

A Critical Thinking Profile of Mathematics Education Students in Solving Ill-Structured Problem based on Mathematical Ability

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ABSTRACT

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This study aims to describe the critical thinking profile of mathematics education students in solving ill-structured problems based on mathematical ability. The critical thinking profile in solving the described ill-structured problem consists of several stages, namely (1) Analyze; (2) Browse; (3) Create; (4) Decision-making; (5) Evaluate; and (6) List or with the term A-B-C-D-E plus L based on critical thinking skills, namely (1) Interpretation; (2) Analysis; (3) Conclusion; (4) Evaluation; (5) Explanation; and (6) Self-regulation. The subjects of this study consisted of one student of mathematics education who had the high mathematical ability and one student of mathematics education who had the moderate mathematical ability. They were interviewed based on the results of the Ill-Structured problem-solving. This research used a descriptive qualitative approach. The data obtained were validated, then analyzed with several steps, namely: reduction, data presentation, categorization, interpretation, and inference. The results showed that the mathematics education students carried out critical thinking processes in solving ill-structured problems through six (6) stages, namely: Analyze, Browse, Create, Decision-making, Evaluate, and List or with the term A-B-C-D-E plus L, those which refer to the stages of critical thinking skills, including Interpretation, Analysis, Conclusions, Evaluation, Explanation, and Self-regulation.



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A. INTRODUCTION

The evaluation of 21st-century abilities has been one of the "hottest" subjects in the community of educational measurement for the past ten years (Geisinger. 2016). It is obvious that the emphasis on memory and routine problem-solving in school needs to change, and the initiatives discussed in this issue herald a bold new era in education. Therefore, it is noted that one's success depends on 21st-century skills (Rotherham & Willingham, 2009). The Partnership for 21st Century Skills identifies 21st century skills including: critical thinking, problem solving Hasbi et al. (2019), communication, and collaboration. Critical thinking means being able to address knowledge critically, solve problems and be able to overcome the problems encountered through the process of learning activities as a vehicle for practicing facing more complex problems in life.

Critical thinking is one of the most important skills in the 21st century in addition to other learning skills, such as creative thinking, communication skills, and collaboration skills (Rotherham & Willingham, 2009; Octaria et al., 2022). These learning skills are recognized as an integral part of students preparing for a more complex life and preparing students for the work environment in the 21st century. Critical thinking is one of the skills needed by everyone. Critical thinking is so important to learn for several reasons. Firstly, critical thinking is a necessary skill in any job because when studying any field of science, problem-solving is an asset for one's career. Secondly, critical thinking is very important in the 21st century because this century is an era of information and technology. A person must respond to changes quickly and effectively, so he needs flexible intellectual skills, and the ability to analyze information and integrate various sources of knowledge for problem-solving (Beswick & Fraser, 2019; Hasbi et al., 2019). Thirdly, critical thinking is an important and necessary skill when studying at university (Thomas, 2011). For a student, critical thinking could shape someone to be a good judge of information, to be able to explain the reasons or arguments, and to be able to solve problems that have not yet been resolved. Thus, critical thinking must be developed from the start to be able to overcome problems in the future, both in studies and in dealing with challenging future jobs (Thomas, 2011). However, students' critical thinking skills in mathematics are still not satisfactory (Arigawati & Kusnandi, 2021; Purwati et al., 2022).

Mathematics education students are specifically prepared to become professional teachers, who require the ability to solve mathematical problems (Jaelani et al., 2022). Because after graduation, they must have professional pedagogical competence as teachers. According to (Hendriana et al., 2014), when viewed from the arrangement of its elements, mathematical problems are divided into two, namely structured problems (well-structured), those which have complete elements so that problems can be solved, and unstructured problems (ill-structured), those which have incomplete elements, so they require the definition of certain relevant elements first.

Critical thinking skill can be seen from a person's ability to identify the information obtained and collaborate it with the knowledge the person has. This skill can direct a person to develop ideas in carrying out the problem-solving process, so that the person gets a solution to the problem faced. Critical thinking skills and problem solving are related to each other. In this case, the stages in the critical thinking process of mathematics education students to solve ill-structured problems based on *A-B-C-D-E plus L* mathematical abilities Jaelani et al. (2022), those which are due to the stages of critical thinking, namely: Interpretation, Analysis, Conclusion, Evaluation, Explanation, and Self-Correction (Facione, 2011).

The contribution of this research is to enrich theories related to critical thinking skills and the development of ill-structured solving abilities of mathematics education students. The results of research on the critical thinking profile of mathematics education students in solving ill-structured problems can be used by lecturers and teachers to develop and encourage students' ill-structured problem-solving abilities and skills. Therefore, based on previous conceptions, the purpose of this study is to describe the critical thinking profile of

mathematics education students in solving ill-structured problems based on mathematical ability.

B. METHODS

The right research approach to achieve the research objectives was qualitative research. The following is the flow of selecting research subjects:

1. Subjects are selected from those who have studied and passed the geometry course. To control the subject's level of mathematical ability, the subject was selected among students who had a temporary grade point average (GPA) above 3.50. The subjects chosen were students with high and moderate mathematical abilities based on the results of the Mathematical Ability Test (TKM).
2. One class was selected from the mathematics education study program, which consisted of three (3) classes, namely the one that was randomly selected, in this case, the class C students in semester III with a total of 21 students. The student is then given a Mathematical Ability Test (TKM), which has been validated by experts.
3. Students who obtain scores above 80 to 100 are categorized as having high mathematical abilities (KMT), and students who obtain scores above 60 to 80 are categorized as having moderate mathematical abilities (KMS).
4. Prospective subjects who meet the requirements are re-selected through confirmation with the lecturer in charge of the geometry course regarding the ability of students to communicate verbally properly and clearly
5. Therefore, the subjects of this study consisted of 1 mathematics education student named KN, who had a GPA of 3.88 (three points eight), obtained a the Mathematical Ability Test (TKM) score of 90, and was in the high mathematics ability category, and 1 mathematics education student named HDI, who had a GPA of 3.72 (three point seven two), got a the Mathematical Ability Test (TKM) score of 80, and is in the category of moderate mathematical ability.

The contribution of this research is to enrich theories related to critical thinking skills and the development of ill-structured solving abilities of mathematics education students. The results of research on the critical thinking profile of mathematics education students in solving ill-structured problems can be used by lecturers and teachers to develop and encourage students' ill-structured problem-solving abilities and skills.

The instrument used to collect data was a set of questions developed with the following steps: writing questions and alternative solutions, submitting the set of questions and their solutions to the validator, revising according to the validators' suggestions, rewriting a set of questions before being used as a basis for data collection. The validators consisted of 2 (two) lecturers with expertise in accordance with the material of the instrument being validated. The two validators provided suggestions for improving the editorial questions and the suggestions were used as input for revising the instrument. The instrument used in this study was the result of a revision in accordance with the advice of the validators and an adaptation of the results of device development (Jaelani & Hasbi, 2022). This instrument includes auxiliary instruments consisting of one item, namely the Ill-Structured Problem-Solving Task

(TPM-IS). Therefore, it is very suitable to be used to explore the critical thinking profile of mathematics education students in solving ill-structured problems, as shown in Figure 1.

It is known that $\triangle ABC$ goes through point A which is parallel to \overline{CD} (\overline{CD} is the bisector of angle C). If $\overline{BC} = a, \overline{AC} = b, \overline{AB} = c, \overline{AD} = c_1,$ and $\overline{BD} = c_2,$ proving that $c_1 : c_2 = b : a!$

Figure 1. Ill-Structured Problem

The subjects that have been chosen are mathematics education students with high and medium mathematics categories. The subject is then given an unstructured task or problem (TPM-IS). An interview process was carried out on subjects related to poorly structured tasks and problems that had just been completed. Time triangulation was carried out until consistently ill-structured task- or problem-based data was obtained, namely, the data had relatively the same structure and content. Data analysis using an interactive model begins with the data collection stage and continues through each research stage until completion Miles & Huberman (1994), namely data reduction, data presentation, and drawing conclusions. The following is a flowchart of the stages of data analysis carried out by the researcher, as shown in Figure 2.

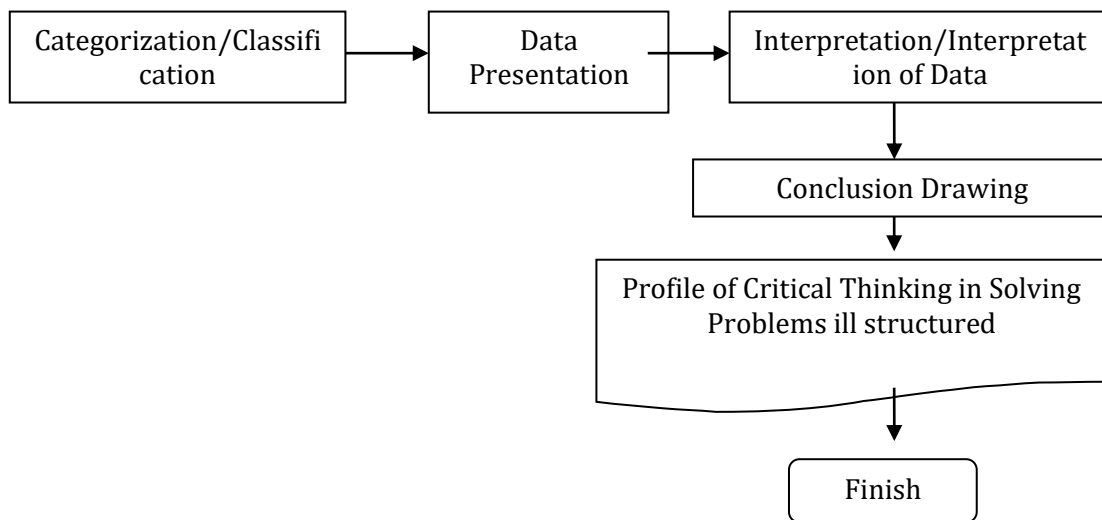


Figure 2. The stages of data analysis

The relationship between the stages of solving ill-structured problems and critical thinking skills in Table 1 is a reference to see the critical thinking skills of mathematics education students in solving ill-structured problems. The components contained in Table 1 are described as follows.

- Step 1 : *Reviewing the problem from an analytical perspective (A1)*
- Step 2 : *Understanding the problem given (A2)*
- Step 3 : *Defining/re-explaining the problem (A3)*
- Step 4 : *Identifying the mathematical content needed to solve the problem (B1)*

- Step 5 : *Gathering the information needed to solve the problem (B2)*
- Step 6 : *Formulating a solution that can satisfy many conditions (C1)*
- Step 7 : *Creating various solutions to the given problem (C2)*
- Step 8 : *Justifying the most appropriate solution to the given problem (D)*
- Step 9 : *Evaluating solutions and reflecting on those solutions (E1)*
- Step 10 : *Identifying an idea, modifying/limiting it, and completing the solution (E2)*
- Step 11 : *Stating the right reasons for the choice of problem-solving procedure (L)*

Table 1. Relationship between Critical Thinking Characteristics and Ill-Structured Problem

Ill-structured troubleshooting indicator	Critical thinking indicator (Facione, 2011)	The relationship between critical thinking and ill-structured problems	Code
<ul style="list-style-type: none"> • Reviewing the problem from an analytical perspective • Understanding the given program/problem • Defining/re-explaining the problems they face/they get with their own words/terms, pictures, and schemas (diversity of problem representation) 	<ul style="list-style-type: none"> • Grouping • Understanding the meaning • Explain the meaning 	Reviewing, understanding, and defining problems with their own words/terms, pictures, and schemas (a variety of problem representations) to express the meaning or significance of various experiences, situations, rules, procedures, or criteria.	<i>A1.i</i>
			<i>A2.i</i>
			<i>A3.i</i>
<ul style="list-style-type: none"> • Identifying the required mathematical content • Gathering the necessary information to solve a given problem 	<ul style="list-style-type: none"> • Testing ideas • Identifying arguments • Identifying reasons and claims 	Identifying and collecting required information regarding the cause issue problem with doing the analytical process, including examining ideas, detecting arguments, and analyzing arguments as a sub-skill of analysis	<i>B1.a</i>
			<i>B2.a</i>
<ul style="list-style-type: none"> • Formulating a solution that satisfies many conditions. • Creating various solutions to the given problem 	<ul style="list-style-type: none"> • Questioning facts • Estimating various alternatives/options • Drawing conclusions based on inductive or deductive reasoning 	Formulating and creating a problem-solving strategy design to concluded. reasoned.	<i>C1.k</i>
			<i>C2.k</i>
<ul style="list-style-type: none"> • Justifying the most appropriate solution to a given problem 	<ul style="list-style-type: none"> • Assessing the credibility of the claim • Assessing the quality of arguments made based on inductive or deductive reasoning 	Implementing a strategy already made to reveal a reason. by detecting a relevant claim	<i>D.e</i>
<ul style="list-style-type: none"> • Evaluating solutions and reflecting/illustrating/thinking about them • Identifying an idea, modifying/limiting it, and 	<ul style="list-style-type: none"> • Declaring results • Justifying procedures • Providing arguments 	Carrying out an evaluation process to assess the credibility of a claim. or information obtained.	<i>E1.p</i>
			<i>E2.p</i>

complementing their own solutions			
<ul style="list-style-type: none"> • Stating the correct reasons for the choice of settlement procedure. • Stating the reason that the answer obtained is the best answer 	<ul style="list-style-type: none"> • Self-monitoring • Self-correction 	Conducting self-correction to confirm the results obtained, which is accompanied by reasons for monitoring cognitive activities (the elements used in these activities), especially by applying analysis and evaluation skills.	<i>L.s</i>

Description: (i) Interpretation; (a) Analysis; (k) Conclusion; (e) Evaluation; (p) Explanation; and (s) Self Correction.

The relationship between the stages of solving ill-structured problems and the stages of critical thinking in Table 1 above uses a combination of capital letters and numbers (which indicate the stages of solving ill-structured problems) which is separated by a period (.) and followed by a lowercase index (which indicates the stages of critical thinking). For example, code (A1.a) is a code used to link the stages of ill-structured problem solving and critical thinking stages. (A1) indicates the stage of reviewing the problem from an analytical perspective, which is separated by a period (.) and followed by an index code (a), indicating the interpretation stage.

C. RESULT AND DISCUSSION

Based on Table 1 and the results of interview analysis, MPMATT was indicated to carry out a critical thinking process in solving ill-structured problems, with the following explanation.

1. MP_{MAT} read the problem then reviewed and mentioned the known information, namely $\triangle ABC$, \overline{CD} as bisector of $\angle C$, a line drawn through point A parallel to the bisector \overline{CD} (A1). In addition, MP_{MAT} interpreted by expressing the opinion that the information in the question might be incomplete with reasons (i).
2. MP_{MAT} mentioned relevant information related to the given problem and presented or related the information through image representation (A2.i).
3. MP_{MAT} again reviewed the problem (A1.i) and understood the problem given (A2.i) by mentioning, writing, and explaining the relationship of the relevant information obtained to define the objectives to be achieved. In this case, the bisector of the triangle was the line dividing the opposite side as the adjacent side (A3.i).
4. MP_{MAT} reviewed the problem again (A1.i) and understood the problem given (A2.i) with the aim of identifying the mathematical content or information needed to solve the problem (B1.a). In this case, MP_{MAT} mentioned one important piece of information related to the problem and explained the information, namely $\triangle BCD$ was congruent with $\triangle BEA$, even MP_{MAT} indirectly mentioned the reason why $\triangle BCD$ was congruent with $\triangle BEA$ based on the angles.

5. MP_{MAT} again understood the problem (**A2.i**) and identified the required mathematical content or information (**B1.a**). It aimed to collect relevant information used as a guideline by MP_{MAT} to solve problems (**B2.a**). In this case, the first information identified by MP_{MAT} was that $\triangle BCD$ was congruent with $\triangle BEA$ based on the angles. The second piece of information that MP_{MAT} identified and mentioned was that $\triangle BCD$ was congruent to $\triangle BEA$ (based on angle-angle-side/side-angle-angle).
6. MP_{MAT} formulated a solution satisfying many conditions (**C1.k**) based on the relevant information collected (**B2.a**) and returned to understanding the problem (**A2.i**). The information was then formulated because they had interconnected each other for a common goal or to find the desired answer. In this case, MP_{MAT} stated that through the similarity of two triangles, it could show the form of the comparison $BD \square AD = BC \square AC$ or what was asked about the problem.
7. MP_{MAT} formulated alternative problem solving (**C2.k**) to obtain reasonable conclusions, namely based on the corresponding angles being congruent to each other in triangle BCD and triangle BEA. Thus, the student could determine proportional corresponding sides by grouping proportional corresponding sides based on pairs of angles that were known to be congruent. However, to reach the stage (**C2.k**), MP_{MAT} again identified the mathematical content or information needed (**B1.a**).
8. MP_{MAT} re-formulated alternative problem solving (**C2.k**) and implemented the strategies that had been made and obtained the results $BD: BC = AD: CE$ (written answer). However, the result of the solution did not match what the student wanted to show, namely $BD: BC = AD: AC$. MP_{MAT} detected the problem (**D.e**) by claiming that CE was equal to AC based on assumptions or conjectures. Solution justification (**D.e**) aimed to find the common thread of a problem, so that the end could be deciphered from the tangled condition. This was shown based on MP_{MAT} conjecture that $CE = AC$. The information revealed by MP_{MAT} was reformulated for problem solving, because the results obtained by MP_{MAT} were related to what to show or rearrange (**C2.k**) for a common goal or to find the desired answer. In this stage (justifying the solution), MP_{MAT} provided alternative answers that were different from what to show but substantially the same.
9. In the solution evaluation stage (**E1.p**), MP_{MAT} again understood the problem (**A2.i**) by mentioning and marking the results obtained then reviewing the results of the work based on the steps and strategies for solving problems. At this stage, MP_{MAT} paid attention to the solution steps based on the information contained in the problem (**A1.i**), namely the corresponding angles of the same size on $\triangle ABC$ and $\triangle ACE$ based on $\angle C_1 = \angle C_2$ (definition of the bisector of the angle). In this case, MP_{MAT} found the relationship of the corresponding angles of equal measure, i.e., $\angle C_1 = \angle C_2 = \angle E$ and showed that the triangle ACE was an isosceles triangle.
10. MP_{MAT} conducted an evaluation process (**E1.p**) when faced with a relatively similar problem, namely MP_{MAT} identified all the information needed to solve the problem. On the other hand, MP_{MAT} assessed the credibility of a claim by completing a written settlement solution and MP_{MAT} identified a problem-solving idea as a follow-up

when faced with a relatively similar problem (**E2.p**). MP_{MAT} mentioned the impression obtained after solving problems in the form of experience in solving problems (**E2.p**).

- MP_{MAT} performed self-correction to confirm the procedure or completion steps, namely through the congruence of two triangles ($\triangle BCD$ and $\triangle BEA$) based on angles on the grounds that if two pairs of congruent corresponding angles were known, then the third angle corresponded congruent (**L.s**). Furthermore, MP_{MAT} revealed that the answer obtained was the best answer accompanied by reasons (**L.s**).

From this explanation, MP_{MAT} was stuck on the given problem, namely the form of an ill-structured problem or a problem had the characteristics of Complexity and Openness. The ill-structured problem given did not contain additional information that $\triangle BCD$ and $\triangle BEA$ were congruent and the second information $\triangle ACE$ was an isosceles triangle. However, MP_{MAT} revealed one piece of information that $\triangle BCD$ and $\triangle BEA$ were congruent based on the information and instructions contained in the problem given. MP_{MAT} only realized that $\triangle ACE$ was an isosceles triangle after looking back at the problem-solving steps and finding the relationship between the large number of angles $\triangle ACE = 180^\circ$ with supplementary angles $m\angle C_1 + m\angle C_2 + m\angle C = 180^\circ$ based on transitive properties. MP_{MAT} only focused on the similarity of two triangles (in this case $\triangle BCD$ and $\triangle BEA$), then MP_{MAT} solved the problem based on the corresponding angles being congruent and determined the corresponding sides were proportional so that the ratio $\overline{BD} : \overline{BC} = \overline{AD} : \overline{CE}$, as shown in Table 2.

Table 2. Data on Subject’s Answer Sheets for Student with High Mathematics Ability (MP_{MAT})

Label	Answer Sheet	Code
LJ1 _{MAT} 01		A1
LJ1 _{MAT} 02		A1.i
LJ1 _{MAT} 03		
LJ1 _{MAT} 04		
LJ1 _{MAT} 05		
LJ1 _{MAT} 06		
LJ1 _{MAT} 07		A2.i
LJ1 _{MAT} 08		B1
LJ1 _{MAT} 09		C1
		D.e
		E1
		A3.i

LJ1_{MAT10}

B1.a

LJ1_{MAT11}

LJ1_{MAT12}

LJ1_{MAT13}

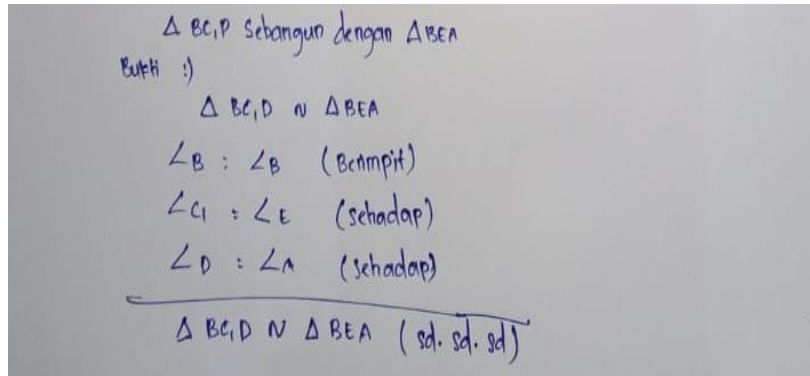
LJ1_{MAT14}

LJ1_{MAT15}

LJ1_{MAT16}

LJ1_{MAT17}

LJ1_{MAT18}



B1.a
B2.a

LJ1_{MAT19}

LJ1_{MAT20}

LJ1_{MAT21}

LJ1_{MAT22}

LJ1_{MAT23}

LJ1_{MAT24}

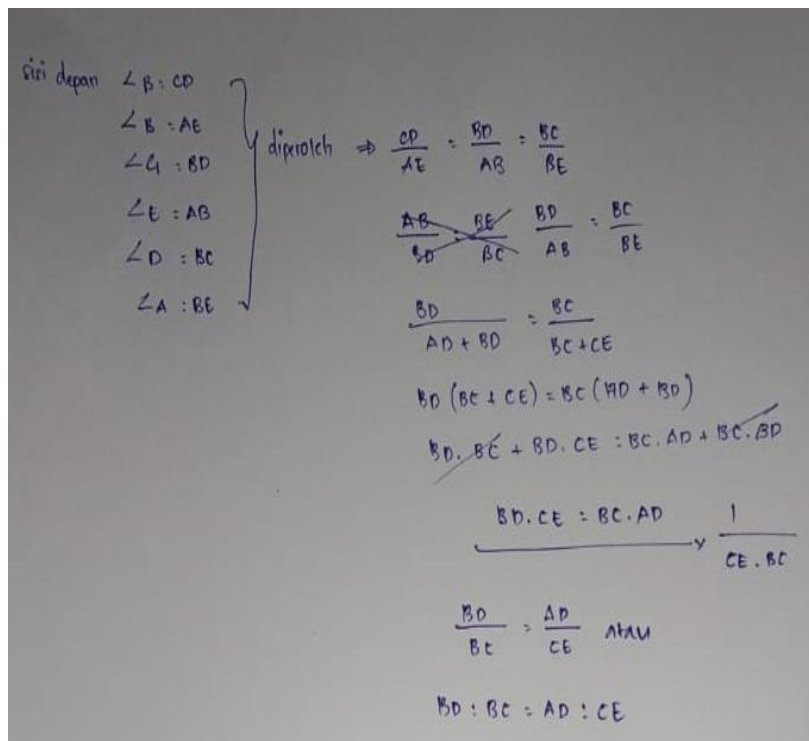
LJ1_{MAT28}

LJ1_{MAT29}

LJ1_{MAT30}

LJ1_{MAT31}

LJ1_{MAT32}



C1.k

C1.k

D.e

Description:

MP_{MAT} : Mathematics Education Student with High Mathematics Ability

$LJ1_{MAT01}$: Letter labels indicate the Subject's Answer Sheet followed by a number label indicating the sequence of completion steps.

Table 3. MP_{MAT} 's Answer Sheet in Solving ill-structured Problem.

Label	Answer Sheet	Code	
$PJ1_{MAT01}$		A1.i	
		A2.i	
		A3.i	
		B1.a	
		B2.a	
		C1.k	
$PJ1_{MAT02}$	<p>Karena CD sejajar TA, sehingga dari $\triangle ABC$ dan $\triangle BTA$ diperoleh:</p> <p>Sudut BCD = Sudut BTA \rightarrow Setudap Sudut BDC = sudut BAT \rightarrow Berimpit setudap Sudut DBC = Sudut ABT \rightarrow Setudap Berimpit atau $\triangle BCD$ sebangun dengan $\triangle BTA$ berdasarkan sudut - sudut dan berlaku perbandingan :</p> $\frac{BD}{AB} = \frac{BT}{BC} = \frac{CD}{AT} \quad \frac{BD}{AB} = \frac{BC}{BT} = \frac{CD}{AT} \quad \text{atau} \quad \frac{c}{c} = \frac{a}{BT} = \frac{CD}{AT} \dots (1)$ <p>karena CD sejajar dengan AT, maka sudut CAT = sudut ATC = sudut ACD = sudut BCD. Sehingga $AC = CT$ atau $b = CT$.</p> $c_2 = \frac{a}{c} = \frac{CD}{AT} \dots (1) \text{ dapat ditulis}$ $\frac{c_2}{c_1 + c_2} = \frac{a}{a + b} = \frac{CD}{AT} \dots (2)$	B1.a	
$PJ1_{MAT03}$		B2.a	
$PJ1_{MAT04}$		C1.k	
$PJ1_{MAT05}$		C2.k	
$PJ1_{MAT06}$		L.s	
$PJ1_{MAT07}$		<p>karena CD sejajar dengan AT, maka sudut CAT = sudut ATC = sudut ACD = sudut BCD. Sehingga $AC = CT$ atau $b = CT$.</p> $c_2 = \frac{a}{c} = \frac{CD}{AT} \dots (1) \text{ dapat ditulis}$ $\frac{c_2}{c_1 + c_2} = \frac{a}{a + b} = \frac{CD}{AT} \dots (2)$	C2.k
$PJ1_{MAT08}$			D.e
$PJ1_{MAT09}$			
$PJ1_{MAT10}$			
$PJ1_{MAT11}$		$\frac{c_2}{c_1 + c_2} = \frac{a}{a + b}$ $c_2 = \frac{a}{\frac{c_1 + c_2}{a + b}}$ $c_2(a + b) = a(c_1 + c_2)$ $ac_2 + bc_2 = ac_1 + ac_2$ $bc_2 = ac_1 \rightarrow \times \frac{1}{c_2 a}$ $\frac{bc_2}{a} = \frac{ac_1}{c_2}$ $\frac{b}{a} = \frac{c_1}{c_2} \text{ atau}$ $b \cdot a = c_1 : c_2$	D.e
$PJ1_{MAT12}$	E1.p		
$PJ1_{MAT13}$	E2.p		
$PJ1_{MAT14}$			
$PJ1_{MAT15}$			
$PJ1_{MAT16}$			
$PJ1_{MAT17}$			

Description:

MP_{MAT} : Mathematics Education Student with Moderate Mathematics Ability

$PJ1_{MAT01}$: Letter labels indicate the Subject's Answer Sheet followed by a number label indicating the sequence of completion steps.

Based on Table 3, $MP_{MAT S}$ was indicated to carry out a critical thinking process in solving ill-structured problem, with the following explanation:

$MP_{MAT S}$ reviewed and mentioned the known information, namely $\triangle ABC$ and through point A, a line was drawn parallel to \overline{CD} (**A1**). In addition, $MP_{MAT S}$ interpreted by expressing the opinion that the bisector of a triangle was a line dividing the angle into two congruent parts (**i**).

1. $MP_{MAT S}$ again reviewed the problem (**A1.i**) with the aim of understanding the problem (**A2.i**), namely by mentioning relevant information related to the problem and presenting the relationship of these information through image representation (**A2.i**).
2. $MP_{MAT S}$ defined the problem (**A3.i**) by reviewing the problem (**A1.i**) and understanding the problem (**A2.i**) to find relevant information relationships, namely $a : b = c_2 : c_1$. As for the information relationship, namely "Comparison of the sides flanking angle C = comparison of the parts of the front side of angle C." (**i**).
3. $MP_{MAT S}$ identified the mathematical content needed to solve the problem (**B1.a**). In this case, $MP_{MAT S}$ identified the problem through the congruence of two triangles.
4. $MP_{MAT S}$ again identified the required mathematical content (**B1.a**) to collect relevant information (**B2.a**). The first information was the similarity of two triangles based on the angle-angle-angle. The second information was the similarity of two triangles based on the angle-side-angle and side-angle-angle.
5. $MP_{MAT S}$ formulated a solution satisfying many conditions (**C1.k**) based on the relevant information collected (**B2.a**). The information was then formulated because they were interconnected for the same purpose. In this case, $MP_{MAT S}$ stated that through congruence, two triangles could show the form of comparison $a \boxtimes b = c_2 : c_1$.
6. $MP_{MAT S}$ identified the required mathematical content (**B1.a**) to formulate alternative solutions to the problem (**C2.k**) and derived a reasonable conclusion, namely the corresponding triangle BCD and triangle BTA were congruent based on the angle-angle, proportional corresponding sides could be determined.
7. $MP_{MAT S}$ formulated alternative problem solving (**C2.k**) and implemented the strategy made, which resulted in $b \boxtimes a = c_1 : c_2$ (written answer) (**D.e**). $MP_{MAT S}$ detected a problem with the claim that "the similarity of two triangles based on the angle-angle-angle and based on the angle-side-angle yields the same ratio, i.e. $b \boxtimes a = c_1 : c_2$ (**D.e**).
8. In the stage of evaluating solutions (**E1.p**), $MP_{MAT S}$ reviewed the steps and strategies for solving problems through stage (**D.e**). At this stage, $MP_{MAT S}$ paid attention to the completion steps until the appropriate results were obtained.
9. $MP_{MAT S}$ conducted an evaluation process (**E1.p**) by assessing the credibility of a claim (**E2.p**) and completing the settlement solution in written form, even $MP_{MAT S}$ had difficulty uncovering and completing the solution. In this case, the DE and DF sides were known to correspond to each other.
10. $MP_{MAT S}$ performed self-correction to confirm the procedure or completion steps, namely through the similarity of two triangles ($\triangle BCD$ and $\triangle BTA$) based on angle-angle-angle by the reason that after knowing the two pairs of corresponding angles were congruent, then the pair of sides that were proportionally congruent could be determined (**L.s**).

From the explanation, it is indicated that $MP_{MAT}S$ carried out a critical thinking process in solving ill-structured problem, even $MP_{MAT}S$ had difficulties in the evaluation and identification stages of ideas. In this case, DE and DF were known to be of equal length. The stages of solving ill-structured problems were not linear stages. This means that someone who is already in the decision-making stage is allowed to return to the previously created step (create) to check the feasibility of strategy or re-analyze the problem situation in depth (analyze & browse). Someone who initially believes in his understanding of the problem and is making a solution strategy may have to re-evaluate his understanding to get a better understanding. Therefore, at each stage of solving ill-structured problem, it is very possible that there is involvement of *A-B-C-D-E plus L* phase, so that in addition to the critical thinking process, mathematics education students in solving ill-structured problems at each stage of problem solving require more detailed information than is generated.

The achievement of ill-structured problem solving indicators for the two subjects was taken through six stages, namely *Analyze, Browse, Create, Decision-making, Evaluates, and List* or in the term *A-B-C-D-E plus L* which refers to the stages of critical thinking skills, namely: *Interpretation, Analysis, Conclusion, Evaluation, Explanation, and Self-regulation* (Facione, 2011). This result is not in line with the findings of previous research, stating that the process of solving ill-structured problem can be carried out in 5 stages called *A-B-C-D-E model (Analyze/Browse/Create/Decision-Making/Evaluate)* (Kim & Cho, 2016). The finding of this research argues that the process of solving the ill-structured problem is divided into 4 stages, namely: *Analyze & Browse, Create, Decision-Making, and Evaluate*, because *Analyze* and *Browse* stages are considered indistinguishable and having many similarities.

This study is in line with the results of a study entitled "*Investigating Elementary Students' Problem Solving and Teacher Scaffolding in Solving Ill-Structured Problem*" (Cho & Kim, 2020). In this case, solving ill-structured problem is conducted through *analyze, browse, create, decision-making, and evaluate* phases. This study provides "*Metacognitive Scaffolding*" to help the subject analyze a number of information in depth by re-identifying information related to the ill-structured problem given (Cho & Kim, 2020). Scaffolding strategy aims to help subjects access a few information in an organized manner and facilitate the solving of ill-structured problem.

Whereas in this study, two main difficulties had been shown in solving ill-structured problem, namely (1) difficulties in the phase of understanding the problem, identifying the problem, and gathering the necessary information from the problem situation. The subjects did not carry out monitoring or did not evaluate the suitability of the final solution chosen. These difficulties occur because the characteristics of the ill-structured problem itself, one or more elements of the problem are not known or vaguely defined so that the problem is not simple; (2) before giving the scaffolding, the subject solved the ill-structured problem without any help, even the purpose of giving each scaffolding was different and generally facilitated the solving of the ill-structured problem. In this study, metacognitive scaffolding helped the subjects to develop solutions for ill-structured problem solving and strategic scaffolding helped subjects identified information and made good use of it to discuss the suitability of the final solution chosen. This is consistent with the finding that the stages of monitoring and justifying solutions are needed to solve ill-structured problem (Jonassen, 1997; Xun & Land,

2004). As reported in the study, scaffolding increases effectiveness in solving ill-structured problem qualitatively (Araiku et al., 2019; Chen & Bradshaw, 2007; Davis, 2000; Hong & Kim, 2016; Jonassen, 1997; Land & Greene, 2000; Lee et al., 2014; Xun & Land, 2004). In fact, the scaffolding is provided due to the subject's circumstances, which means that with the help of the scaffolding the subject can do what he or she cannot do alone. The results of the study indicate that "*Metacognitive Scaffolding*" assisted the subject in facilitating the solving of ill-structured problem, to explore the problem situation in depth, leading to efforts to find the best solution.

D. CONCLUSION AND SUGGESTIONS

Based on the results of the analysis and discussion, it can be concluded that the subjects from mathematics education carried out critical thinking processes in solving ill-structured problems through six (6) stages, namely: analyze, browse, create, make decisions, evaluate, and list, or, with the term A-B-C-D-E plus L, those which refer to the stages of critical thinking skills, including interpretation, analysis, conclusions, evaluation, explanation, and self-regulation. The following is a student's profile in mathematics education, including: (1) By defining the facts required and being able to explain the problem, subjects may comprehend the issue; (2) Subjects were able to group pertinent material in their writing or mention it to address the situation at hand; (3) Subjects were able to design and develop alternate problem-solving techniques in writing, along with justifications; (4) The subjects were able to provide evidence for their solutions by connecting the data they had gathered to identify a problem's common thread, which allowed them to characterize the problem's outcome from its muddled beginning; and (5) The subjects were able to assess the procedures involved in solving the issue and come up with solutions that matched their desired outcomes. Future work needs to explore the process of critical thinking skills and ill-structured problems as problem-solving strategies, for observers of education in general and mathematics education.

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