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Modelling Dependencies of Stock Indices During Covid-19 Pandemic by Extreme-Value Copula

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ABSTRACT

Quantifying dependence among variables is the core of all modelling efforts in financial models. In the recent years, copula was introduced to model the dependence structure among financial assets return, and its application developed fast. A large number of studies on copula have been performed, but the study of multivariate extremes related with copulas was quite behind in comparison with the research on copulas. The COVID-19 pandemic is an extreme event that has caused the collapse of various economic activities which resulted in the decline of stock prices. The modelling of extreme events is therefore important to mitigate huge financial losses. Extreme-value copula can be suitable to quantify dependencies among assets under an extreme event. In this paper, we study the modelling of extreme value dependence using extreme value copulas on finance data. This model was applied in the portfolio of the IDX Composite Index (IHSG), *Straits Times Index* (STI) and Kuala Lumpur *Stock Exchange* (KLSE). Each individual asset return is modelled by the ARMA-GARCH and the joint distribution is modelled using extreme value copulas. This empirical study showed that Gumbel copula is the most appropriate extreme value copulas for the three indices. The results of this study are expected to be used as a basis for investors in the formation of a portfolio consisting of 2 financial assets and a portfolio consisting of 3 financial assets.

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A. INTRODUCTION

In recent years, financial risk management becomes a highly discussed topic in finance. The goal of financial risk management is to measure and manage risk in various activities in the sector of security and insurance. An example of risk management is the formation of a portfolio, and the measure of risk based on the distribution of returns is Value-at-Risk (VaR). An investor on a financial asset is expecting to get a return. In the estimation of portfolio risk there are two key problems, namely (1) modelling the individual distribution of return of each asset; and (2) modelling the joint distribution of return of multiple financial assets forming the portfolio, or modelling the dependence structure between the financial assets' returns in the portfolio. The model of the dependence structure between the financial assets' returns provides an important consideration for the investor to form his/her portfolio. The formation of portfolio is part of an effort to minimize risk (Bodie et al., 2014). Modelling dependence is the main element in almost all financial applications such as portfolio management, risk evaluation, pricing, and hedging (Mashal & Zeevi, 2005).

Regarding the first key problem, namely the modelling of individual asset's return, generally many researcher, academician, and practitioner assume that the individual asset's return is normally distributed. This assumption is unrealistic. Normality assumption cannot capture the event of extreme loss which is the main interest for an investor or financial institution. According to McNeil et al. (2005), the distribution of return for financial assets are in general fat-tailed, not independent and identically distributed (i.i.d), and has a volatility clustering. Extreme Value (EV) is more suitable to describe the fat-tailed characteristic of the asset return's distribution.

Regarding the second key problem, namely the modelling of dependence structure between multiple financial assets in a portfolio, a commonly used measure of dependence is Pearson correlation. However Poon et al. (2004) expressed concerns that Pearson correlation is not very good in describing dependence at the tail of a distribution (extreme event). Therefore, recent years have seen the introduction of copula as a tool to model the dependence structure between multiple financial assets in a portfolio. The measure of dependence between random variables using copula was first introduced by Sklar (Nelsen, 2006). The use of copula in finance is relatively recent, introduced by Embrechts et al. (1997) and grows rapidly in this subject. Many studies have used copula based models, for example thera are several studies in the field of insurance (Ghosh et al., 2022; Jafry et al., 2022; Mung'atu, 2015; Shi et al., 2016; Soto et al., 2015) and many studies in the field of climatology and agriculture (Ballarin et al., 2021; Liu et al., 2019; Mesbahzadeh et al., 2019; Nguyen-Huy et al., 2018; Su et al., 2014; Tavakol et al., 2020; Wong et al., 2008; Wu et al., 2021; Yang et al., 2020; Zhang et al., 2022).

The following are some studies in the field of finance that have used copula based models and ARMA-GARCH models. In Engle (2009) the GARCH model was used to overcome the problem of volatility clustering in the return data of financial asset return. In several studies Budiarti et al. (2018); Jafry et al. (2022); Jondeau & Rockinger (2006); Shams & Haghighi (2013) the copula-GARCH model was used to overcome the problem of volatility clustering, non-linear relationship, and non-normal distribution. In (Longin, 2000) extreme value was used to model the tail of a distribution. In Hsu et al. (2012), the approach of copula and extreme value were combined to model a portfolio in Asia financial market. The approach of copula-GARCH-extreme value was used in Ghorbel & Trabelsi (2009) to estimate risk with a parametric approach (Maximum Likelihood Estimator / MLE).

The Covid-19 pandemic was caused by a coronavirus first detected in Wuhan, China. The epidemic has spread to almost all nations with more than 40 million confirmed cases and 1 million death worldwide up to October 2020 (World Health Organization, 2020). The pandemic and the following lockdowns have destabilized world economies. Stock prices plummeted which contributed to the decline of composite stock indices. When this extreme event occurred, the modelling of dependence between financial assets plays an important role to predict massive loss.

Regarding the dependence model between extreme asset returns, the notion of extreme value copula plays an important role. This is because of the special characteristic of extreme value copula that allows it to extrapolate to the tail of a distribution, so it can be used to estimate the probability of extreme events. Some researchers have conducted studies of dependence model using extreme value copula on insurance data Haug et al. (2011), financial data in Asia financial market Hsu et al. (2012), and agricultural commodity data (Gródek-Szostak et al., 2019). The use of extreme value copula in modelling the dependence between financial assets is still rarely studied. In this study, extreme value copule will be used to model (1) the dependence between three composite stock indices (IDX Composite Index (IHSG), Straits Times Index (STI) and Kuala Lumpur Stock Exchange (KLSE)); and (2) to model the pairwise dependence of IHSG-STI, IHSG-KLSE, and STI-KLSE, during the Covid-19 pandemic. The results of this study are expected to be used by investors in the formation of a portfolio consisting of 2 financial assets and in the formation of a portfolio consisting of 3 financial assets.

B. METHODS

We use the data of daily returns of the IDX Composite Index (IHSG), Straits Times Index (STI) and Kuala Lumpur Stock Exchange (KLSE) in the time period from March 1st to October 1st 2020. The data was intentionally chosen to coincide with the COVID-19 pandemic to get extreme data, because the returns were highly fluctuative during the period. The main tools of analysis are the Autoregressive Moving Average – Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) models, copula, and extreme-value copula, which we describe below. We also describe Sklar's theorem, a theorem that is widely used in applied statistics, to explain the role of copula in connecting a joint distribution to its marginal distributions.

1. ARMA Model

Let $(\varepsilon_t)_{t\in\mathbb{Z}}$ be a white noise $WN(0,\sigma^2)$, having mean 0 and variance σ^2. A process $(X_t)_{t\in\mathbb{Z}}$ is called ARMA(p,q) if for each $t\in\mathbb{Z}$ the following holds

$$X_t-\phi_1X_{t-1}-\cdots-\phi_oX_{t-p}=\varepsilon_t+\theta_1\varepsilon_{t-1}+\cdots+\theta_q\varepsilon_{t-q} \qquad (1)$$

(Box et al., 2016; Enders, 1996; McNeil et al., 2005; Pham, 2013)

If $(\varepsilon_t)_{t\in\mathbb{Z}}$ is not a white noise, then $(\varepsilon_t)_{t\in\mathbb{Z}}$ should be modelled by GARCH.

2. GARCH Model

Let $(Z_t)_{t\in\mathbb{Z}}$ be a strict white noise $SWN(0,1)$, having mean 0, variance 1, We call $(\varepsilon_t)_{t\in\mathbb{Z}}$ GARCH(r,s) if

$$\varepsilon_t=\sigma_tZ_t,\ \ \sigma_t^2=\alpha_0+\sum_{i=1}^{r}\alpha_iX_{t-i}^2+\sum_{i=1}^{s}\beta_j\sigma_{t-j}^2 \qquad (2)$$

where $\sigma_t>0,\alpha_0>0$, and $\alpha_i\geq 0,\beta_j\geq 0$ for all $i=1,\dots,p$ and $j=1,\dots,q$.

(Box et al., 2016; Enders, 1996; McNeil et al., 2005; Pham, 2013)

3. Bivariate Copula

A 2-copula is a map $C: I^2\rightarrow I$ where $I=[0,1]$ such that:

a. C is *grounded*, namely $C(a,0)=0=C(0,b)$ for all $a,b\in I$.

b. $C(a,1)=a$ and $C(1,b)=b$ for all $a,b\in I$.

c. C is 2-increasing, namely if $a_1,a_2,b_1,b_2\in I$ and $a_1\leq a_2, b_1\leq b_2$, then

$$C(a_2,b_2)-C(a_2,b_1)-C(a_1,b_2)+C(a_1,b_1)\geq 0$$

(Joe, 2015; Nelsen, 2006)

4. Multivariate n-dimensional Copula

An n-copula is a map $C: I^n \to I$ where $I = [0,1]$ such that:

a. $C(\mathbf{a}) = 0$ if at least one entry of $\mathbf{a} \in I^n$ is 0.

b. $C(\mathbf{a}) = 1$ if all but one entries of $\mathbf{a} \in I^n$ is 1.

c. C is n-increasing, namely if $\mathbf{a}, \mathbf{b} \in I^n$ and $\mathbf{a} \leq \mathbf{b}$, then $V_C(\mathbf{u}, \mathbf{v}) \geq 0$.

(Joe, 2015; Nelsen, 2006)

5. Sklar's Theorem

For any joint distribution H with marginals $F_1, \dots, F_n$, there is a copula C with

$$H(\mathbf{x}) = C\big(F_1(x_1), \dots, F(x_n)\big) \tag{3}$$

for all $\mathbf{x} = (x_1, \dots, x_n) \in I^n$. When $F_1, \dots, F_n$ are continuous functions, C is unique.

(Joe, 2015; Nelsen, 2006)

6. Extreme-Value Copula

According to (Haug et al., 2011), C is an extreme-value copula if

$$C(\mathbf{u}) = \left(C\left(u_1^{1/m}, \dots, u_n^{1/m}\right)\right)^m$$

for all $\mathbf{u} = (u_1, \dots, u_n) \in I^n$ and $m \in \mathbb{N}$. According to (Gudendorf & Segers, 2010) in the bivariate case, C is called an extreme-value copula when there is a function $A: [0,1] \to [1/2,1]$ such that

$$C(u_1, u_2) = \exp\left[\log(u_1 u_2) \cdot A\left(\frac{\log u_1}{\log(u_1 u_2)}\right)\right]$$

See also (Ghorbel & Trabelsi, 2009; Marcon et al., 2014; Vettori et al., 2018). The function A, called Pickands Dependence Function, is convex and $\max(t, 1-t) < A(t) < 1$ for each $t \in [0,1]$. According to (Ghorbel & Trabelsi, 2009), parametric forms of A for several extreme-value copulas are given as follows:

a. Gumbel Copula

$$C(u,v) = exp\left(-\left((-\log u)^\theta + (-\log v)^\theta\right)^{\frac{1}{\theta}}\right), \quad 1 \leq \theta \leq \infty \tag{4}$$
$$A(t;\theta) = (t^\theta + (1-t)^\theta)^{1/\theta}$$

The parameter $\theta \geq 1$ measures the degree of dependence. If $\theta = 1$, then X_1 and X_2 have low dependence and can be considered independent. When $\theta \to \infty$, the dependence is nearly perfect.

b. Galambos Copula

$$C(u,v) = uvexp\left(\left((-\log u)^{-\theta} + (-\log v)^{-\theta}\right)^{-\frac{1}{\theta}}\right),\ 0 \leq \theta < \infty \quad (5)$$
$$A(t;\theta) = 1 - (t^{-\theta} + (1-t)^{-\theta})^{-1/\theta}$$

c. Husler-Reiss Copula

$$C(u,v) = exp\left\{\log(u)\,\Phi\left[\frac{1}{\delta} + \frac{1}{2}\delta\log\left(\frac{\log u}{\log v}\right)\right] + \log(v)\,\Phi\left[\frac{1}{\delta} + \frac{1}{2}\delta\log\left(\frac{\log v}{\log u}\right)\right]\right\} \quad (6)$$
$$A(t) = t\Phi\ \left[\frac{1}{\delta} + \frac{1}{2}\delta\log\left(\frac{t}{1-t}\right)\right] + (1-t)\Phi\ \left[\frac{1}{\delta} - \frac{1}{2}\delta\log\left(\frac{t}{1-t}\right)\right]$$

Here $0 \leq \delta < \infty$, while Φ is the conditional excess distribution of standard normal distribution.

d. Tawn Copula
This is an assymetric extension of Gumbel copula, with

$$A(t;\theta) = 1 - \beta + (\beta - \alpha)t + [\alpha^r t^r + \beta^r (1-t)^r]^{1/r} \quad (7)$$

where $\alpha \geq 0, \beta \leq 1$, and $1 \leq r < \infty$.

e. t-Copula
The t-EV or t-Extreme Value copula with degree of freedom r and correlation coefficient ρ is defined as follows

$$C_{r,\rho}^{tEV}(u,v;\rho) = \exp\left\{T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u}{\ln v}\right)^{\frac{1}{r}}\right]\ln u + T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u}{\ln v}\right)^{\frac{1}{r}}\right]\ln v\right\} \quad (8)$$

Here T_r is the t-Student distribution function having degree of freedom r, and the parameter θ is defined by $\theta^2 = \frac{1-\rho^2}{r+1}$. The Pickands Dependence Function is

$$A_{r,\rho}^{tEV}(t) = tT_{r+1}\left((1+r)^{\frac{1}{2}}\frac{\left(\frac{t}{1-t}\right) - \rho}{\sqrt{1-\rho^2}}\right) + (1-t)T_{r+1}\left((1+r)^{\frac{1}{2}}\frac{\left(\frac{1-t}{t}\right) - \rho}{\sqrt{1-\rho^2}}\right)$$

(Gródek-Szostak et al., 2019)

7. Steps of This Study

The aims of this study are to analyze the return data of each index, identify a suitable model for the distribution of return data of each index, and identify a suitable extreme value copula for the indices. The following diagram summarizes the steps of this study, as shown in Figure 1.

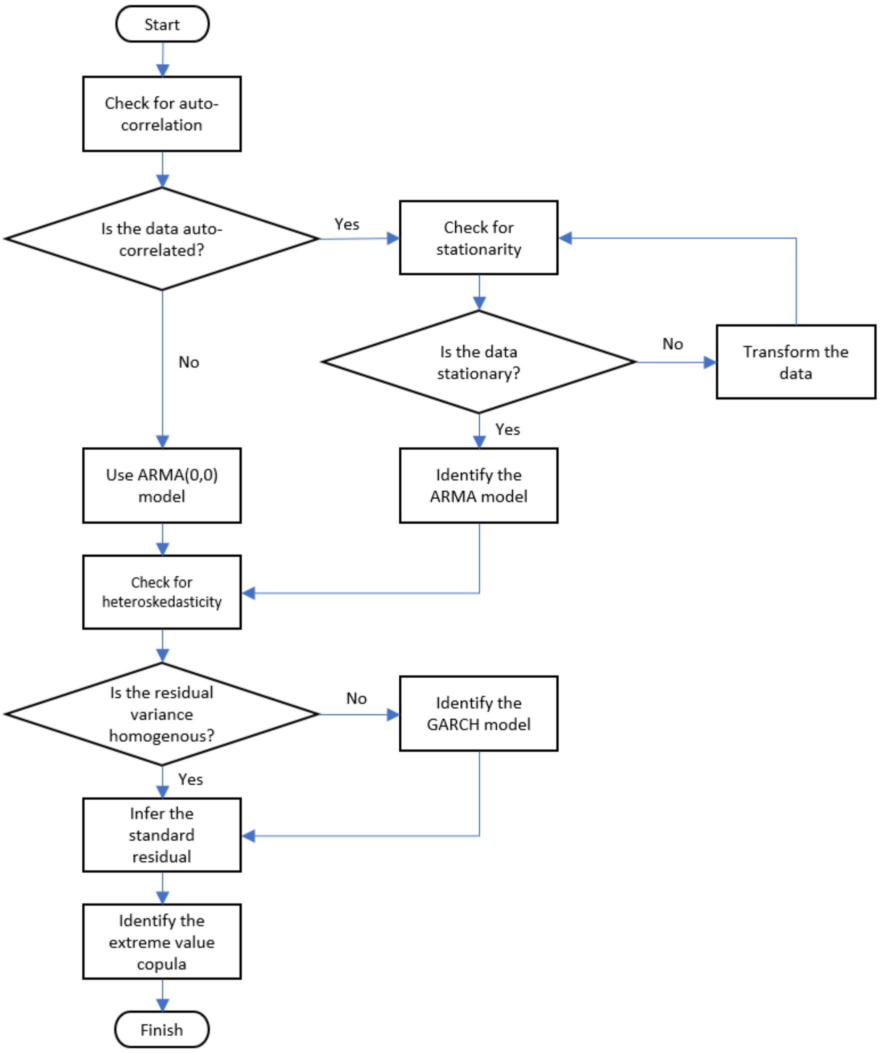

Figure 1. Flow chart of this study.

The following points explain the steps in more details.

a. Since this study uses a time-series data, there is a possibility of correlation between observations at different times, which will cause a problem with maximum likelihood estimation (MLE). This is checked by using the Ljung Box test.
b. If there is auto-correlation, it needs to be filtered by a time series model (ARMA) which requires stationarity. This is checked by using the Augmented Dickey Fuller (ADF) test. If the data is not stationary, the data needs to be transformed for example by differencing. If the data is already stationary, the next step is to identify possible candidates for the order of the ARMA model by looking at the ACF and PACF. The suitable model is then chosen by using Akaike Information Criteria (AIC) (Snipes & Taylor, 2014).
c. In the ARMA model there is an assumption that the residual should be white noise (independent and have a homogenous variance). This is checked by using the ARCH-LM test.
d. If there is heteroskedasticity (namely, if the residual's variance is not homogenous) then the residual is further modelled by GARCH. Several candidates for the order of the GARCH model are considered, and the suitable model is chosen by using AIC.

e. The next step is inferring the distribution of standard residual by comparison with normal distribution, t-student distribution, and logistic distribution.
f. Next, to model the dependence between indices, some extreme value copula are considered and a suitable model is chosen by using AIC.

C. RESULT AND DISCUSSION

1. Autocorrelation

Since this research uses a time-series data, there is a possibility of correlation between observations at different times. This disallows the use of maximum likelihood estimation (MLE). If the data are not independent, it needs to be filtered by a time series model (ARMA) which requires stationarity. By using the Ljung Box test, we found autocorrelation in STI, while there is no correlation in IHSG and KLSE, as shown in Table 1.

Table 1. The results of Ljung-Box test for all three indices.

Index	Lag	Test Statistic	χ^2	*p-value*
IHSG	12	16.324	21.026	0.1768
	24	26.022	36.415	0.3520
	36	32.858	51.000	0.6188
STI	12	36.144	21.026	0.0003
	24	38.848	36.415	0.0283
	36	42.465	51.000	0.2124
KLSE	12	18.390	21.026	0.1044
	24	27.905	36.415	0.2641
	36	33.722	51.000	0.5774

2. Stationarity

The ARMA model requires the data to be stationary. Looking at Figure 2 it is visually apparent that the data is stationary. This is supported further by a statistical test. Using Augmented Dickey-Fuller (ADF) test, it was found that all three data are stationary, as shown in Figure 2.

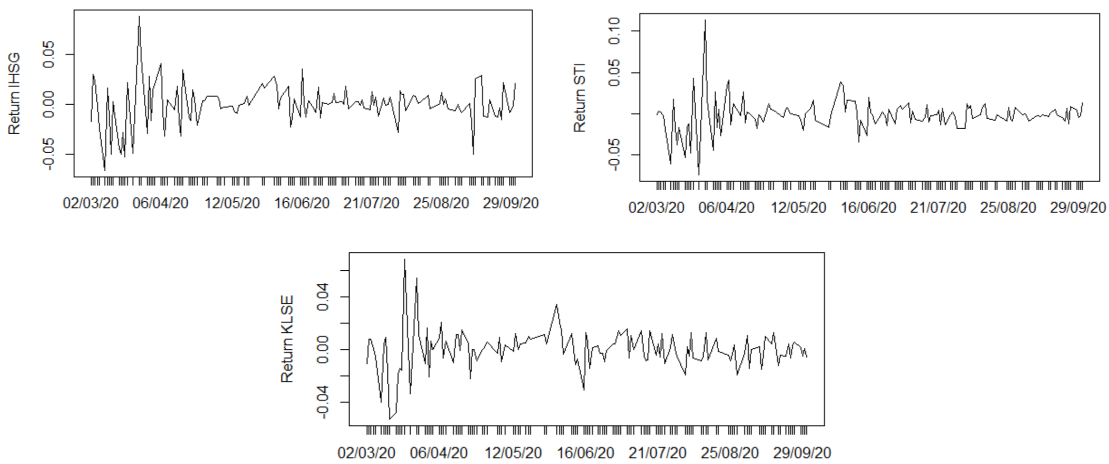

Figure 2. Daily returns of three indices during COVID-19 pandemic.

3. ARMA Model

According to the Ljung Box test, only the STI data is autocorrelated. Therefore, we use ARMA model. The ACF and PACF for STI daily return are shown in Figure 3. The observed cutoff is on 1st lag, so the starting model for STI is estimated to be AR(1), MA(1), as shown in Figure 3.

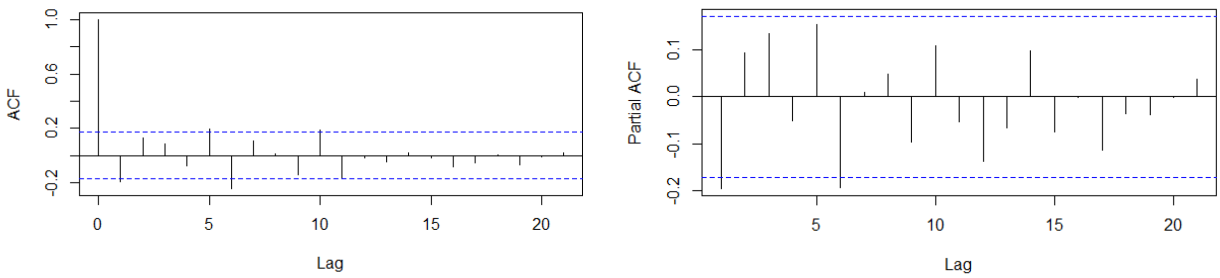

Figure 3. ACF and PACF plots of STI daily return.

The next step is choosing a suitable model based on parameter significance and smallest AIC. After trying out several choices for the order p and q, Table 2 shows that the AR(1) has the smallest AIC compared to other models, and the parameters are significant. Therefore, we infer that AR (1) is the best model to overcome autocorrelation in STI data, as shown in Table 2.

Table 2. Tentative ARMA models for STI.

Model	Parameter	Estimate	*p-value*	AIC
AR(1)	Constant	-0.0012417	0.3828	-655.36
	AR(1)	-0.1940008	0.0234	
MA(1)	Constant	-0.0012415	0.3897	-654.33
	MA(1)	-0.1537981	0.0392	
ARMA(1,1)	Constant	-0.0012383	0.3955	-653.80
	AR(1)	-0.3675191	0.1278	
	MA(1)	0.1755749	0.4755	

The next step is checking whether the residual is white noise. By the ARCH LM test, the residuals of all three indices have a non-homogeneous variance. In other words, there is heteroskedasticity. Therefore, the residuals are modelled by GARCH.

4. ARCH/GARCH Model

To overcome heteroskedasticity, we consider ARCH/GARCH model. After trying out several choices, it was found that GARCH (1,1) has the smallest AIC and all the parameters are significant, for all three indices (IHSG, STI, KLSE), as shown in Table 3.

Table 3. GARCH model estimation

Index	Model	Parameter	Estimate	***p-value***	AIC
IHSG	GARCH(1,1)	ω	0.000041	0.035703	-5.3905
		α_1	0.626319	0.016600	
		β_1	0.372678	0.041807	
STI	GARCH(1,1)	ω	0.000013	0.455164	-5.6755
		α_1	0.390253	0.004677	
		β_1	0.608747	0.005082	
KLSE	GARCH(1,1)	ω	0.000013	0.000023	-5.8673
		α_1	0.154782	0.000001	
		β_1	0.764509	0.000000	

The estimated models are as follows:

a. IHSG

ARMA(0,0)-GARCH(1,1)

$R_t = -0.00050065 + \varepsilon_t$

$\varepsilon_t = \sigma_t Z_t$

$\sigma_t^2 = 0.000041 + 0.626319\varepsilon_{t-1}^2 + 0.372678\sigma_{t-1}^2$

b. STI

AR(1)-GARCH(1,1)

$R_t = -0.0012417 - 0.1940008R_{t-1} + \varepsilon_t$

$\varepsilon_t = \sigma_t Z_t$

$\sigma_t^2 = 0.000013 + 0.390253\varepsilon_{t-1}^2 + 0.608747\sigma_{t-1}^2$

c. KLSE

ARMA(0,0)-GARCH(1,1)

$R_t = 0.00018089 + \varepsilon_t$

$\varepsilon_t = \sigma_t Z_t$

$\sigma_t^2 = 0.000013 + 0.154782\varepsilon_{t-1}^2 + 0.764509\sigma_{t-1}^2$

5. Standard Residual

Next, we estimate a distribution that approximates the empirical distribution of all three indices. We compare the standard residuals of three distributions, namely normal distribution, t-Student distribution, and Logistic distribution, with the empirical distributions, as shown in Figure 4.

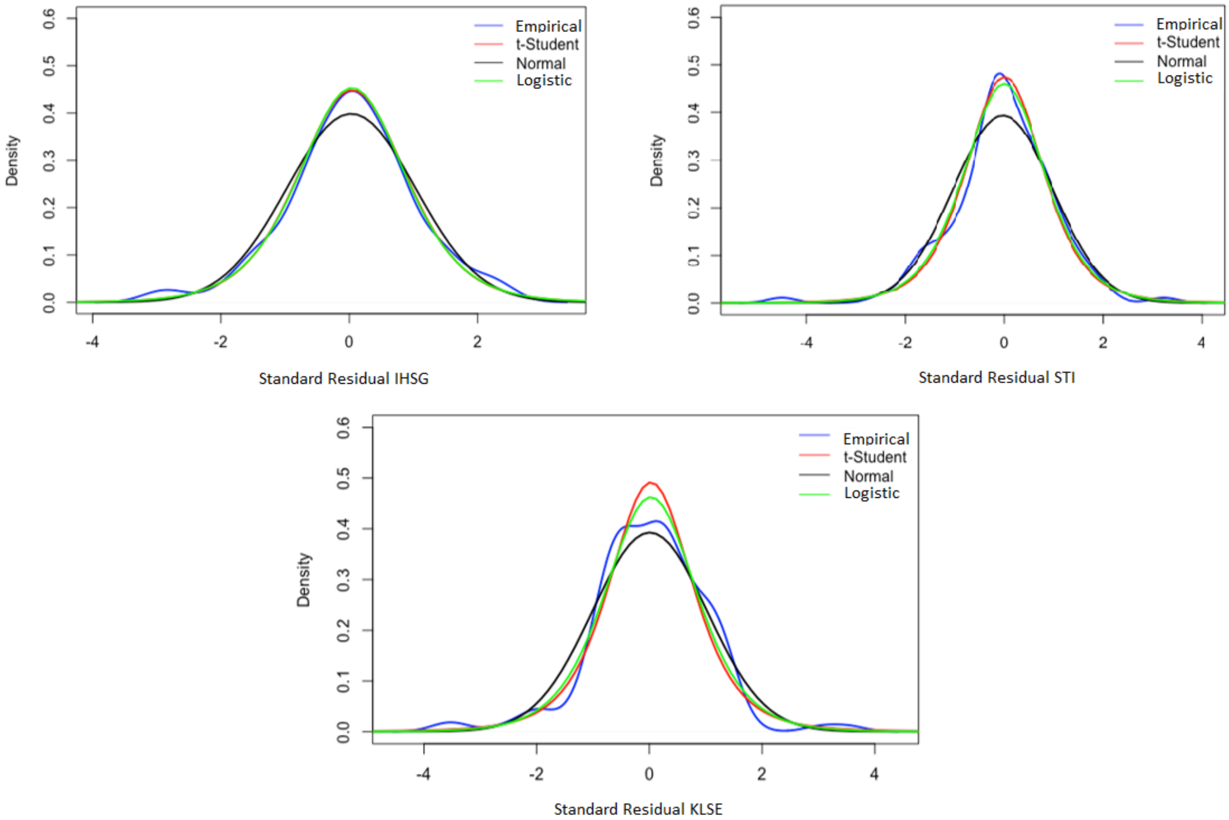

Figure 4. Comparison of standard residuals between the empirical distribution and normal distribution, t-Student distribution, and logistic distribution.

Anderson-Darling test shows that the most suitable distribution for STI and KLSE data is t-Student, while the most suitable distribution for IHSG is the logistic distribution, as shown in Table 4.

Table 4. Anderson-Darling test results for each index.

Index	Distribution	W_n^2	*p-value*
IHSG	Normal	0.4888	0.7580
	t-*Stude*nt	0.1770	0.9955
	Logistic	0.1692	0.9966
STI	Normal	0.8382	0.4541
	t-*Student*	0.3150	0.9263
	Logistic	0.3449	0.9008
KLSE	Normal	1.0606	0.3268
	t-*Student*	0.4212	0.8272
	Logistic	0.4484	0.7994

6. Extreme Value Copula

Next we choose the most suitable extreme-value. The parameters are estimated using Maximum Likelihood. It was found that the most appropriate extreme value copulas for modelling the dependency of the three indices is the Gumbel copula. Table 5 shows that the t-EV copula is chosen for the IHSG-STI pair and STI-KLSE pair, while the Galambos copula is chosen for the IHSG-KLSE pair because it has the greatest maximum likelihood, as shown in Table 5.

Table 5. Estimated parameters of the extreme-value copulas

Indices	Copula	Parameter	Maximum Likelihood
IHSG, STI, KLSE	Gumbel	$\alpha = 1.4650$	41.80
IHSG, STI	Gumbel	$\alpha = 1.5130$	19.77
	Galambos	$\alpha = 0.7791$	19.38
	Husler Reiss	$\alpha = 1.1650$	18.58
	Tawn	$\alpha = 0.8523$	19.73
	t-EV	$\rho = 0.6747$	19.83
		$df = 3.0913$	
IHSG, KLSE	Gumbel	$\alpha = 1.4540$	16.15
	Galambos	$\alpha = 0.7169$	16.27
	Husler Reiss	$\alpha = 1.1140$	16.22
	Tawn	$\alpha = 0.8097$	15.28
	t-EV	$\rho = 0.8983$	16.26
		$df = 14.4232$	
STI, KLSE	Gumbel	$\alpha = 1.5400$	21.93
	Galambos	$\alpha = 0.8014$	20.72
	Husler Reiss	$\alpha = 1.1800$	19.55
	Tawn	$\alpha = 0.8152$	22.32
	t-EV	$\rho = 0.3261$	24.51
		$df = 0.8514$	

The estimated Gumbel copula for three indices is as follows

$$
\begin{aligned}
H\left(F^{-1}(u_1), F^{-1}(u_2), F^{-1}(u_3)\right) &= C(u_1, u_2, u_3) \\
&= exp\left(-((-\log u_1)^{\alpha} + (-\log u_2)^{\alpha} + (-\log u_3)^{\alpha})^{\frac{1}{\alpha}}\right)
\end{aligned}
$$

where $\alpha = 1.4650$ measures the dependencies between the indices. The t-EV copula is chosen for the IHSG-STI pair and STI-KLSE pair because it has the greatest maximum likelihood. The t-EV copula model for IHSG and STI is as follows

$$
\begin{aligned}
H\left(F^{-1}(u_1), F^{-1}(u_2)\right) &= C_{12}(u_1, u_2; \rho) \\
&= \exp\left\{T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u_1}{\ln u_2}\right)^{\frac{1}{r}}\right]\ln u_1 + T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u_1}{\ln u_2}\right)^{1/r}\right]\ln u_2\right\}
\end{aligned}
$$

with degree of freedom $r = 3.0913$, $\rho = 0.6747$, and $\theta^2 = 0.1332$. The joint density of IHSG and STI can be expressed as follows, whose 3-dimensional plot and contour map are shown in Figure 5.

$$
h_{12}(z_1, z_2) = c_{12}(F_1(z_1), F_2(z_2)) f_1(z_1) f_2(z_2)
$$

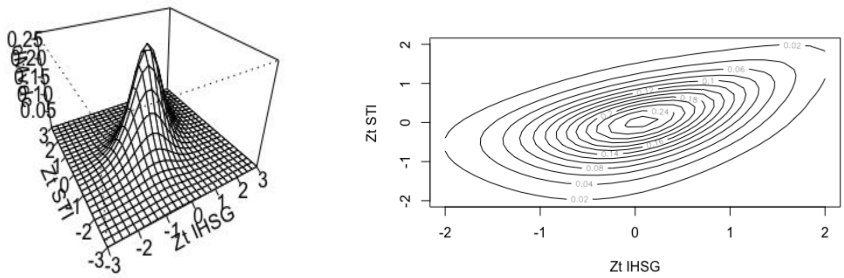

Figure 5. 3D plot (left) and contour map (right) of joint density of IHSG and STI.

The contour map provides some insight into the correlation between the indices. If the contour is skewed to the right, the correlation is positive. If the contour is skewed to the left, the correlation is negative. If the contour shape is nearing a perfect ellipse and more flat, then the indices are linearly correlated and the correlation is stronger. Based on the figure above, IHSG and STI have a positive correlation that is not linear and not very strong. For the STI-KLSE pair, the estimated t-EV copula is

$$\begin{aligned} H\left(F^{-1}(u_2), F^{-1}(u_3)\right) &= C_{23}(u_2, u_3; \rho) \\ &= \exp\left\{T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u_2}{\ln u_3}\right)^{\frac{1}{r}}\right]\ln u_2 + T_{r+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u_2}{\ln u_3}\right)^{1/r}\right]\ln u_3\right\} \end{aligned}$$

with $r = 0.8514$, $\rho = 0.3261$, and $\theta^2 = 0.4287$. The joint density function of STI and KLSE can be expressed as shown in Figure 6.

$$h_{23}(z_2, z_3) = c_{23}(F_2(z_2), F_3(z_3))f_2(z_2)f_3(z_3)$$

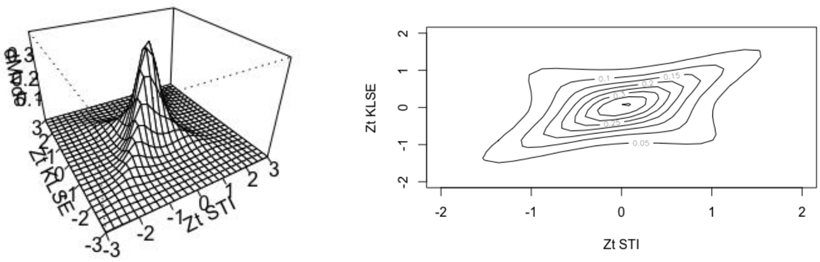

Figure 6. 3D plot (left) and contour map (right) of joint density of STI and KLSE.

For the IHSG-KLSE pair, Galambos copula is chosen. The estimed Galambos copula is

$$H\left(F^{-1}(u_1),\ F^{-1}(u_3)\right) = C_{13}(u_1, u_3) = u_1 u_3 exp\left(((-\log u_1)^{-\alpha} + (-\log u_3)^{-\alpha})^{-\frac{1}{\alpha}}\right)$$

with $\alpha = 0.7169$ and joint density function as shown in Figure 7.

$$h_{13}(z_1, z_3) = c_{13}(F_1(z_1), F_3(z_3))f_1(z_1)f_3(z_3)$$

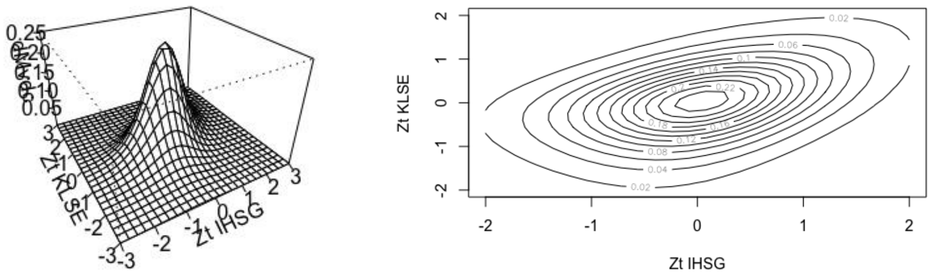

Figure 7. 3D plot (left) and contour map (right) of joint density of IHSG and KLSE.

The parameter of extreme value copula by itself is often inadequate to describe how tight is the relation between the asset returns. For example, equation (5) shows that the parameter of Galambos extreme value copula has a range of $0 \leq \theta \leq \infty$. If, say $\theta = 2$, it is not clear whether the dependence between asset returns is weak or strong. The strength of the dependence can be measured by tranforming the copula parameter into Tau-Kendall coefficient, as shown in Table 6.

Table 6. Tau-Kendall correlation coefficient between every pair of indices.

	IHSG	STI	KLSE
IHSG	1		
STI	0.13636	1	
KLSE	0.11004	0.12622	1

As can be seen from Table 6, the highest correlation is between IHSG and STI, indicating that they are more dependent (compared to other pairs). On the other hand, the lowest correlation is between IHSG and KLSE. These correlation coefficients are useful for investors. If the investor has to choose one of the three possible pairs, the pair with the smallest correlation should be chosen to minimize risk.

D. CONCLUSION AND SUGGESTIONS

In this study, it was found that the best extreme-value copula to model the dependencies of all three indices at once (IHSG, STI, KLSE) is the Gumbel copula. To model the dependencies of the pair IHSG and STI, the best extreme-value copula was found to be t-EV. The same copula was also the best choice for the STI-KLSE pair. As for the IHSG-KLSE pair, the best extreme-value copula was found to be Galambos copula. Based on these copulas, the Tau-Kendall correlation coefficients were computed. It was found that the highest correlation occured between IHSG and STI, indicating that IHSG and STI have the highest dependencies compared to the other two pairs.

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