

Callable Bond's Value Analysis Using Binomial Interest Rate Tree Considering Early Redemption and Default Risks

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ABSTRACT

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Bonds are known as one of low-risk investments and worth to be considered as a part of an investor's portfolio, however there are still underlying risks that could affect its price. This paper focuses on the effect of early redemption risk and default risk to a bond's value. Using binomial interest rate tree method and its adjusted for default risk version, this paper wants to analyse how these risks affect Indonesian bonds' values through simulations, while showing how these bonds can be used to construct the binomial interest rate trees. In the default risk simulation, more assumptions are made because of data limitations, which causes the first period recovery fraction to soar higher than the other periods. The analysis shows that, compared to present value of standard bonds, early redemption risk tends to cause the bond's present value to drop, while on the contrary, default risk tends to cause the bond's present value to rise. The cause of higher present value of bonds with default risk is explained by the high first period recovery fraction.



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A. INTRODUCTION

Binomial tree is widely known as an option pricing method, with the most notable models are the ones proposed by Cox et al. (1985) and Cox et al. (1979) They are still used in recent papers, for example as a base for models developed to price specific type of option like European option (He et al., 2019) and American option (Appolloni et al., 2013). This paper applies a bond valuation method called binomial interest rate tree, proposed by Kalotay et al. (1993), that follows the same up-and-down movement of the option pricing method.

The up-and-down movement of binomial interest rate tree relies on interest rate volatility (Kalotay et al., 1993). As the centre of economic theories, models, and systems, interest rate volatility plays a major role in the development of valuation method of bond and its derivatives (Markellos & Psychoyios, 2018). Skalický et al. (2022) note that, in the event of declining interest rates, the risk of a lower return on investment might be posed to investors. However, callable bond investors are exposed to one more risk, that is early redemption risk. Therefore, the issuance firms are motivated to issue bonds with the right for the issuer to redeem the bond early. This type of bonds allows the issuer to pay off the bonds with higher interest (coupon)

rate and reissue a new bond at a lower interest rate (Baker et al., 2019). Additionally, note that a firm that issues a lot of bonds indirectly signify that it has loads of interests to pay. If the issuer fails to pay them, they are at risk of default.

Due to the early redemption risk in bonds (i.e., callable bonds), they depend on the future value of interest rates, which in turn makes their future cash flows uncertain. Valuing a bond with this characteristic requires a model that's capable of explaining the fact that future interest rates are uncertain. The binomial interest rate tree is commonly used to value this type of bonds. Some recent papers used the more popular models. Díaz & Tolentino (2020) examined interest rate risk in bonds with embedded options, comparing both the Ho & Lee (1986) model which follows the binomial distribution and utilizes Wiener process that can still produce negative interest rates and Black et al. (1990) model that follows the lognormal distribution resulting in nonnegative interest rates. Skalický et al. (2022) proposed an approach to valuing bonds with an embedded European option based on models describing the evolution of interest rates (Cox et al., 1985; Marsh & Rosenfeld, 1983; Vasicek, 1977). By extending the Ho-Lee model, Kim et al. (2017) improved the model to produce a more realistic and flexible model. Some recent papers also use more complex method to value bonds while considering default risk, specifically for convertible bonds (Batten et al., 2013; Park et al., 2017). In this paper, simpler models were chosen as it focuses on analysing Indonesian callable bonds and their early redemption risk and default risk.

Default risk occurs when the bond issuer fails to meet the payment of interest and repayment of the amount borrowed on time (Baker et al., 2019; Fabozzi & Fabozzi, 2021). Skalický et al. (2022) summarized how callable bonds and default risk are connected. First, callable bonds are predominantly issued by firms facing higher default risk since they suffer from the lack of positive NPV projects (Elsaify & Roussanov, 2016). Second, firms with higher default risk are more likely to issue callable bonds because of stronger incentives to engage in risk-shifting activities. Callable bonds are also more flexible to liquidate from conventional assets, then to be invested in riskier assets. Third, they note that hypothetically callable bond issuers might be private and non-transparent firms, hence they are subject to a higher default risk. To counter this, Hsu et al. (2015) found that the more innovative a firm is, the more it can withstand from defaulting.

This paper addresses early redemption risk in callable bonds and default risk in both callable and standard bonds. Specifically, the objective is to create binomial interest rate trees, which are explained in detail in the following section. Utilizing data from Indonesian bonds, the trees will be used to evaluate the value of Indonesian corporate bonds while also considering the potential risks involved, which are discussed in the third section. The primary goal of this paper is to analyse how these two types of risks could impact the prices of Indonesian bonds. Moreover, the study examines which types of Indonesian bonds (whether on-the-run government bonds or corporate bonds) are utilized to create the binomial interest rate tree.

B. METHODS

For the early redemption risk analysis, this paper follows the bond valuation model by Kalotay et al. (1993), which explains that the value of any bond can be expressed as the present value of its future cash flows. They viewed each cash flow as a zero-coupon instrument maturing on the date it will be received. Consequently, each cash flow needs to be discounted by fluctuating rates from different time periods, not a single discount rate. The main objective in constructing a binomial interest rate tree is to find the different rates called the one-year-rates.

The model proposed by Finnerty (1999), who developed the Kalotay et al. (1993) model to accommodate default risk would be suitable for the default risk analysis. This model captures not only the default risk, but the interest rate risk and early redemption risk as well. The model also considers expected recovery fractions i.e., the residual amount of the principal that investors are likely to receive (Baker et al., 2019).

1. Data and Data Sources

The purpose of this research is to assess callable bonds, and as such, the information utilized in this study pertains to Indonesian corporate bonds that contain embedded call options. The data covers bonds that were issued on or prior to 2022 and includes bonds that will mature annually over a five-year period. The study aims to demonstrate how the value of the bond changes each year in relation to the call provision. The relevant data was obtained from the prospectuses of each of the involved bonds.

Before constructing the binomial interest rate tree for each bond, the one-year rates are collected from (Investing.com, 2022b). Data of on-the-run government-issued bonds were used to calculate the one-year rates for the first simulation. There are no bonds available with maturities other than 1, 3, and 5 years. Hence, to estimate the bond rate for the remaining maturities, the method of linear interpolation is utilized. In the second simulation, corporate bonds with two to ten years to maturity are used to construct the binomial interest rate tree. To assess the risk of early redemption, additional corporate bonds with maturities of 3, 6, and 9 years are employed for analysis. This data is obtained from (Indonesian Central Securities Depository (KSEI), 2022).

The default risk analysis uses bonds that have matured before the valuation date. It is time consuming to find bonds with the same credit risk that were active in that same period, so the third simulation only uses one bond issued by PT ANTAM Tbk in 2011 and matured in 2021 to construct the binomial tree (PT ANTAM Tbk, 2022a). The company's credit rating is B+ according to Standard & Poor's Global Ratings. The data were obtained from PT ANTAM Tbk (2022) website. Table 1 shows the initial data used to construct this binomial interest rate tree.

Table 1. Data Overview of Default Risk Analysis

Coupon Rate (%)	F	r_0 (%)
9.05	100	5.83

This is the sole tree that has been created, assuming that the bond prices and coupon rates remain constant for each maturity throughout the life of the bond. There is a valid reason for using this approach: bonds that have already matured do not possess any historical data pertaining to their price or yield-to-maturity (YTM). The data for Indonesian on-the-run

government bonds in that period, that is, 2011 until 2021, are unavailable as well. To compute the r_0 , historical data of one-year Indonesian government bond yield in 2011 are used.

2. Binomial Interest Rate Tree Construction

To evaluate a bond using a binomial interest rate tree, the initial step is to identify the one-year rates for each year. Only the one-year rates located at the bottom of each year are required since the one-year rates at adjacent nodes for a given year are related via the equation shown below.

$$r_U = r_L e^{2\sigma} \quad (1)$$

with σ as the assumed volatility of the forward one-year rates through all time period.

To find the one-year rates, the bond's value at each of the nodes is required in the calculation, which could be found using the equation

$$V = \frac{1}{2} \left[\frac{V_U + C}{1+r^*} + \frac{V_D + C}{1+r^*} \right] \quad (2)$$

with

- V : the bond's value at the node in question,
- V_U : the bond's value in one year if the one-year rate rises,
- V_D : the bond's value in one year if the one-year rate drops,
- C : the coupon payment,
- r^* : the one-year rate at the node in question.

The goal in constructing a binomial interest rate tree is to find the one-year rates that make the bond's value at the node in "This Year" the same as the target value, which is assumed to be 100 in this study, while still consistent with the volatility assumption. This is done through an iterative process i.e., trial-and-error. Using the Solver function in Ms. Excel, the steps are as follows:

- a. Calculation is done per year by discounting the bond's value in one year. Suppose r_t for Year 1 ($t = 1$) is being calculated, use the coupon rate from a bond that would mature in Year 2 ($t = 2$) for each node.
- b. Set 100 as the value for "This Year".
- c. To keep the bond's value non-negative, set the condition r_t is positive for all years.
- d. Use Solver to find r_t for the period concerned to obtain the value that is consistent with steps 2 and 3.
- e. Repeat step 1 to 4 to find r_t until the $n - 1$ -th period. The n -th period is the last period before the bond matures, therefore r_n is not needed.

3. Valuation of Callable Bond

The existence of call provision in a callable bond leads to the bond's value at a node must be changed to reflect the lesser of its value if it is not called or the call price:

$$V = \min \left(P, \frac{1}{2} \left[\frac{V_U + C}{1+r^*} + \frac{V_D + C}{1+r^*} \right] \right) \quad (3)$$

with P notating the bond's redemption value. Analysis of early redemption risks is also closely related to the bond's value, which could be seen in the equation

$$\text{Value of Call Option} = \text{Value of Option-Free Bond} - \text{Value of Callable Bond.} \quad (4)$$

4. Default Risk Analysis

Before diving into the valuation, there are some new symbols that need to be introduced to incorporate the default risk in the valuation. The first one is R_t , the expected recovery fraction. For the first period, R_1 is calculated using the formula:

$$\frac{(1-p_1)(100+C)+(p_1 \times (100+C) \times R_1)}{1+s_1} = 100. \quad (5)$$

The p_1 is the unconditional probability that it will default in one year and s_1 is the spot rate at Year 1. After estimating R_1, R_2, \dots, R_{n-1} , the expected recovery fractions for the time n are calculated using the equation:

$$\frac{C(1-p_1)+(100+C) \times R_1 \times p_1}{1+s_1} + \frac{C(1-p_1-p_2)+(100+C) \times R_2 \times p_2}{(1+s_2)^2} + \dots + \frac{(100+C) (1-\sum_{i=1}^n p_i)+(100+C) \times R_n \times p_n}{(1+s_n)^n} = 100. \quad (6)$$

The second one is q_{n+1} , the conditional probability that the issuer would default on the cash interest payment at time $n + 1$, given that it has not yet defaulted as of time n . This conditional probability found by the equation

$$q_{n+1} = \frac{p_{n+1}}{U_{n+1}}. \quad (7)$$

The new variable U_{n+1} is the proportion of population has not defaulted (undefaulted) at time $n + 1$. After finding the two values described above, the bond's value at time n and node m with default risk is obtained by:

$$V_{n,m} = \frac{\left(\frac{V_u+V_D}{2}+C\right)(1-q_{n+1})+(100+C)q_{n+1}R_{n+1}}{1+r_{n,m}} \quad (8)$$

with $r_{n,m}$ notating the one-year rate at time n and node m .

Even further, the early redemption risk could be analyzed on the bond's value with default risk too, using the modified version of equation (3)

$$V = \min\left(P, \frac{\left(\frac{V_u+V_D}{2}+C\right)(1-q_{n+1})+(100+C)q_{n+1}R_{n+1}}{1+r_{n,m}}\right). \quad (9)$$

C. RESULT AND DISCUSSION

This study involves three simulations of constructing binomial interest rate trees. The first simulation uses standard on-the-run government bonds, the second simulation uses callable corporate bonds, and the third simulation uses corporate bonds that carry default risk. These simulated interest rate trees are then used to evaluate the value of both standard and callable corporate bonds. For the third simulation particularly, there is a valuation of bonds with default risk.

1. Simulation of Binomial Interest Rate Tree Construction

Assumptions were made in this study to simplify the research method and to make up for the lack of data available. For the first simulation, for each of the bonds being used, the assumptions are:

- a. The bond's par value is 100.
- b. The bond's market value equals its par value, so that the bond's yield to maturity (YTM) equals its coupon rate.
- c. The bond's redemption value is 100.

- d. One-year rate’s volatility is 10%.
- e. Coupons are paid annually.

The binomial interest rate tree in this simulation is constructed using on-the-run yield of government bonds with different maturities as shown in Table 2. As discussed in the last section, a bond that will mature at time $t + 1$ is needed to find r_t . Thus, the results are the one-year-rate for each year except for the last year. Observing the data reveals that the one-year rate for each maturity follows the annual coupon rate, indicating a positive correlation between the two. Nevertheless, it cannot be stated that they are genuinely comparable. For example, the two-year bond’s coupon rate used to calculate the one-year rate was 6.5% and for the next maturity the coupon rate was 5.8125%. The discrepancy between the two is only 0.6875%, whereas the difference between the one-year rates amounts to 3.4906%, as shown in Table 2.

Table 2. Data Overview and One-Year-Rate Results

Maturity Year	On-the-Run Yield Rate (%)	One-Year-Rate (%)	Coupon Rate (%)	
			Case I	Case II
2023	5.6250	5.8815	-	-
2024	6.0625	6.0681	9.2500	5.6000
2025	6.5000	2.5775	9.3000	5.7000
2026	5.8125	1.3746	9.6000	6.0000
2027	5.1250	-	9.4000	6.6500

Early redemption risk analysis in this simulation is divided into two cases. The first one used corporate bonds with an average coupon rate greater than the average coupon rate of the on-the-run government bonds. Meanwhile, the other case used corporate bonds with an average coupon rate smaller than the average coupon rate of the on-the-run bonds. These bonds are assumed to have embedded American call options with redemption value 100. These coupon rates are all provided in Table 2 as well.

The bonds for the first case are issued by PT XL Axiata Tbk, PT Medco Energi Internasional Tbk, PT Pembangunan Jaya Ancol Tbk, and Indonesia Eximbank with their time to maturity are 2, 3, 4, and 5 respectively. The second case’s bonds are issued by PT Pupuk Indonesia, PT Toyota Astra Financial Services, PT Sarana Multigriya Finansial (Persero), and PT Waskita Karya (Persero) Tbk. The values of standard bonds, callable bonds, and call options for both cases are presented in Table 3.

Table 3. Bond Valuation Results

Maturity Year	Case I			Case II		
	Standard	Callable	Call Option	Standard	Callable	Call Option
2024	105.85	103.43	2.42	99.15	99.15	0
2025	107.46	105.94	1.52	97.87	97.87	0
2026	113.31	103.76	9.35	100.65	98.67	1.98
2027	118.14	103.57	14.57	106.47	100.09	6.38

Each of the values obtained in Table 3 need a dedicated binomial tree, but this paper will only show some examples of the trees. In Case I, the standard bonds are increasing in value as the maturity increases, all higher than their callable counterparts. The values of callable bonds

do not appear to exhibit a clear pattern, resulting in the value of the three-year bond call option being lower than that of the two-year bond.

The call option of the four-year bond has a value of 9.35, significantly higher than the call options of the two-year and three-year bonds. This demonstrates the replacement of values to its redemption value at the nodes that occurs more frequently in four-year bonds than the other maturities. Comparing the trees in Figure 1 and Figure 2, it is observable that in addition to having more nodes than the shorter terms, most of the bond's values at the nodes are also replaced by 100. The replacement of certain terms has an impact on the value of callable bonds, which experiences a more significant shift compared to shorter terms, leading to an increase in the value of the option. Most of the one-year-rates obtained in this simulation are also visible in Figure 1 and Figure 2, placed at the bottom of each year. Since the one we're discussing in these two figures is a four-year bond, the one-year-rate for Year 4, that is 1.3746% is not used, as shown in Figure 1 and Figure 2.

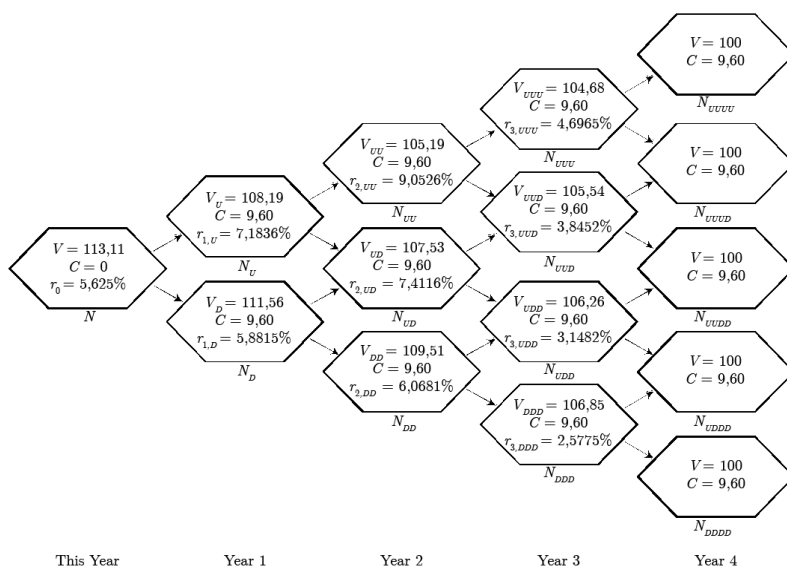


Figure 1. Binomial Interest Rate Tree, Valuation of the 9.6% Four Years to Maturity Standard Bond in Case I, Simulation 1

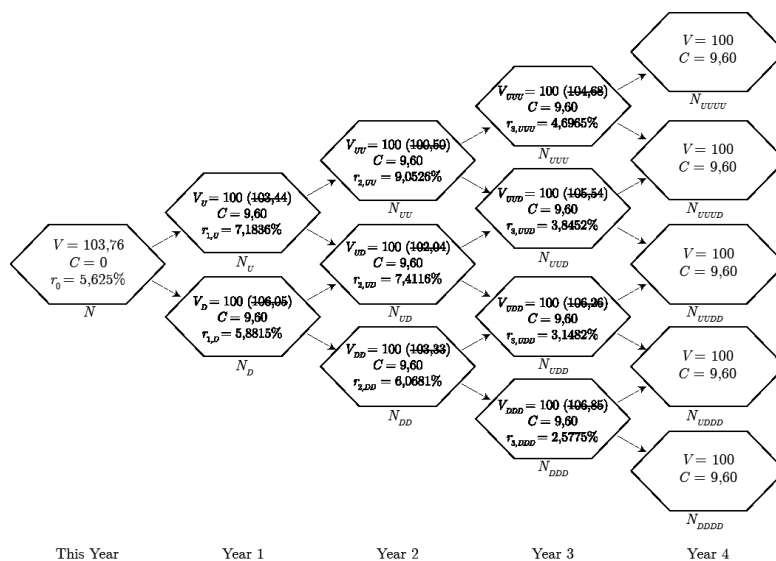


Figure 2. Binomial Interest Rate Tree, Valuation of the 9.6% Four Years to Maturity Callable Bond in Case I, Simulation 1

For Case II, as provided in Table 3, the two-year bond and the three-year bond’s standard and callable values are identical. This is more evident if observed from the binomial interest rate trees, that is Figure 3 and Figure 4. In both trees, none of the nodes have a value greater than 100, indicating that no value replacement has occurred. This might imply that no interest rate is low enough for the issuer to consider early redemption, causing the call options to have no value. Note that the present values of the callable bonds are also less than 100, as shown in Figure 3 and Figure 4.

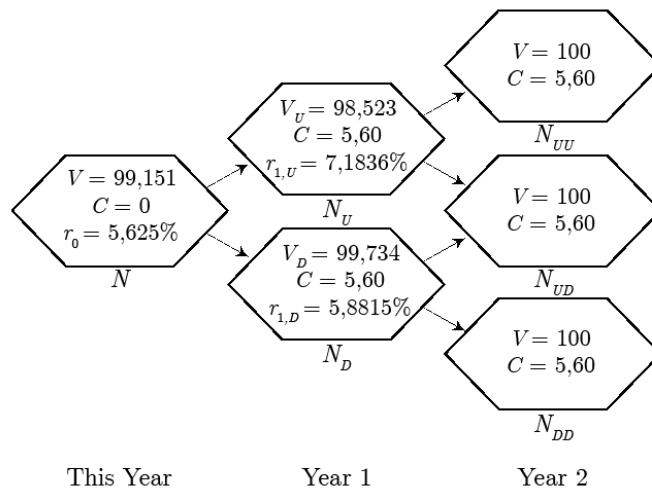


Figure 3. Binomial Interest Rate Tree, Valuation of the 5.6% Two Years to Maturity Standard Bond and Callable Bond in Case II, Simulation 1

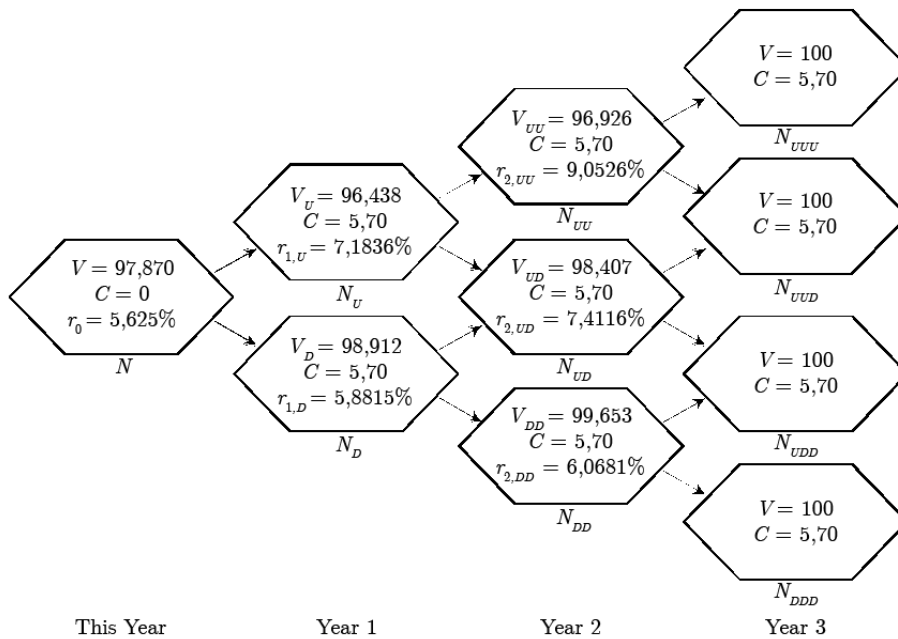


Figure 4. Binomial Interest Rate Tree, Valuation of the 5.7% Three Years to Maturity Standard Bond and Callable Bond in Case II, Simulation 1

2. Simulation of Binomial Interest Rate Tree Construction using Corporate Bonds

Using the same assumptions as before, the valuation year for this simulation is 2021. The construction used bonds issued before or at 2021 and will mature at 2023 to 2031. The value of the one-year rate on "This Year", r_0 is obtained from the geometric mean return of the one-year government bond at 2021, which is 3.68%. The data for this is collected from Investing.com (2022a), as shown in Table 4.

Table 4. Data Overview and Results of Simulation 2

Period	Maturity Year	Coupon Rate (Construction) (%)	One-Year-Rate (%)	Coupon Rate (Valuation) (%)	Bond Value		Call Option Value
					Standard	Callable	
1	2022	-	13.434	-	-	-	-
2	2023	8.9000	8.1947	-	-	-	-
3	2024	9.2500	6.4459	9.5000	100.2400	100.0300	0.2100
4	2025	9.3000	7.9288	-	-	-	-
5	2026	9.6000	4.8312	-	-	-	-
6	2027	9.4000	6.9572	8.3000	95.0790	94.4420	0.6370
7	2028	9.7500	3.3956	-	-	-	-
8	2029	9.5000	2.7705	-	-	-	-
9	2030	9.2500	2.2322	10.6000	107.9900	101.3600	6.6300
10	2031	9.0000	-	-	-	-	-

In Table 4, the first period shows higher r_t than the other periods. This one-year rate needs to discount the bond's value small enough in one year, so that the bond's value discounted by r_0 would reach 100. From the third to fifth period, the coupon rates increase, followed by r_2 , r_3 , and r_4 . If the coupon rates experience fluctuations in the upcoming periods, the one-year rates demonstrate a similar pattern. This strengthens the argument made in the first simulation that the two have a positive correlation. The one-year rates in this simulation are relatively higher than those in the first simulation. However, some of these rates are unlikely to occur, which explains why most research studies use on-the-run government bonds for constructing binomial interest rate trees. Furthermore, the analysis of early redemption risks produces results like those of the first simulation, with standard bonds having higher valuations than callable bonds.

3. Simulation of Bonds with Default Risk's Valuation

Unlike the previous simulations, this section uses default rate and credit rating to value the bonds. Therefore, it requires some new assumptions:

- Only one matured bond is used to construct the binomial interest rate tree.
- The bond's value is set to 100 for each r_t calculated.
- The bond's market value is constantly at 100.
- The spot rates in the expected recovery fractions calculation are estimated from the one-year rates output using standard binomial interest rate tree. Chosen one-year rates are the ones with highest probability.
- In the binomial interest rate tree, "This Year" period is the issue year of the bond used to construct.
- Bonds valuation used active bonds in "This Year" period.

- g. Geometric mean of one-year government bond return is used for r_0 .
- h. The issuers' credit ratings are constant in the observed years.

The bond used in this simulation was issued by PT ANTAM Tbk with a coupon rate of 9.05%. Meanwhile the r_0 is calculated from historical data of one-year Indonesian government bond return in 2011 and amounted to 5.83%. Before jumping in to the construction of a binomial interest rate tree, q_t and R_t for each t must be estimated. From Equations (5), (6), and (7), we need additional data. In this study, as written on the assumptions, the spot rate is estimated. Table 5 provides the data used in these calculations as well as the results. Note that the one-year rates are the chosen one-year rates to estimate the spot rates, not r_t , as shown in Table 5.

Table 5. Data Overview and Results of Simulation 3

Period	One-Year-Rate (%)	Default Probability (%)	Undefaulted Percentage of Population (%)	Spot Rate (%)	Conditional Default Probability (%)	Default Recovery Fraction (%)
1	11.424	1.5800	100	11.424	1.5800	237.78
2	8.9850	1.5200	98.420	10.198	1.5444	96.142
3	8.1097	0.7900	96.900	9.4973	0.8152	5.7601
4	8.9479	2.4200	96.110	9.3597	2.5179	92.1833
5	8.0899	3.7600	93.690	9.1045	4.0132	78.061
6	8.9411	1.0000	89.930	9.0773	1.1120	91.018
7	8.0975	0.9400	88.930	8.9368	1.0570	17.363
8	8.9647	1.4900	87.990	8.9402	1.6934	95.380
9	8.1326	3.5300	86.500	8.8502	4.0809	79.385
10	-	0.5200	82.970	-	-	-

In some instances, such as at times 3 and 7, the anticipated recovery fractions are below 20%. This suggests that companies may be spending a significant amount of money to delay default, as doing so would decrease the expected recovery fractions for debenture holders. However, in period 1, the expected recovery fraction reaches an extraordinarily high value of 237.78% compared to the other fractions. If a bond has just been issued on the market, investors may anticipate high returns in the future. Unfortunately, in the following year, the issuer is unable to pay the interest and is even on the brink of bankruptcy, which explains the exceedingly high expected recovery fraction. However, given the amount of data required to obtain this result, this interpretation should be processed with caution, as shown in Table 6.

Table 6. Coupon Rates and Bond Valuation Results of Simulation 3

Time to Maturity	Coupon Rate (%)	Bond Value			
		Standard	Early Redemption Risk	Default Risk	Early Redemption Risk and Default Risk
3	14.250	113.273	107.953	115.422	110.303
5	6.050	88.287	88.285	90.032	90.030
7	8.375	96.597	95.604	97.572	96.892
10	9.050	100.000	96.778	99.967	98.033

By utilizing Equation (8), the bond's value can be determined for each node. This binomial interest rate tree is capable of valuing bonds that have a maturity period of 1 to 10 years. The research presents outcomes of bond valuations with 3, 5, 7, and 10 years to maturity in Table 6. All issuers of the valued bonds possess a credit rating of B+. In comparison, the standard and callable values are also included.

The values of standard bonds are greater than the values of callable bonds. However, it is more complicated when default risk is involved. When bonds only have a risk of default, the values are surprisingly greater than the standard bond values, except for the 10-year bond. This is reasonable because the 10-year bond is used to construct the binomial interest rate tree. Additionally, these findings indicate that longer maturity periods result in smaller differences between standard and default-risked bond values. However, Figure 6 shows that the bond values at nodes are mostly lower than the values at the nodes of Figure 5. This applies to all nodes except for the fourth year, considering that the same calculation method is utilized for both trees, as shown in Figure 5.

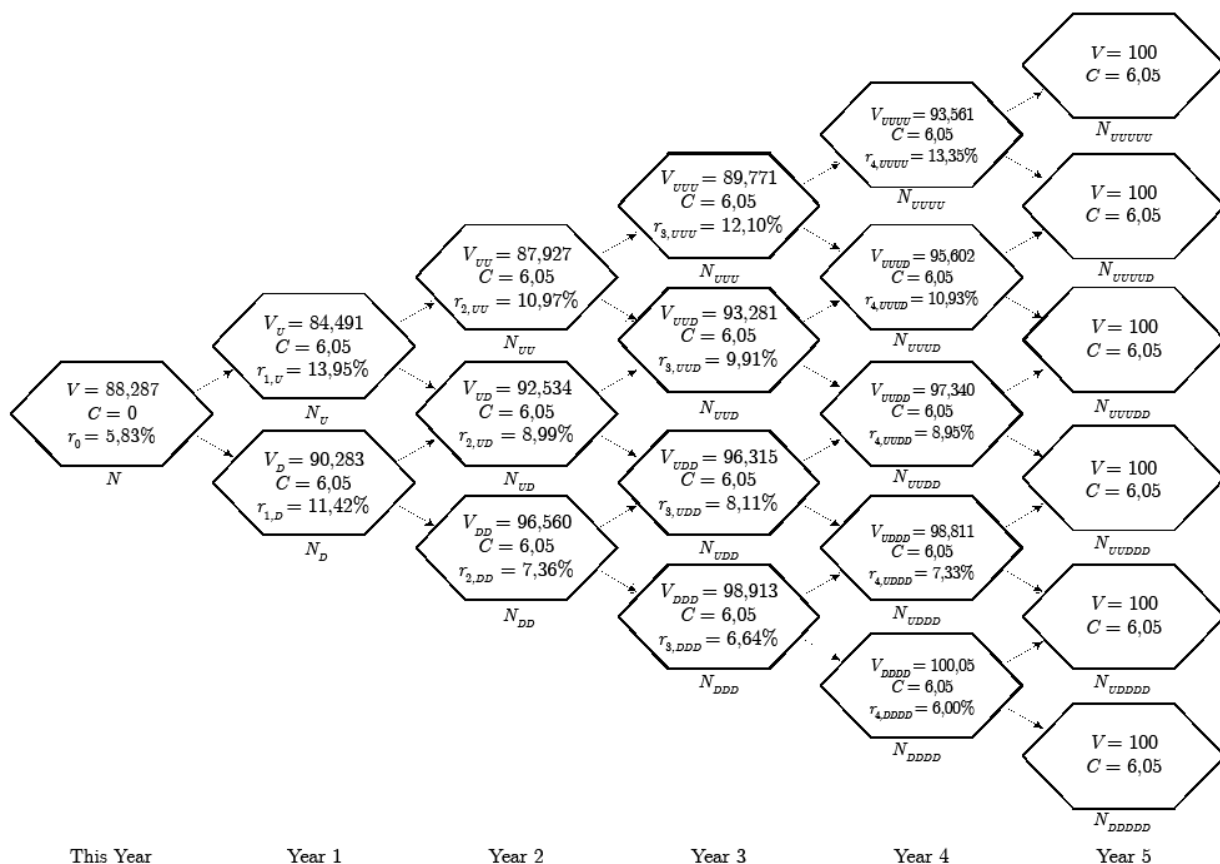


Figure 5. Valuation of the 6.05% Five Years to Maturity Standard Bond in Simulation 3

On the contrary, when there are two risks involved, both the 3-years and 10-years bonds values are lower than the standard bonds values, while the rest are unusually higher. This is due to less nodes having their value replaced by 100 for the 5-years and 7-years bonds, since even their standard and default risked values are lower than 100. It should be noted that the double risked bonds went through default risk analysis first before undergoing early redemption risk analysis, meaning the default risked bond valuation like in Figure 6 can show

how many nodes with values exceeding 100. Meanwhile, the 3-years bonds all have their values, with risk(s) and no risk, over 100. Despite the small number of nodes in the 3-years bond valuation, due to the many value replacements, the present value of the double risked bond ended up lower than the standard value, as shown in Figure 6 and Figure 7.

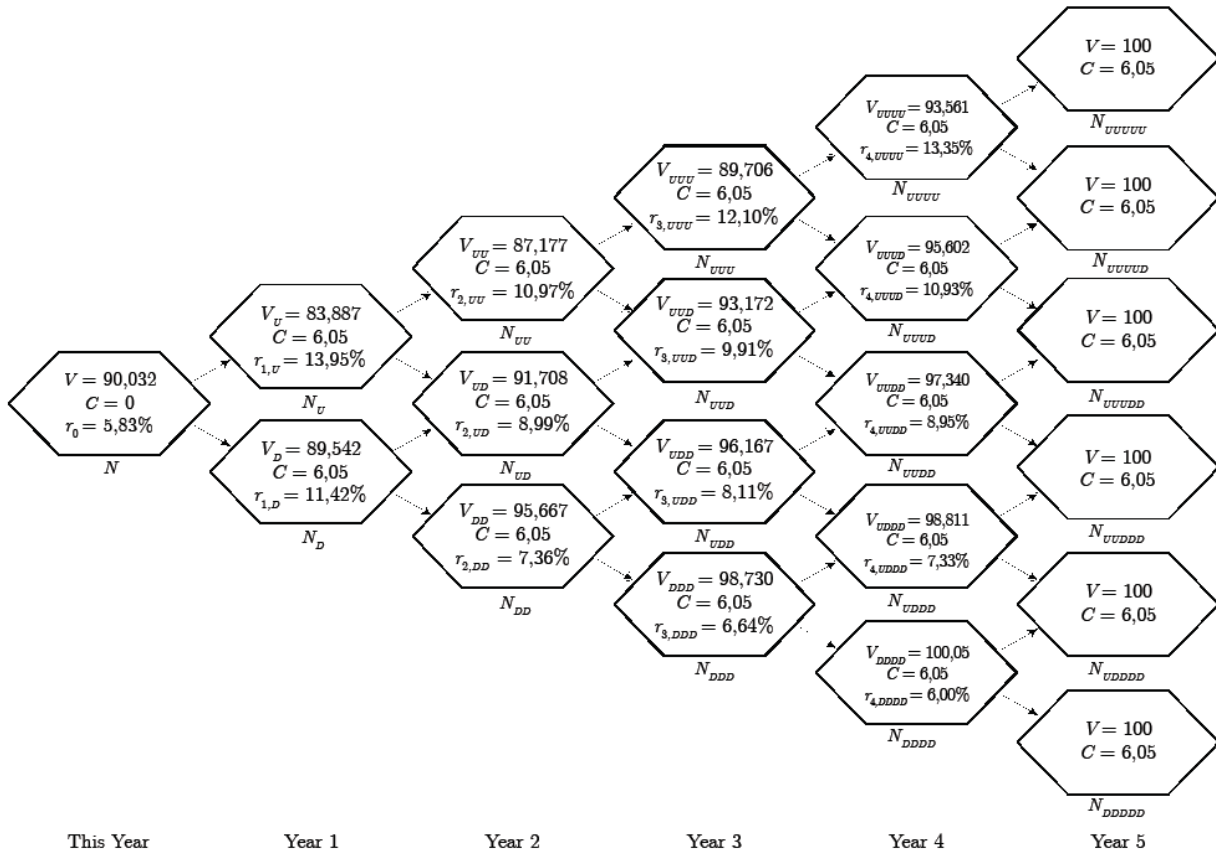


Figure 6. Valuation of the 6.05% Five Years to Maturity Default Risked Bond in Simulation 3

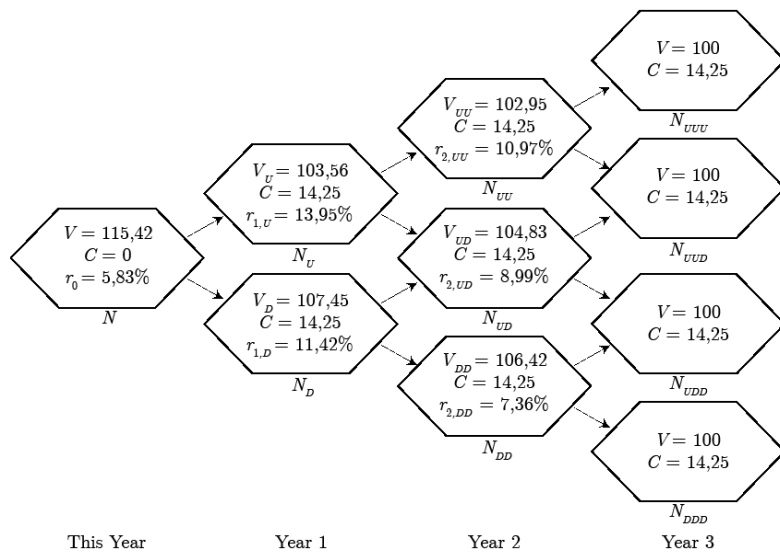


Figure 7. Default Risked Bond Valuation with Three Years to Maturity in Simulation 3

From these simulations it can be concluded that default risk and early redemption risk do affect a bond's value. The impact of the risk associated with early redemption is relatively more

foreseeable, as it consistently diminishes the value of a bond, even though the precise percentage of alteration may not be immediately apparent. Conversely, the default risk exhibits a dissimilar trend, often leading to an upsurge in a bond's value, which is atypical for a risk. The difference in amount of data and assumptions between these simulations might be the cause to this abnormal result. Despite its imperfections, this study introduces a new method of identifying various risks associated with Indonesian bonds that have not been previously explored. While previous research has analysed factors that affect government bond values through hypothesis testing, determining which factors generate a statistically significant impact on bond values (Azizah & Hidajat, 2016; Ichsan et al., 2013). The factors analyzed in this paper is not the same as these recent papers, albeit some might be related, thus exhibiting another perspective in evaluating a bond's value. Additionally, while a lot of recent papers study Indonesian government bonds, this paper focuses more on Indonesian corporate bonds.

There are also papers that focuses more on developing models for corporate bonds rather than applying to data. Chen et al. (2014) develops a model examining credit risk on corporate bonds which focuses on the interaction of liquidity and default over the business cycle. Y. Zhang & Chen (2021) use a machine learning approach in forecasting default risk of corporate bond issuers in China. Park et al. (2017) applies the Longstaff-Schwartz Least Squares simulation method to price convertible bond while considering default risk. While these recent papers demonstrate a more sophisticated approach to the study of default risk, it may be a little too intricate to follow and apply. Therefore, this paper offers a method to study default risk in a more holistic way.

D. CONCLUSION AND SUGGESTIONS

This paper focuses on constructing binomial interest rate trees with Indonesian bonds and valuing Indonesian corporate bonds, while also analyzing the early redemption risk and default risk effect on the bonds' values. Mostly, the early redemption risk cause the bond's value to decrease, with the difference being the call option value. The first simulation shows that when the bond being valued has significantly higher coupon rate than the one-year-rates, the call option value would be higher as well. If the coupon rate is similar to the one-year-rates, it could even result in no value to the call option. This may be interpreted as interest rates are never low enough for the issuer to call the bonds back, so that the option has no value to them. The second simulation shows why on-the-run government-issued bonds should be used to construct binomial interest rate tree, since the one-year-rates are bizarrely high if corporate bonds are used.

Meanwhile, the default risk is more troublesome to deal with. While rationally risk would bring the bond's value down, the results show that is not the case, except for the 10 years to maturity bond. The double-risked bond analysis results are even more indefinite, there is little to no pattern to them. There appears to be an influence from the bond data itself too, like the coupon rate and time to maturity, since those affect the nodes' amount and also the values in the nodes.

The main limitation in this paper is the insufficiency of the data, particularly regarding the default risk analysis. While this shortfall is offset to some extent by the use of assumptions, excessive reliance on such assumptions may negatively impact the final results. It is a challenge

to find issuers with the same credit rating who issue bonds at around the same time with consecutively different time to maturity.

Possible development of this paper is to find spot rates using real historical data or other methods like Monte Carlo or bootstrapping. Also, this research may produce better results if the volatility are not constant, this could be achieved by using more assumptions of volatility or use other models more suitable. When using this model, it is also advisable to take more time to gather data, since sometimes even when the issuers have the same credit rating, if the coupon rates are too far apart between each time to maturity, the one-year-rate could not be non-negative.

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