raphson method has a high level of accuracy. However, other research results explain that the Halley and Olver method has a higher accuracy rate than the Newton-Raphson method (Mandailina et al., 2020).

On the other hand, the Chebyshev Method is an improved method of the Newton-Raphson method. This method uses a more stable approach and has a better convergence speed in some cases (Fanani et al., 2024). Chebyshev method is a numerical technique used for interpolation and approximation of mathematical functions utilizing Chebyshev polynomials as a basis for approximating functions given in an interval (I.R et al., 2022). In principle, the Chebyshev method utilizes a series of iterations to produce a more accurate solution in a few iterations compared to the Newton-Raphson method. The main advantage of this method is more effective numerical error reduction, although in some specific cases, it can be more complex than the Newton-Raphson method (Darussalam et al., 2024).

This study aims to compare the accuracy of Newton Rapshon, and Chebyshev methods in selecting solutions to non-linear equations using a systematic and comprehensive approach. The assessment criteria that will be used include convergence, stability, and computational speed of each method (Fanani et al., 2024). results expected from this research is to know the comparison of convergence in solving the system of Non-linear equations with Newton Rapshon and Chebyshev methods using Matlab applications. By analyzing the comparison of these two methods, it is hoped that the best solution can be found for practical applications in various fields of science that require numerical solving of the roots of non-linear equations.

B. METHODS

In this section, we will explain in detail about the two methods used to solve the roots of non-linear equations, namely the Newton-Raphson Method and the Chebyshev Method. Each method will be explained starting from the basic concepts, iterative formulas, to its numerical application to solve the roots of non-linear equations. In this study, the researcher compares the solution procedures (Newton Rapshon and Chebyshev) in selecting the roots of non-linear equations. The equations used include trigonometric, pilinomial, exponential and logarithmic. Experiments were conducted 8 times with an error of 0.001 and using a maximum of 100 iterations. Of the four equations, the first step taken is to draw a graph of each equation with the aim of determining the starting point or x0. then the researcher performs a simulation using Matlab software. The description of the two methods used is explained below.

1. Newton Rapshon Method

The Newton-Raphson method is an iterative method used to find the roots of non-linear equations. The basic principle of this method is to improve the root estimate by using the first derivative information of the function in question. For a function f(x), the root of the equation f(x)=0 is found using iteration expressed in the following form:

Where:

xn+1

= xn

− $f(xn)$

$f'(xn)$

(1)

- xn is the root estimate value at the nth iteration,
- $f(xn)$ is the function value atxn ,
- $f'(xn)$ is the first derivative of the function atxn ,
- xn + 1 is the estimated root value at the next iteration

This iteration process will continue until the difference between two consecutive root estimates $|xn + 1 - xn|$ is smaller than a predetermined tolerance value. The speed of convergence of the Newton-Raphson method depends largely on the choice of initial guess$x0$. If the initial guess is close enough to the root, the method tends to converge quickly.

2. Chebyshev Method

The Chebyshev method is an improvement of the Newton-Raphson method and is used to improve efficiency and convergence stability in finding the roots of non-linear equations. It uses an iteration based on the Chebyshev series, which produces a more accurate solution with fewer iterations than the Newton-Raphson method. The iteration formula for the Chebyshev method is as follows:

Where:

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xn
$+ 1 = xn$
$- \ 2f(xn)$
$f'(xn)+\sqrt{(}\ f'(xn))2-2f(xn)f''(xn)$
(2)

- $f''(xn)$ is the second derivative of the function atxn ,
- $f'(xn)$ is the first derivative of the function atxn ,
- $f(xn)$ is the function value at .xn

Both methods are then used to solve non-linear equation problems. Furthermore, the problems used for simulation consist of:

- $f(x) = 2x2 \sin(3x + 2)$ (trigonometry)
- $f(x) = 2x3 + 5x - 2$ (polynomial)
- $f(x) = 2xe{-}4x + 2$ (exponential)
- $f(x) = 2x \log(2x{+}1 + 2)$ (logarithm)

The following is the flow of research conducted. In the initial stage, the researcher compiled a script to simulate each method, then in the second stage, the simulation results were compared by looking at the percentage of error and the number of iterations produced. The research procedure is described in Figure 1 below.

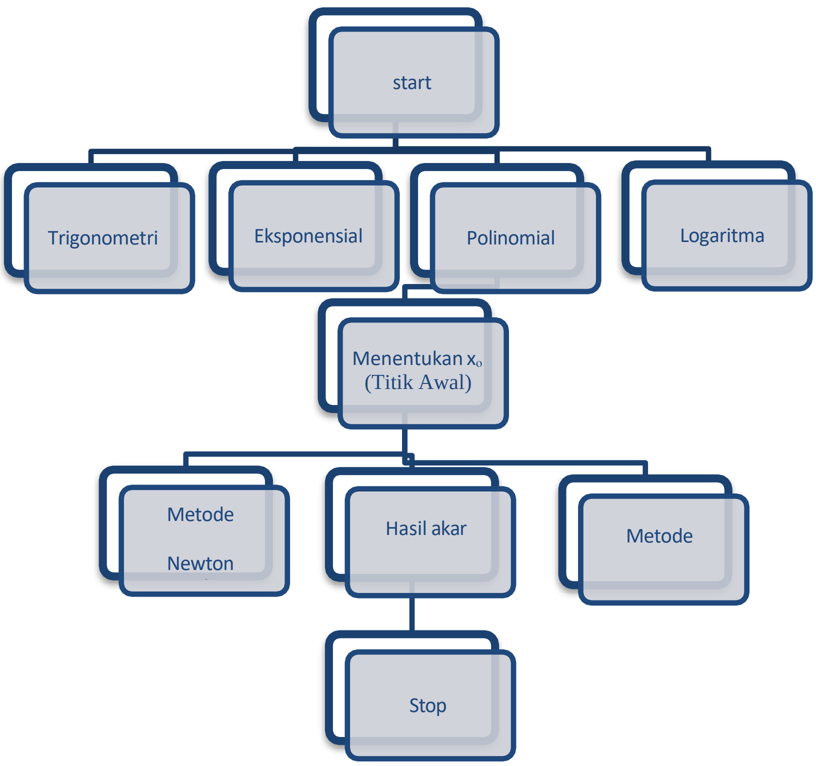

Figure 1. shows the flow of research conducted

Based on Figure 1, it is known that the research process is carried out in stages and continuously. Initially, researchers developed programming algorithms by conducting literature studies to find models or equations, determine the parameters used, and design algorithms for each method. After that, the algorithm was developed into a computational algorithm in the form of a MATLAB M-file. The researcher then determined the non-linear equation to be used as a test, by considering the similarity to Wilkinson polynomials, and determining the initial value of x. In this study, we used four types of problemsnamely trigonometric, polynomial, exponential, and logarithmic equations. Each of these problems is solved using two numerical methods, namely the Chebyshev method and the Newton-Raphson method. Furthermore, simulations of each method were carried out on each type of equation that had been determined, and the researcher recorded the root value obtained, the number of iterations, and the percentage of error. The results of the simulation were further analyzed by calculating and comparing the error rate and the number of iterations of each method. Based on the analysis, the researcher interpreted the results and drew conclusions about which method was most effective based on the smallest error value and the least number of iterations.

C. RESULTS AND DISCUSSION

Researchers used 4 non-linear equation problems consisting of trigonometric, polynomial, exponential, and logarithmic equations. Based on the steps that have been taken, the following are the graphical results of each equation according to Figure 1, Figure 2, Figure 3, and Figure 4 below.

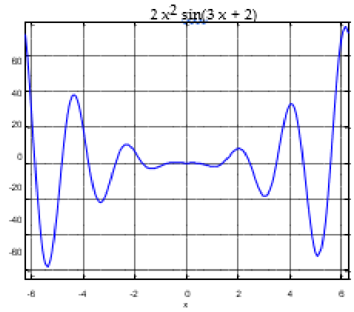

Figure 1 graph of a trigonometric function.

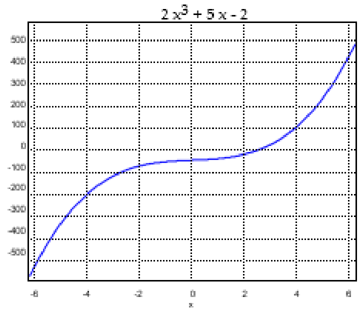

Figure 2. Graph of a polynomial function

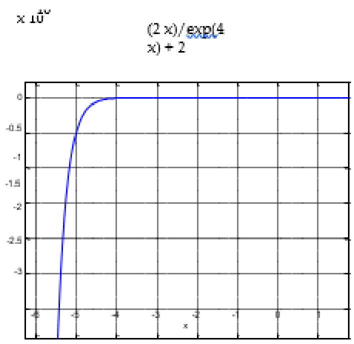

Figure 3: Graph of the exponential function

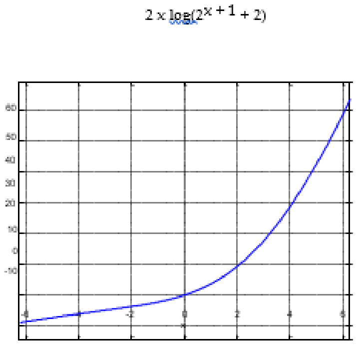

Figure 4: Graph of logarithmic function

In Figure 1 above, it can be seen that the roots of the equation are in the interval [-6,6], in this case the starting point or x0 = [-1] is the starting point for finding the roots of trigonometric equations. Figure 2 above shows that the roots of the equation are in the interval [-6,6], in this case the starting point x0 = [-4] or x0 = [-2] becomes the starting point to find the roots of the polynomial equation. In Figure 3 above, it can be seen that the roots of the equation are in the interval [-6,1], in this case the starting point x0= [-4] becomes the starting point for finding the roots of the exponential equation. Figure 4 above shows that the root of the equation is in the interval [-6,6], in this case the starting point x0= [-2] becomes the starting point to find the root of the logarithm equation. The following is the script used in each method. This script is used for simulation in the matlab application. Simulation on matlab application.

Table 1. Scrip of Each Method

Methods	Script

Newton Rapshon	for k=1:imax
	iter=iter+1;
	x2=x1-(feval(f,x1)/feval(f_diff,x1)); error=abs((x2-x1)/x2);
	x1=x2; y=feval(f,x1);
	fprintf('%10.0f %6.10f %6.10f %6.10f\n', [iter;x1;y;error])
	if (error< error1 || (iter> imax)), break,
	end
	end
	fprintf('The root is = %6.10f\n',x1)

Chebyshev	for k=1:imax
	iter=iter+1;
	%Chebyshev formula
	L=feval(f_diff2,x1)*feval(f,x1)/feval(f_diff1,x1)^2; x2=x1-(1+0.5*L)*(feval(f,x1)/feval(f_diff1,x1));
	error=abs((x2-x1)/x2); x1=x2;
	y=feval(f,x1);
	fprintf('%10.0f %6.10f %6.10f
	%6.10f\n',[iter;x1;y;error])
	if (error< error1 || (iter> imax)),break,end
	end
	fprintf('The root is = %6.10f\n',x1)

Using the script, the researchers then simulated 8 times with the matlab script. The simulation was carried out with the aim of calculating the iteration and the root of the equation, then obtained the simulation results in Table 1 below.

Table 2. Simulation Results

No.	Case	Methods	Iterations	X	F(x)	Error
1.	$2x^2 \sin(3x + 2)$	Newton rapshon	2	-4.8554571522	-0.0000397129	0.0001695392
		Chebyshev	3	-4.8554568715	0	0.0000000433
2	$2x^3 + 5x - 2$	Newton rapshon	4	0.3783379446	0.0000000075	0.0001515603
		Chebyshev	3	0.3783379433	0	0.0000060060
3	$2xe^{-4x} + 2$	Newton rapshon	21	-0.3005419683	-0.0000000003	0.0000080872
		Chebyshev	14	-0.3005419683	-0.0000000001	0.0002237906
4	$2x\log(2^{x+1} + 2)$	Newton rapshon	100	0	0	inf
		Chebyshev	100	0	0	inf

According to the results of the above computation using the Newton Rapshon method, for case 1 it is known that the value of x such that f(x) < 0.001 is x = -4.8554571522 with f(x) = -0.0000397129, the value was obtained after computing up to 2 iterations. By using Chebyshev's method, for case 1, it is known that the value of x such that f(x) < 0.001 is x = -4.8554568715 with f(x) = 0, the value is obtained after computing up to 3 iterations. Thus it shows that for solving the root equation of f(x) =$2x2$ sin($3x$ + 2) using the Newton Rapshon method the rate of convergence is faster when compared to the Chebyshev method.

For case 2, it is known that the value of x such that f(x) < 0.001 is x = 0.3783379446 with f(x)=0.0000000075, the value is obtained after performing computation up to 4 iterations. By using Chebyshev's method, for case 2, it is known that the value of x as a result of f(x) < 0.001 is x = 0.3783379433 with f(x)=0, the value is obtained after computing up to 3 iterations of repetition. Thus it shows that for solving the root equation of f(x) = $2x3 + 5x - 2$ using the Newton Rapshon method the rate of convergence is slower when compared to the Chebyshev method. For case 3, it is known that the value of x such that f(x) < 0.001 is x = -0.3005419683 using f(x) = -0.0000000003, the value was obtained after computing up to 21 iterations. By using Chebyshev's method, for case 3, it is known that the value of x such that f(x) < 0.001 is x = -0.3005419683 using f(x) = 0.0000000001, the value is obtained after computing up to 14 iterations. Thus it shows that for solving the root equation of f(x) =$2xe-4x + 2$ using the Newton Rapshon method the rate of convergence is slower when compared to the Chebyshev method.

For case 4, it is known that the value of x as a result f(x) < 0.001 is x = 0, using f(x) = -0, the value is obtained after computing up to 100 iterations. By using Chebyshev's method, for case 4 it is known that the value of x as a result of f(x) < 0.001 is x = 0 using f(x) = 0, the value is obtained after computing up to 100 iterations. Thus it shows that for solving the root equation of f(x) =2x log(2x+1 + 2) using Newton Rapshon and Chebyshev methods the rate of convergence is error or does not find results. Several studies have been conducted by several authors who simulate numerical methods. one of the studies conducted by (Salwa, 2022), examines the comparison of the Newton Midpoint Halley Method, the Olver Method and the Chabysave Method in completing the roots of Non-Linear Equations. Furthermore, (Akmala et al., 2022), examines the Bisex Method using Matlab GUI: A Simulation and Solution of Non-Linear Equations. Furthermore, there is (Mandailina, 2020) examining the Wilkinson Polynomial: Accuracy Analysis according to the Taylor Series Derivative Numerical Method. And the last one is (M. Putri & Syaharuddin, 2019), examining the numerical implementation of open and closed methods: Convergence Test of Nonlinear Equation Solutions.

D. CONCLUSION AND SUGGESTIONS

From the four cases of solving equations (trigonometric, polynomial, exponential, and logarithmic) tested, it can be concluded that the Chebyshev method has a faster convergence rate and higher accuracy than the Newton-Raphson method in most cases, although the iteration calculation is more complex. Therefore, it can be concluded that the Chebyshev method is the best method in solving the roots of non-linear equations.

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