

Problem-Based Tasks in Mathematics Learning: Opportunities and Challenges for Teachers

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ABSTRACT

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Mathematics education and teaching should not only include routine tasks; but tasks that need practice and memorization are mathematically more prosperous and more difficult because they promote problem-solving, investigation, and the discovery of the actual value of mathematical assertions. Therefore, the teacher must be able to select, create, and modify assignments under the established goals. This study attempts to examine opportunities and challenges that teachers encounter while adopting problem-based tasks. The subjects involved in this study were three secondary school mathematics teachers that are acceptable, in East Java, Indonesia. The teachers involved have more than five years of teaching experience and have good academic qualifications. The data collection consisted of observing three classes of secondary school during the implementation of a problem-based task related to the topic of rectangles and algebra. The study begins with the introduction and giving of several examples of problem-based tasks in secondary school mathematics designed to foster advanced learners' critical thinking and creativity. The findings show that teachers gain new knowledge about problem-based assignments and pedagogical insights on how they explain them to students. The results also provide an overview of the challenges for teachers to design and implement problem-based tasks.



A. INTRODUCTION

Since many mathematics teaching and learning activities rely on completing math tasks, math tasks are crucial to the learning process (NCTM, 2000; Krauss et al., 2008; Shimizu et al., 2010; Boesen et al., 2014). Mathematical tasks significantly impact students' learning chances (Boston & Smith, 2009). They significantly impact what students need to do to solve problems or participate in mathematics learning (Stein & Lane, 1996). Henningsen & Stein (1997) suggest that the task's design may also affect and structure students' thought processes. The relationship between mathematics assignments and students' opportunities to learn can be seen in Figure 1.



Figure 1. Relationship between mathematics tasks and students' opportunities to learn

The competence of the instructor or educator in creating the assignment will significantly impact the nature of the duties assigned to students. According to statistics, many educators create repetitive exercises while creating problem-based tasks (Yeo, 2017; D. Clarke & Roche, 2018). This occurs because most teachers lack a strong foundation to properly develop problem-based assignments, and even the description of such tasks is not well understood (Merritt et al., 2017). Different definitions of problem-based tasks are shown in various research. As an illustration, research by (Tarhan & Acar 2007; Tseng et al., 2008). According to their research, problem-based assignments were more structured, and the teacher provided more support to keep the investigative process moving toward beneficial learning. Problem-based tasks are assignments given to students with defined guidelines, which establish what students already know, what information they need to know, and the best approach to obtain this information. This definition comes from another researcher, Drake & Long (2009).

Since researchers' definitions are imitated and embraced by educators, the inconsistency of these definitions results in ending mistakes. Additionally, teachers employ assignments from textbooks that they use as resources in their classrooms, according to Casa et al. (2019), which found that the majority of teachers create assignments on the spot by drawing inspiration from some but not all current math textbooks. It offers a good variety of work (Glasnovic Gracin, 2018). According to research by Zanten & Heuvel-Panhuizen (2018), available school mathematics books do not provide problems that provide opportunities for students to learn. Despite being used as the primary reference in the classroom, school mathematics books do not significantly impact learning opportunities for students.

Suryanti et al. (2022) discovered similar findings on the subpar teacher's ability to provide tasks that can give students significant learning opportunities. According to his research,

teachers with teaching backgrounds ranging from 3 to 15 years who have participated in professional development programs are still struggling to create engaging assignments. Most professors create routine duties when creating assignments. Therefore, it is crucial to establish a model for problem-based tasks used by both mathematics educators and other academics as a starting point when creating problem-based tasks, as will be discussed in this work. So the questions to be answered in this study are as follows:

- 1. What are some examples of problem-based tasks in geometry and algebra that can provide a meaningful learning experience for students?
- 2. What kinds of pedagogical activities can produce meaningful learning experiences for students in the context of problem-based tasks?
- 3. What challenges do teachers have in carrying out problem-based tasks?

1. Tasks in mathematics and the development of mathematical knowledge

Knowing clearly how and why math exercises are used in the classroom can help us determine the characteristics that make for effective homework. It does not, however, immediately endow us with the capacity to create worthwhile work. A blend of theory and practice, planned and actual, assignments and students, is necessary for good assignment design. A mathematical task's design is a recursive process that includes the development of a brand-new task and the modification (or improvement) of an existing task (Liljedahl et al., 2007). According to Sierpinska (2003), one of the most significant duties of mathematics education is the design, analysis, and empirical testing of mathematical tasks, whether for research or teaching objectives.

The tasks students are given significantly impact what they learn (Stein & Smith, 1998). The activities given to the students in the classroom provide a starting point for their mathematical inquiry Doyle (1988), and the efficiency with which they complete the assignments indicates their cognitive ability. Its nature significantly impacts the kind of work students complete in math class; they need to be varied in nature and context, elicit multiple representations, employ various resources, and promote debate. Discussions centered on mathematical activities requiring flexible thinking, reasoning, and problem-solving are crucial tools for fostering conceptual comprehension of mathematics (Smith et al., 2009). These discussions give students the opportunity to communicate their thoughts, make their understanding clear, make compelling arguments, acquire the language necessary to convey mathematical concepts and build the ability to consider different viewpoints (NCTM, 2000).

Discussions on higher-level assignments offer valuable opportunities for students to learn and foster creativity in problem-solving, but they also pose difficulties for teachers who must figure out how to structure discussions that draw from various responses. Teachers must choose which components of the assignment to emphasize, how to structure student work, what questions to provide to challenge students with varying ability levels, and how to assist students without controlling their thought processes and removing obstacles (NCTM, 2000). Reduced task cognition needs can arise from providing students with too much, too little, or too much direction (Stein & Smith, 1998). While most of the challenges can be provided by the teacher through problem-based tasks Suryanti et al. (2022), where students do not just solve problems but also conduct investigations, determine the veracity of a mathematical statement, or compare solutions, challenge math activities are not only tricky tasks or have a higher level of mathematicization (Swan, 2011).

In order to better prepare students for future learning, rectangular tasks are a particular sort that provides rich and varied connections with all arithmetic topics (NCTM, 2000). Researchers Yuniato et al. (2021) found that teachers frequently have misconceptions regarding rectangles. We will therefore focus mainly on this subject. The algebra assignment comes next. In order to obtain generalizable formulations and support the approach to algebraic thinking, we develop relevant didactic experiences. According to Freudenthal (1977), algebra is the study of equations, the capacity to explain how to solve problems, and the mathematical links that underpin this process. This description is still valid today since it encompasses both the relational thinking that drives algebraic reasoning and the symbolic features of algebraic activity (Freudenthal, 1977). According to research, algebra is seen as a challenging subject in teaching and learning mathematics in secondary schools (Prendergast & Treacy, 2018). As a result, we are particularly interested in quadrilaterals and algebra.

2. Problem-based Tasks

Although they are a component of problem-based learning, problem-based assignments have a different emphasis on giving students learning experiences. Not every learning challenge may be a problem-based task, especially those associated with the problem-based assignment (van Barneveld & Strobel, 2009; Bosica et al., 2021). One mathematical instruction method that seeks to do more than merely solving the mathematical problem is the problem-based task (Uden, 2003; Takahashi, 2008). These instructions can take the form of rebuilding similar difficulties or creating representations of problems in which the context of the problem is relevant and related to the student's life. The conceptual structure of the math activity in this model highlights the value of being involved in issues that affect students today and could affect how well they can handle issues in the future. The activities of math teachers who have participated in professional development programs are used to generate problem-based assignments for this study. The courses are designed to give math students practice in handling challenging, contextual, and real-world problems (Hmelo-Silver, 2004).

There have been various studies on related subjects that have produced both favorable and unfavorable findings. The encouraging findings suggest that assignments positively impact students' autonomy, motivation, creativity, and learning outcomes (Nicol & Krykorka, 2016; B. Clarke et al., 2009; Wijnen et al., 2017). The disadvantages of problem-based tasks are criticized despite the implementation of problem-based tasks (van Barneveld & Strobel, 2009; Bosica et al., 2021). We draw attention to the lack of robust theoretical frameworks on a problem-based assignment in the available studies, which might have affected students' processes and learning outcomes. These gaps show that there are still a lot of problem-based task research holes that previous academics and educators have not looked into. Therefore, research on problem-based learning models at the primary and secondary education levels has been significantly underrepresented in mathematical problem-based tasks (Clarke et al., 2009; Merritt et al., 2017). The success of task implementation, failures, and difficulties associated with problem-based tasks have not been extensively investigated, even though the research results on PBL

and task-based learning have been generally acknowledged. This research focuses on how math teachers perceive and use problem-based tasks within teacher professional development.

Mathematical tasks and mathematical problems are two critical elements of problem-based task frameworks. Delisle (1997) introduced the first mathematical problem; he clearly explained problem creation and the teacher's role in problem-based learning. This study adopts the Delisle (1997) framework for mathematics problems, which states that: (a) problem-based tasks should take students' prior experiences into account; (b) problems should be relevant to students' daily lives; (c) problems should be poorly structured; and (d) instructions or questions should encourage students' engagement.

The discussion of mathematical tasks, which become a key component of mathematics teaching and learning, is provided in this section. The task here refers to what the students are expected to complete. The task activity will influence the students' learning results, as well as their participation and interaction (Sullivan et al., 2012). Another determining factor in boosting students' learning engagement is the type of task the instructor selects or creates for their instructional activities (Grevholm et al., 2005). Swan (2011) has created five distinct categories of mathematical tasks, including classifying mathematical objects (requiring students to design or apply classification), interpreting multiple representations (requiring students to draw relationships between representations), evaluating mathematical statements (requiring students to determine the veracity of statements), generating problems for others to solve (requiring students to generate problems for their class), and analytical reasoning and solutions (students identify errors and compare solutions). The problem-based task approach was applied to this study using the following standards: issues that are (1) poorly structured; (2) relevant to students' real-world situations and curricula; and (3) task instructions that meet one or more of the five requirements for arithmetic assignments (designing classifications, interpreting multiple representations, evaluating the validity of mathematical statements, creating problems, or identifying errors and comparing solutions).

B. METHODS

We started this research by providing examples of problem-based tasks in the domains of geometry and algebra. The author made observations of three secondary school teachers who were involved in solving problem-based tasks before they were involved in discussion activities. Then the next observation is done after conducting discussion activities and implementing them in class. The instrument used in this study consists of class observation sheets, teacher responses to problem-based tasks before implementation, and teacher responses after implementation.

1. Participants

Participants in the study were three mathematics teachers at a secondary school in Gresik, East Java, Indonesia. These three teachers already have professional teacher certificates, and came from various schools, 2 teachers from private schools, and one teacher from a public school. In detail from participant data as shown in Table 1.

| Tuble II anticipante bata | | | |
|----------------------------------|-------------|----------------------|--------------------------|
| No. | Participant | Teaching experiences | Qualification |
| 1 | WA | More than 5 years | S1 Mathematics Education |
| 2 | NW | More than 20 years | S2 Mathematics Education |
| 3 | ОМ | Less than 3 years | S1 Mathematics Education |
| | | | |

Table 1. Participant Data

C. RESULTS AND DISCUSSION

1. Examples Of Problem-Based Tasks For The Classroom

According to the constructivist viewpoint, problem-based exercises in various situations help students develop their mathematics knowledge and abilities. Teachers in this situation should provide students with a range of exercises that allow them to acquire mathematical content and emphasize and develop mathematical processes, including guessing, researching, and contributing to more effective mathematics learning. However, a good task must call for the mobilization, fusion, and application of various expertise. According to NCTM (2000), assignments are practical when they are related to or act as a primary for fundamental mathematical concepts that present students with an intellectual challenge and permit a variety of methods.

We give two examples of assignments from the lesson design that cover two distinct subjects, namely algebra and geometry for secondary students. Algebraic includes relations and number patterns, ratios and proportions, as well as non-formal algebra in the form of picture symbols and formal algebra in the form of letter symbols that represent specific numbers. While this is going on, the sub-area of measuring geometric and non-geometric quantities is where the field of Measurement studies addresses quantities measurement, how to measure specific quantities, and demonstrates principles or theorems linked to specific quantities.

These tasks are designed to foster students' capacity for higher-level thinking and reasoning. Specifically, doing Algebraic tasks by students is authentic evidence that students are able to use the properties of operations (commutative, associative, and distributive) to produce equivalent algebraic forms, through complex activities, including students narrating the steps for a given solution, then determining correctness of simplification steps and analyze the causes of errors or inconsistencies. Students are also asked to use critical thinking skills, be creative in their analysis of various kinds of shapes, and examine proportional or area comparisons between planes with the same circumference when completing the geometry project. Students illustrate issues to represent them, enhancing their literacy and numeracy. We briefly go through various methods for task investigation.

a. Task 1. Algebraic simplification

The tasks in the algebra area often result in simple or routine exercises. We offer problem-based tasks for algebraic subjects, like Figure 2.

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| Problem: | | | | |
|--|--|--|--|--|
| Almira simplifies algebra with the following steps | | | | |
| $a^2 - a^2 = (a + a) (a - a)$ i) | | | | |
| a(a - a) = (a + a)(a - a)ii) | | | | |
| a (a - a) = (a + a) (a - a)iii) | | | | |
| a = a + aiv) | | | | |
| $a = 2a \dots v$ | | | | |
| a = 2avi) | | | | |
| 1 = 2vii) | | | | |
| Tasks: | | | | |

- a) Describe the steps Almira took to simplify.
- b) Is this step of simplification, in your opinion, valid?
- c) What, in your opinion, explains for the inconsistent final simplification outcomes of Almira?

Figure 2. Algebraic Simplification Tasks

b. Task 2. Rectangle

We claim that activities on rectangular topics that have been developed by teachers or found in textbooks so far tend to be straightforward tasks. Here, we offer a problembased task that focuses on quadrilaterals, as shown in Figure 3.

Problem. Rectangle ABCD has a length of 15 cm and a width of 8 cm. Tasks: a) Draw a rectangle or any other rectangular shape whose perimeter is the same as the rectangle. b) Verify the size of the quadrilateral you created. c) What do you obtain when you divide the area of the quadrilateral you created by the area of the rectangle ABCD?

Figure 3. Rectangle Tasks

These tasks, which range from calculations to creating representations, provide students with a range of learning possibilities. We anticipate that creativity will show through this work in a variety of original ways that the different rectangles are made. The goal should be for students to create as many rectangles as they can and then suggest a square with the largest area.

Students are expected to use mathematical symbols to express the intricate stages needed in completing their mathematical assignments, to provide sentence explanations, and to support their arguments with a variety of representations. We anticipate that students will identify holes in the evidence and initiate new justifications as they observe and describe the actions. According to Swan (2011), one of the objectives in math tasks is for students to demonstrate the accuracy of a claim through argumentation and the use of various representations.

2. Teachers' Responses to The Tasks

This section reports on the types of responses to two types of problem-based tasks, the rectangular task and the algebraic simplification task.

a. Response to rectangular tasks

I think students will be challenged with this assignment. So far, it has never occurred to me to do a task like this. From very simple problems in rectangles, assignments can be made that really make students think, and even train their creativity (WA & OM). When I think about the task in part a, how many quadrilaterals will be produced by

students, but I doubt whether students are able to (NW).

b. Response to algebraic simplification tasks

When looking at this assignment, first a contradiction will appear in the students' minds, because $1 \neq 2$, so students will be challenged to find out why the simplification result is in 1=2. (NW)

Students who have good mathematical abilities will believe that there is an error in the simplification step because one cannot be equal to two. (WA)

3. The Challenge of implementing tasks

a. The results of the implementation of problem-based tasks

According to observations, there are a number of difficulties in implementing problembased tasks, including the question of whether or not students would be able to complete them. Teachers must therefore create the necessary mechanisms to guide learners toward solutions. Out of 37 students in one of the WA classes, one person was unable to complete a rectangle task. Here are some samples of students' work from Figure 4.

| Persegi panlang ABCO memiliki ukuran panlang 15 cm dan lebar b cm. • Keliling persegi panlang • Luas persegi panlang K: 2 (p + L) L: p × L : 2 (15 + 8) • 15 × 8 | (C.) Perbandingan luas bangun datar dengan luas persegi panlang • Luas persegi panlang L : p x L L : p x L · L : p x L |
|---|--|
| : 2 (23) : 120 // | 15 X 5 7 X (20) X 6 |
| : 46 . | = 120 = 15 x 6 = 90 |
| (a) Tradosium siku -siku | Perbandingan Luas kedua bangun |
| p 12 c · Keliling trapesium | = 120 : 90 |
| AB + BC + CO + DA | = 4 : 3 , Jadi perbandingannya adalah 4 : 3, |
| 6 10 : 18 + 10 + 12 + 6 | |
| i 46// A 18 0 | (d.) bangun datar yang memiliki luas maksimum adalah persegi pantang dengan 1. Juas, 120 cm/ |

Figure 4. The results of students' rectangle tasks (AMA)

Because this is a public school with strict admission requirements for students, the students in the WA class are generally capable. However, based on observations of these 37 students, it appears that none of them drew the different quadrilaterals as instructed in the task; instead, they all drew a single flat figure whose perimeter matched the ABCD rectangle. These results indicate a variety of options. First of all, problem-based tasks with essential tasks are unfamiliar to learners. Second, because the teacher has not been able to create tasks that are problem-based, they frequently offer learners repetitive

tasks. According to (Suryanti et al., 2022), new teachers encounter a variety of complex issues while creating tasks, including those relating to context and optional task instructions. Senior teachers choose and create tasks that are straightforward because they are concerned that their students won't be able to finish them (Jaffe, 2020); (Szczygieł & Pieronkiewicz, 2020).

- b. Teachers' Views on Advantages and Difficulties in Using Problem-based Tasks Teachers were asked to list the "advantages of using this type of task in their teaching" after implementing and piloting the rectangle task. Typical comments include:
 - 1) More hands-on.
 - 2) Some were beneficial for students who struggled with mathematics.
 - 3) Enhances the student's ability to think critically.
 - 4) Facilitates students to draw on a variety of perspectives.
 - 5) Involves advanced students. This method combines both creative and critical thinking.

Teachers were also asked, "What makes teaching this type of task difficult." In the comments below, "support students" refers to those students in the classes in which students of "lower ability" were grouped. Typical responses were the following:

- 1) Some of the tasks were too difficult to support students.
- 2) The students' varying learning needs and abilities.
- 3) Students who are less confident have little idea where to start if left to their own devices rather than being assisted. These tasks can adversely affect their negative feelings about themselves and math.
- 4) Students are more focused on the solution than the method.

The findings from interviews about what makes learning challenging when employing problem-based tasks, one of which is that students are more focused on the outcomes of their work than on the manner in which the strategies for completing the tasks are provided. Students frequently only read the instructions before focusing on the task's design (Warshauer, 2015; Livy et al., 2018; Regier & Savic, 2020).

D. CONCLUSIONS AND SUGGESTIONS

Mathematical challenges are just one aspect of math activities. The effectiveness of this curriculum depends on the teacher's encouragement of the students' mathematical learning. Teachers must overcome the issue of presenting challenging times to students. As they move forward, they must choose the suitable task and offer the possible help. In order to appreciate what students are saying and assist them in working on it in various contexts, teachers must have a deep understanding of the mathematics they teach in addition to the didactic one.

We aim to learn more about the effects of teaching and learning mathematics to secondary school students through these tasks, as well as whether there is a chance for innovative solutions that accelerate the learning process. We heavily focused on classroom engagement, which included questions, oral presentations, written work, and discussion, to assess this goal. We also gave teachers' evaluation and interpretation of their students' mathematical thinking with a lot of attention

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