

# The Odd Harmonious Labeling of Layered Graphs

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## ABSTRACT

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Graphs that have the properties of odd harmonious labeling are odd harmonious graphs. The research objective of this paper is to obtain odd harmonious labeling on layered graph  $C(x,y)$  and layered graph  $D(x,y)$ . The research used in this paper is a qualitative method. The research flow consists of data collection, processing, and analysis. The data collection stage consists of constructing the definition of the new class graph, the data processing stage consists of constructing the vertex labeling and edge labeling, and the data analysis stage consists of constructing the theorem and proving it. The research results show that the layered graph  $C(x,y)$  and layered graph  $D(x,y)$  fulfill odd harmonious labeling. Such that the layered graph  $C(x,y)$  and layered graph  $D(x,y)$  are odd harmonious graphs. The benefit of this research is to add new properties of odd harmonious graphs. In addition, it does not rule out the possibility that this research can be developed again both in theory and application.



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## A. INTRODUCTION

Graph labeling has been a highly developed graph theory topic in recent years, in addition to researchers interested in developing the theory, some have also found applications of graph labeling in communication network problems, data security, or cryptography. Graph labeling is basically labeling vertices and edges with specific properties (Gallian, 2019). There are several types of graph labeling, and one type of graph labeling studied by researchers is the odd harmonious labeling. The graph  $G(p,q)$  with  $p = |V(G)|$  and  $q = |E(G)|$  is an odd harmonious graph if it fulfills the injective vertex labeling function  $f: V(G) \rightarrow \{0,1,2,3, \dots, 2q - 1\}$  and the bijective edge labeling function  $f^*: E(G) \rightarrow \{1,3,5,7, \dots, 2q - 1\}$  defined by  $f^*(ab) = f(a) + f(b)$  (Liang & Bai, 2009).

Here are some odd harmonious graph classes that have been found by researchers. Abdel Al has obtained odd harmonious labeling of cyclic snake graphs (Abdel-Aal, 2013). Saputri et al have obtained dumbbell graphs are odd harmonious graphs (Saputri et al., 2013). Jeyanthi and Philo have proved that shadow graphs are cycles graphs with sharing a common vertex and edge are odd harmonious graphs (Jeyanthi & Philo, 2016). Abdel-Aal and Seoud have proved the odd harmonious labeling of splitting graphs (Abdel-Aal & Seoud, 2016). Firmansah have obtained odd harmonious graph classes, namely snake net graphs (Firmansah & Yuwono,

2017a) and amalgamation of double quadrilateral windmill graphs (Firmansah & Syaifuddin, 2018).

Renuka and Balaganesan have proved odd harmonious labeling of complete bipartite graphs (Renuka & Balaganesan, 2018). Seoud and Hafez are introducing strongly odd harmonious graphs (Seoud & Hafez, 2018). Kalaimathi and Balamurugan obtained computation of even odd harmonious labeling (Kalaimathi & Balamurugan, 2019). Jeyanthi and Philo have obtained odd harmonious labeling of pyramid graphs Jeyanthi & Philo (2019), and line and disjoint union of graphs (Philo & Jeyanthi, 2021). In another paper, Jeyanthi et al have proved that super subdivision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019) and grid graphs are odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019).

Febriana and Sugeng have proved squid graphs and double squid graphs are odd harmonious graphs (Febriana & Sugeng, 2020). Govindarajan and Srividya have obtained even cycles graphs and dragons graphs are odd harmonious graphs (Govindarajan & Srividya, 2020). Furthermore, the multiply net snake graphs Firmansah, (2020b), and double triangular snake graphs (Senthil & Ganeshkumar, 2020). Firmansah and Giyarti have obtained an amalgamation of the generalized double quadrilateral windmill graph (Firmansah & Giyarti, 2021).

Zara et al have proved that even odd harmonious labeling of some graphs (Zala et al., 2021). Mumtaz and Silaban have obtained snake graphs with hair (Mumtaz & Silaban, 2021). In another paper, Mumtaz et al proved that matting graphs are odd harmonious graphs (Mumtaz et al., 2021). Sarasvati et al have obtained odd harmonious labeling of  $P_nC_4$  and  $P_nD_2(C_4)$  (Sarasvati et al., 2021). Firmansah has proved that string graphs are odd harmonious graphs (Firmansah, 2022). The relevant research results about odd harmonious graph classes that have been found can be seen in (Jeyanthi & Philo, 2015), (Jeyanthi et al., 2015), (Firmansah & Yuwono, 2017b), (Firmansah, 2017), (Sugeng et al., 2019), (Firmansah, 2020a) and (Pujiwati et al., 2021).

In previous studies Firmansah and Tasari have proven that edge amalgamation from two double quadrilateral graphs Firmansah & Tasari (2020), this research is a development for  $n$  graphs double quadrilateral graphs, namely the layered graphs  $D(x, y)$ . In addition, the author constructs new graph classes, namely the layered graphs  $C(x, y)$  and furthermore, the author has proved that the layered graph  $C(x, y)$  and layered graph  $D(x, y)$  satisfy the properties of odd harmonious labeling such that they are a new family of odd harmonious graphs. It is possible that this result can also be used to solve graph labeling problems, especially odd harmonious graph labeling.

## B. METHODS

The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. After the definition of the graph class is formed, it is continued with the vertex labeling construction and edge labeling construction. Furthermore, the construction of theorem and its proof are formed. The research method is as follows.

Based on Figure 1, the stages of the research method are as follows. The data collection stage consists of finding as much information as possible related to graph labeling, especially

odd harmonious graphs. The data processing stage consists of constructing a new graph definition, point labeling construction, and edge labeling construction. The data analysis phase consists of constructing theorems about odd harmonious graphs and mathematical proofs, as shown in Figure 1.

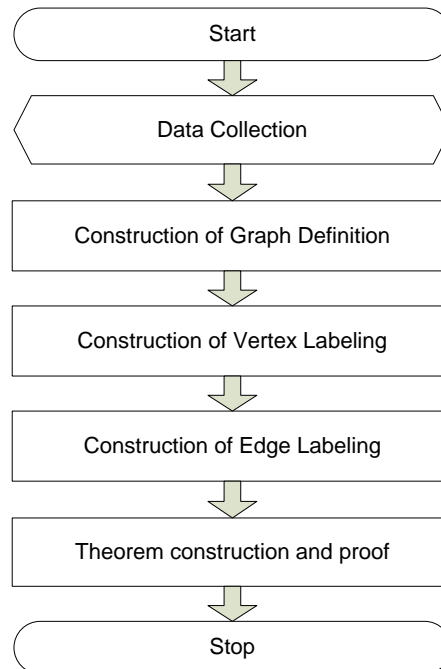


Figure 1. Flowchart research methodology

### C. RESULT AND DISCUSSION

#### 1. Data Collection

The data collection stage consists of finding odd harmonious graphs.

##### Definition 1.

The graph  $G(p, q)$  with  $p = |V(G)|$  and  $q = |E(G)|$  is an odd harmonious graph if it fulfills the injective vertex labeling function  $f: V(G) \rightarrow \{0, 1, 2, 3, \dots, 2q - 1\}$  and the bijective edge labeling function  $f^*: E(G) \rightarrow \{1, 3, 5, 7, \dots, 2q - 1\}$  defined by  $f^*(ab) = f(a) + f(b)$  (Liang & Bai, 2009).

#### 2. Construction of Graph Definition

The following definition is given for a layered graph  $C(x, y)$

##### Definition 2.

Layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is a graph with  $V(C(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$  and  $E(C(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$ .

In such a way that it is obtained  $p = |V(C(x, y))| = 3xy + x$  and  $q = |E(C(x, y))| = 4xy + x - 1$ . The following is given the construction of the layered graph  $C(x, y)$ .

The following definition is given for a layered graph  $D(x, y)$

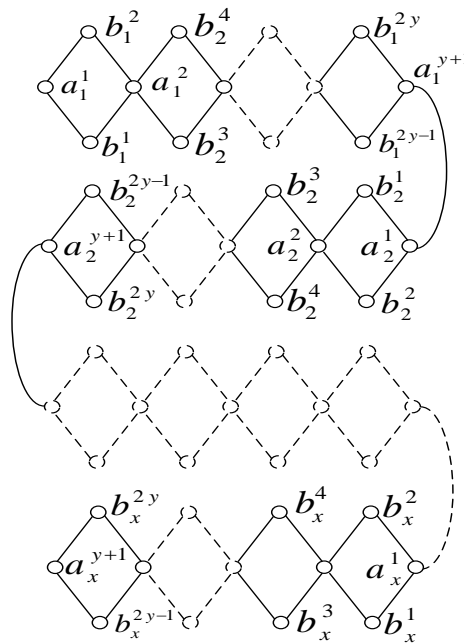
**Definition 3.**

Layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is a graph with

$$V(D(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y + 1\} \cup \{c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\}$$

$$\text{and } E(D(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{c_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} c_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{c_{i-1}^{y+1} b_i^1 | 2 \leq i \leq x\}.$$

In such a way that it is obtained  $p = |V(D(x, y))| = 4xy + 2x$  and  $q = |E(D(x, y))| = 6xy + 2x - 1$ , as shown in Figure 2 and Figure 3.



**Figure 2.** Construction of a layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$ .  
The following is given the construction of the layered graph  $D(x, y)$ .

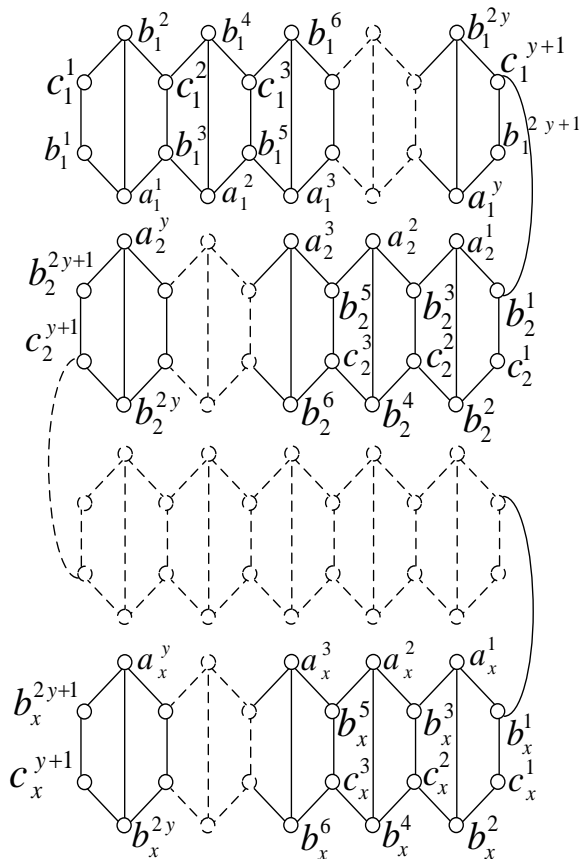


Figure 3. Construction of a layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$ .

### 3. Construction of Vertex Labeling

Define vertex labeling function of a layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$  as follows

$$f(a_i^j) = (4y + 1)i + 4j - 4y - 5, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{1}$$

$$f(b_i^j) = (4y + 1)i + 2j - 4y - 2, 1 \leq i \leq x, 1 \leq j \leq 2y \tag{2}$$

Define vertex labeling function of a layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$  as follows

$$f(a_i^j) = (8y + 2)i + 2j - 8y - 4, 1 \leq i \leq x, 1 \leq j \leq y \tag{3}$$

$$f(b_i^j) = (4y + 2)i + 2j - 4y - 3, 1 \leq i \leq x, 1 \leq j \leq 2y + 1 \tag{4}$$

$$f(c_i^j) = (8y + 2)i + 2j - 2y - 4, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{5}$$

### 4. Construction of Edge Labeling

Next, define edge labeling function of a layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$  as follows:

$$f^*(a_i^j b_i^{2j-1}) = (8y + 2)i + 8j - 8y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{6}$$

$$f^*(a_i^j b_i^{2j}) = (8y + 2)i + 8j - 8y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{7}$$

$$f^*(b_i^{2j-1} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{8}$$

$$f^*(b_i^{2j} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 3, 1 \leq i \leq x, 1 \leq j \leq y \tag{9}$$

$$f^*(a_{i-1}^{y+1} a_i^1) = (8y + 2)i - 8y - 3, 2 \leq i \leq x \tag{10}$$

Define edge labeling function of a layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$  as follows:

$$f(a_i^j b_i^{2j-1}) = (12y + 4)i + 6j - 12y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{11}$$

$$f(a_i^j b_i^{2j}) = (12y + 4)i + 6j - 12y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{12}$$

$$f(a_i^j b_i^{2j+1}) = (12y + 4)i + 6j - 12y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{13}$$

$$f(b_i^{2j-1} c_i^j) = (12y + 4)i + 6j - 6y - 9, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{14}$$

$$f(c_i^j b_i^{2j}) = (12y + 4)i + 6j - 6y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{15}$$

$$f(b_i^{2j} c_i^{j+1}) = (12y + 4)i + 6j - 6y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{16}$$

$$f(c_{i-1}^{y+1} b_i^1) = (12y + 4)i - 12y - 5, 2 \leq i \leq x \tag{17}$$

### 5. Theorem Construction and proof

#### Theorem 4.

Layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is an odd harmonious graph.

#### Proof.

Based on (1) and (2), a different label is obtained at each vertex and  $V(C(x, y)) \subseteq \{0, 1, 2, 3, \dots, 8xy + 2x - 3\}$  so the function  $f$  is injective. Based on (6), (7), (8), (9) and (10) a different label is obtained at each edge and  $E(C(x, y)) = \{1, 3, 5, 7, \dots, 8xy + 2x - 3\}$  so the function  $f^*$  is bijective. Consequently the layered graph  $C(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is an odd harmonious graph ■

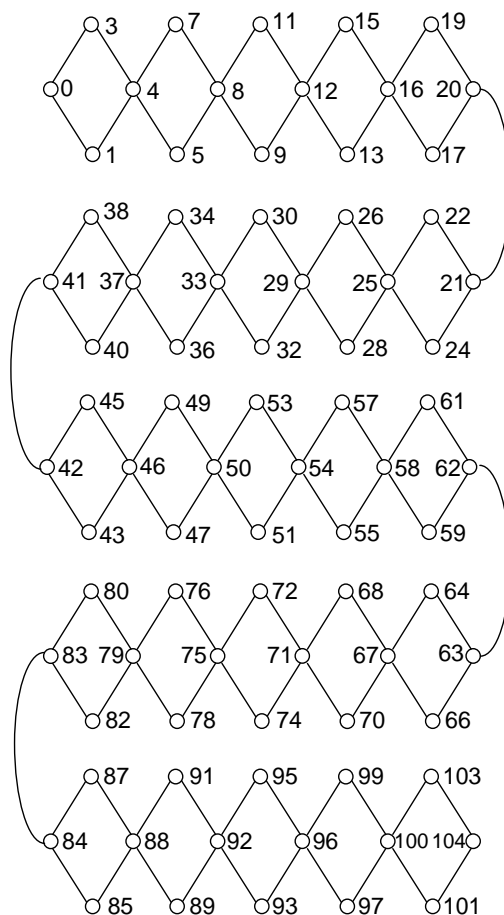
#### Theorem 5.

Layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is an odd harmonious graph.

#### Proof.

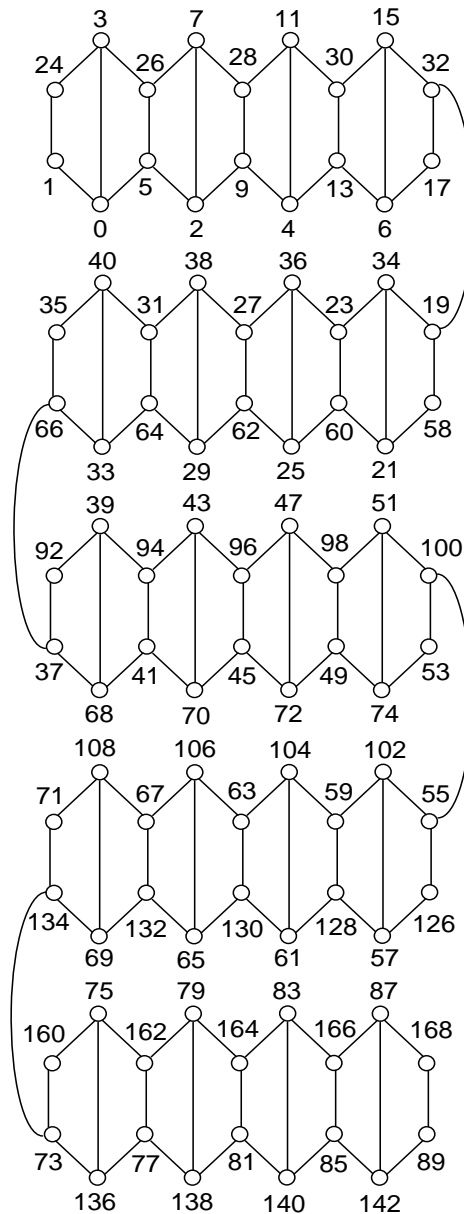
Based on (3), (4) and (5) a different label is obtained at each vertex and  $V(D(x, y)) \subseteq \{0, 1, 2, 3, \dots, 12xy + 4x - 3\}$  so the function  $f$  is injective. Based on (11), (12), (13), (14), (15), (16) and (17) a different label is obtained at each edge and  $E(D(x, y)) = \{1, 3, 5, 7, \dots, 12xy + 4x - 3\}$  so the function  $f^*$  is bijective. Consequently the layered graph  $D(x, y)$  with  $x \geq 1$  and  $y \geq 1$  is an odd harmonious graph ■

Here is the odd harmonious graph of the layered graph  $C(5, 5)$ . Based Figure 4, a different label is obtained at each vertex and  $V(C(5, 5)) = \{0, 1, 3, \dots, 104\} \subseteq \{0, 1, 2, 3, \dots, 207\}$  so the function  $f$  is injective. Based Figure 4, a different label is obtained at each edge and  $E(C(5, 5)) = \{1, 3, 5, 7, \dots, 207\}$  so the function  $f^*$  is bijective. Consequently the layered graph  $C(5, 5)$  is an odd harmonious graph, as shown in Figure 4.



**Figure 4.** The layered graph  $C(5,5)$

Here is the odd harmonious graph of the layered graph  $D(5,4)$ . Based Figure 5, a different label is obtained at each vertex and  $V(D(5,4)) = \{0,1,3, \dots,168\} \subseteq \{0,1,2,3, \dots,257\}$  so the function  $f$  is injective. Based Figure 4, a different label is obtained at each edge and  $E(D(5,4)) = \{1,3,5,7, \dots,257\}$  so the function  $f^*$  is bijective. Consequently the layered graph  $D(5,4)$  is an odd harmonious graph, as shown in Figure 5.



**Figure 5.** The layered graph  $D(5,4)$

Based on Definition 1, the definition of a new graph class is the layered graphs  $C(x, y)$ . Furthermore, based on Theorem 3, it is obtained that the layered graphs  $C(x, y)$  satisfies the odd harmonious labeling function so are odd harmonious graphs. On the other hand, based on Definition 2, a new class definition is obtained, namely the layered graphs  $D(x, y)$ . The layered graphs  $D(x, y)$  is a development of the previous graph found by (Firmansah & Tasari, 2020). In line with this result by Theorem 4, it has been proven that the layered graphs  $C(x, y)$  are odd harmonious graphs. This result shows that a new class of graphs has been discovered which is a family of odd harmonious graphs.



## D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs  $C(x, y)$  in Definition 2 and the layered graphs  $D(x, y)$  in Definition 3. Furthermore, it has been proven that layered graphs  $C(x, y)$  in Theorem 4 and the layered graphs  $D(x, y)$  in Theorem 5 fulfill odd harmonious labeling so that they are odd harmonious graphs. Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

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