The MODWT-ARIMA Model in Forecasting The COVID-19 Cases

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ABSTRACT

The Maximal Overlap Discrete Wavelet Transform-Autoregressive Integrated Moving Average (MODWT-ARIMA) is a forecasting method that uses the ARIMA model generated from MODWT data. The purpose of this study is to analyze an investigation into the MODWT-ARIMA model with regard to the total number of COVID-19 cases in DKI Jakarta. For this study, daily data on cases of COVID-19 in DKI Jakarta were obtained. The model is trained with data from April 3, 2022, to June 11, 2022 (referred to as the "in-sample"), and the outcomes of the prediction are tested with data from June 12, 2022, to June 18, 2022 (referred to as the "out-sample"). These data exhibit trends and are organized into four data series using MODWT. The ARIMA modeling technique is applied to each of the produced sequences. When using the MODWT-ARIMA approach, the RMSE value obtained from the in-sample data is found to be lower than the RMSE value obtained from the out-sample data. It became clear that MODWT-ARIMA is better suited for estimation than prediction. The fact that the RMSE value for the data acquired from the in-sample is lower than the RMSE value for the data collected from the out-sample demonstrates that this is the case.

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A. INTRODUCTION

In the present time, COVID-19 remains to be a significant issue in Jakarta, which is the capital city of Indonesia. Jakarta is one of the provinces in Indonesia with a sizeable number of people infected with the COVID-19 virus. The number of people diagnosed with the disease in Jakarta has skyrocketed ever since the pandemic was first reported (Ministry of Health of the Republic of Indonesia, 2021). The Provincial Government of DKI Jakarta initiated a comprehensive vaccination against COVID-19. Vaccinations against COVID-19 have been administered to several populations in Jakarta, including health workers, the elderly, and other susceptible groups. The Provincial Government of DKI Jakarta has enacted several rules and limitations to prevent the virus from spreading further. It includes limits on travel, the obligatory use of masks, restrictions on social and public activities, and restrictions on closing public spaces. These restrictions are imposed according to the circumstances and danger level of the spread of COVID-19. Because of the significant impacts of the pandemic on public health and the economy, it is essential to model the case of the COVID-19 virus. Whenever modeling the case of COVID-19, the aim is to determine patterns and trends in the spread of the virus,

analyze the dynamics of transmission, and provide more accurate forecasts to assist in making appropriate decisions to manage and overcome the pandemic. Specifically, the intention is to identify patterns and trends in the spread of the virus. The combination of MODWT and ARIMA is one technique that can be utilized in a modeling endeavor.

Implementing MODWT-ARIMA in case modeling for the COVID-19 research can confer several improvements. First, wavelet decomposition disentangles short-term and long-term patterns, which paves the way for improved identification of cyclical changes and trends in the case data for COVID-19. Second, employing ARIMA models on each time scale component makes it possible to choose an approach that is more flexible toward fluctuations in the behavior of the data. It can improve the ability to predict outcomes and make it easier to make decisions based on accurate information in the face of shifting patterns of virus spread. MODWT is a method that is used to break down data into a variety of various time-scale components (Spelta & De Giuli, 2023). This decomposition can be done automatically or manually. It makes it possible to simultaneously observe patterns on both short-term and long-term time scales. After the time scale components have been decomposed, the ARIMA model can be applied to each component to depict patterns and trends at the appropriate level. The wavelet method uses the data analysis process to model non-stationary data (Rhif et al., 2019). Wavelet decomposition analysis is an essential function that provides a new tool as an approach that can be used to represent data or other functions. The wavelet function is a type of mathematical function that exhibits a number of distinct qualities, including oscillating around the value zero and being localized in the temporal domain (Srivastava et al., 2019). Wavelet transforms are produced as a result of time and frequency representations, and these transforms can be utilized to evaluate non-stationary data (Dautov & Ozerdem, 2018). Because wavelets are localized in the time domain, the representation of functions with wavelets becomes more efficient. This is because there are relatively few non-zero wavelet coefficients in reconstructing functions with wavelets (Hendrickson et al., 2020).

The wavelet transform is used as a pre-treatment on the data before forecasting, which improves the predictive behavior of forecasting techniques such as ARIMA (Hu et al., 2018). Suppose the wavelet transform is applied to data with an average value and variance that is not constant, contains outliers, or is seasonal. In that case, the transformed data will have generally better behavior than the original data, have a more stable variance, and have no outliers (Deng et al., 2018). Thus, the future value of the data can be predicted more accurately by using forecasting techniques such as ARIMA. The wavelet transform is composed of two subcomponents, which are referred to as the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT) (Alickovic et al., 2018). When modeling time series data with the DWT, it is essential to assume that the sample size N is divisible by 2^J where J is an integer positive (Liu et al., 2019). Therefore, applying DWT for filtering cannot be done with any sample size. In order to circumvent the constraints imposed by DWT concerning sample size, concepts and techniques known as the MODWT were devised (Juliza et al., 2019). There are advantages to using MODWT. It is feasible to use ODWT for any sample size, and it divides the data in half (a technique known as "downsampling") in order to ensure that at each level of decomposition, there is a wavelet coefficient and a scale that is proportional to the amount of the data (Adib et al., 2021).

Research studies have been conducted on the MODWT, which were carried out by Ashok et al. (2019), who performed illustrative signal processing techniques. The MODWT has been shown to cite faulty signal attributes in cross-country cases and growing errors, which are complex. Furthermore, Yaacob et al. (2021) have researched the application of employing discrete wavelet transform filters, a combination of maximum overlap, modeling, and forecasting the time-dependent mortality index of the Lee-Carter model. Yousuf's research Yousuf et al. (2021) integrates the MODWT with the ARIMA and adjusts the moving window MC. The research published to date makes it abundantly evident that the MC model is an appropriate tool for forecasting sequence residuals. So that MODWT combined with ARIMA (MODWT-ARIMA model) can be used to deal with non-stationary data and LRD (Long Range Dependence) time series. Combining simple forecasting models such as ARIMA with modern models such as MODWT can improve forecasting results. As a result, the purpose of this study was to determine the viability of the MODWT-ARIMA model as a fresh approach to the age-old problem of estimating the total number of COVID-19 cases.

B. METHODS

This research will examine the MODWT-ARIMA method for modeling time series data to determine the number of COVID-19 cases. The study case used is in DKI Jakarta, because DKI Jakarta is the province with the most cases of COVID-19 in Indonesia. The data is daily from April 3, 2022, to June 11, 2022, with 70 observations (referred to as the "in-sample"). The results of the prediction are evaluated using data spanning from the 12th to the 18th of June 2022 (this period is referred to as the "out-sample"). The MODWT method was utilized to decompose the time series data included in this research into exact values, such as various scale values at each level. MODWT coefficients, also known as wavelet and scale coefficients, will be generated throughout the decomposition. Before progressing with the decomposition process, the wavelet filter, including the level of decomposition, must be determined. Time series data modelling is carried out using the ARIMA model. The final results of forecasting the data collected across time series are obtained by mixing the values of the smooth and detailed forecasts or by performing an inverse-MODWT (IMODWT) of the smooth and detailed forecast values, depending on the wavelet filter used. For more details, modelling time series data using MODWT-ARIMA in this study shown in Figure 1.
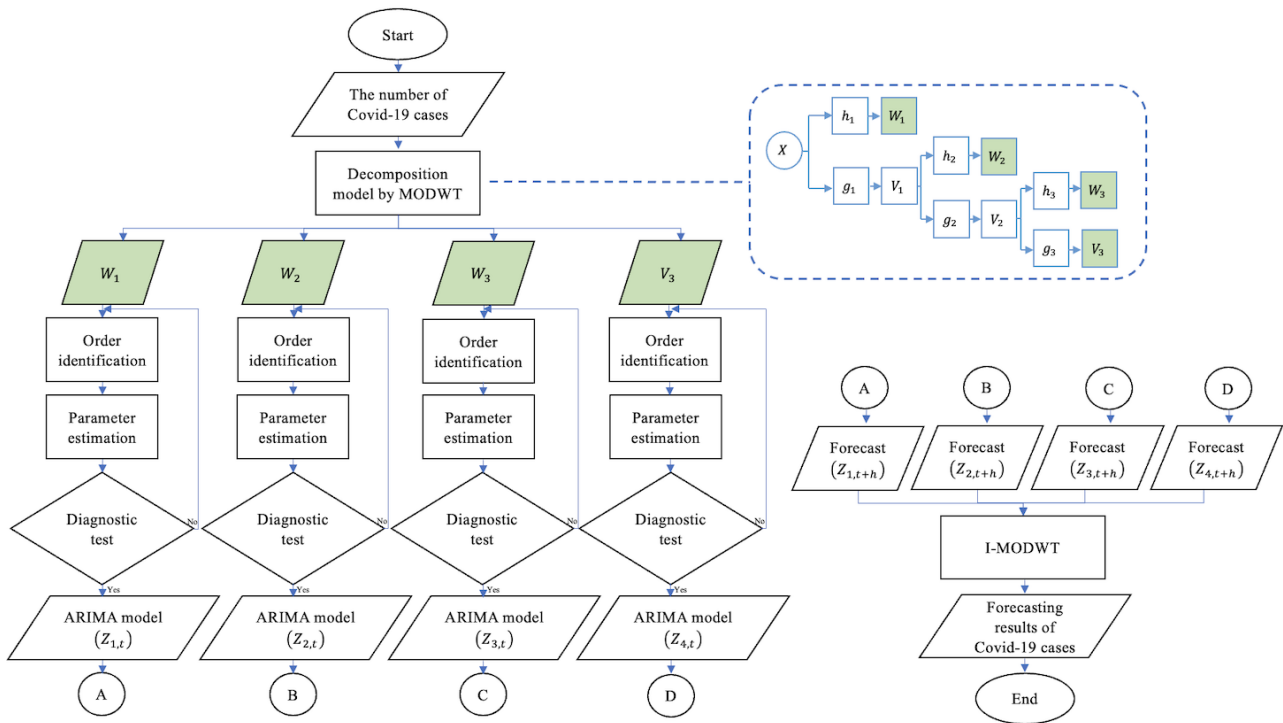
Figure 1. Flowchart of MODWT-ARIMA Model

1. Maximum Overlap Discrete Wavelet Transform (MODWT) Method

DWT is the component for which the MODWT was constructed. Within wavelet and MODWT scale filters, a connection occurs between TWD and MODWT (Gomes & Blanco, 2021). TWD is defined as $h = [h_0, h_1, h_2, \cdots, h_{L-1}]$ whenever the wavelet filter is active; hence, the MODWT wavelet filter can be written as (Ahanonu et al., 2019) $\tilde{h} = [\tilde{h}_0, \tilde{h}_1, \tilde{h}_2, \cdots, \tilde{h}_{L-1}]$ where $\tilde{h}_l = \frac{h_l}{\sqrt{2}}$. While it occurs to the MODWT scale filter, the same thing applies: if the TWD scale filter is written as (Meglic & Goic, 2021) $g = [g_0, g_1, g_2, \cdots, g_{L-1}]$ then the MODWT scale filter can be written as $\tilde{g} = [\tilde{g}_0, \tilde{g}_1, \tilde{g}_2, \cdots, \tilde{g}_{L-1}]$ where $\tilde{g}_l = \frac{g_l}{\sqrt{2}}$. The following conditions must be fulfilled for the terms of the MODWT wavelet filter to be valid (Abdulwahab et al., 2021):

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0; \sum_{l=0}^{L-1} \tilde{h}_l^2 = \frac{1}{2}; \text{and} \sum_{l=0}^{L-1} \tilde{h}_l \tilde{h}_{l+2n} = 0$$

Furthermore, the scale filter must fulfill the following conditions (Strömbergsson et al., 2019):

$$\sum_{l=0}^{L-1} \tilde{g}_l = 0; \sum_{l=0}^{L-1} \tilde{g}_l^2 = \frac{1}{2}; \text{and} \sum_{l=0}^{L-1} \tilde{g}_l \tilde{g}_{l+2n} = 0$$

where $n = 1, 2, \cdots, \left(\frac{L}{2}\right) - 1$. A time series data set X_t with any sample size N, the wavelet coefficient, and the j-th level MODWT scale is defined as (Wang et al., 2021):

$$W_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_l X_{t-lmodN}$$

and

$$V_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_l X_{t-lmodN}$$

where $t = 0,1,\cdots,N-1$. Filters constructed on the MODWT method have a width $L_j = (2^j - 1)(L-1) + 1$ (Sasal et al., 2022).

In MODWT, the number of wavelet coefficients at each level is always the same, so it is more suitable for modeling in a time series than DWT (Gusman et al., 2020). The time series prediction data is modeled linearly based on the wavelet coefficients of the decomposition results at previous times (Jang et al., 2021). Suppose we have signal $Z = (Z_1, Z_2, \cdots, Z_t)$. The prediction of one step ahead from an autoregressive process of order p or $AR(p)$ can be written as (Montgomery et al., 2015):

$$\hat{Z}_{t+1} = \sum_{k=1}^{p} \hat{\phi}_k Z_{t-(k-1)}$$

2. ARIMA Time Series Model

Suppose the sequence of random variables Z_t follows the ARIMA process, then Z_t can be defined as follows (Wei, 2006)

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B)a_t \tag{3}$$

where $\phi_p(B) = (1 - \phi_1 B - \cdots - \phi_p B^p), \theta_q(B) = (1 - \theta_1 B - \cdots - \theta_q B^q)$, and a_t is error at time t. The model in Equation 3 is called the ARIMA model with the order (p, d, q) and is written as the ARIMA model (p, d, q) (Srivastava et al., 2019). If $p = 0$, then ARIMA (p, d, q) model is also called IMA model with order (d, q). Likewise, if $q = 0$, then the ARIMA (p, d, q) model is called an ARI model with order (p, d) (Montgomery et al., 2015). ARIMA model is a model used for data stationery with the following three main stages (Imro'ah & Huda, 2022):

a. Order identification. Examining the ACF and PACF plots based on transformed and differencing time series data is required to construct the ARIMA (p, d, q) model. This is required so that the model can be constructed.

b. Parameter estimation. Estimating the value of the AR and MA parameter's value requires performing a parameter estimation process. The Ordinary Least Squares method is the one that is utilized for parameter estimation.

c. Diagnostic test. The diagnostic test is divided into two parts: the residual test, which models white noise, and the residual test is normally distributed.

C. RESULT AND DISCUSSION

1. Data Analysis

The data on the number of cases of COVID-19 in DKI Jakarta, which encompasses East Jakarta, South Jakarta, West Jakarta, North Jakarta, Central Jakarta, and the Thousand Islands, were used in this study. This study's period is daily, from April 3, 2022, to June 11, 2022, with 70 observations. The highest number of cases during the observation period occurred on April 6, 2022, namely 462 cases. This makes DKI Jakarta the province with the most cases of COVID-19 in Indonesia. Some of the things the government is doing in this regard are continuing to vaccinate the first, second, and booster doses. In addition, the government also stipulates Eid al-Fitr joint holidays on April 29, 2022, and May 4-6, 2022. Most cases were below the average during the observation period, namely 135.6 (shown by the dotted line in Figure 2). The COVID-19 cases in DKI Jakarta experienced an increase again on June 9, 2022. One reason is the impact of a new variant due to the high mobility of people in the capital city of Jakarta. Descriptive statistics of the data are presented in Figure 2. In addition, the shape of the distribution of the number of cases accumulates at a smaller value, meaning that the shape of the data distribution is not symmetrical (see histogram in Figure 2).
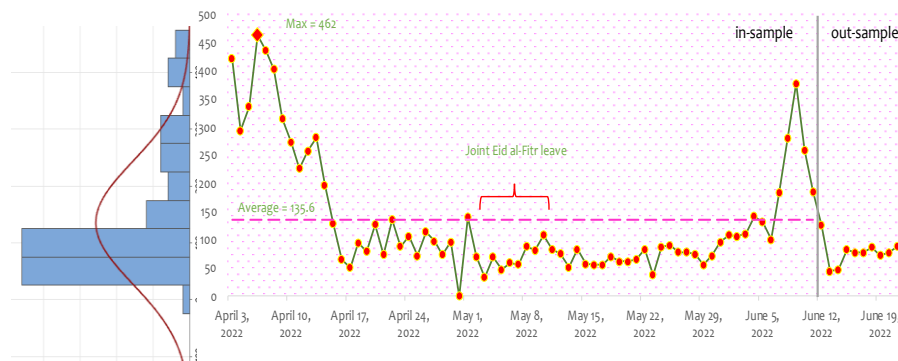
Figure 2. Plot of Data in this Study

In this study, modeling time series data using MODWT-ARIMA, as shown in Figure 1, provides additional information. When MODWT is carried out, the Daubechies4 ($db4$) filter is utilized. This produces four data rows, three containing wavelet coefficient data and one containing scale coefficient data. The data rows are denoted as the following: W_1, W_2, W_3 and V_3. The coefficients W_1, W_2, and W_3 are all stationary, but the coefficient V_3 is not. Because V_3 is not stationary, it is necessary to make these differences. Afterwards, the ACF and PACF plots are analyzed to determine the order in which each coefficient should be represented in the ARIMA (p, d, q) model. The plots of each data series and ACF PACF plots are presented in Figure 3. Table 1 contains additional details regarding these models.

The red text in Table 1 is the smallest AIC value for each possible model for each coefficient. So that the residual diagnostic test is applied to those models. Plots related to the residuals of each model are presented in sixth column (Table 1), the residuals of all models subjected to diagnostic tests fulfill the assumptions of normality and are independent of each other concerning time lag. Thus it can be concluded that ARIMA (3,0,3) model for W_1 coefficients

(Equation 4), ARIMA (2,0,3) for W_2 coefficients (Equation 5), ARIMA (3,0,2) for W_3 coefficients (Equation 6), and ARIMA (1,2,4) for the coefficient V_3 (Equation 7) is the best model.

$$Z_t = -0.0044 + 0.5087Z_{t-1} - 0.3410Z_{t-2} + 0.1404Z_{t-3} - 2.7688e_{t-1} + 2.5782e_{t-2} - \\ -0.8050e_{t-3} \tag{4}$$

$$Z_t = -0.0261 + 0.6026Z_{t-1} - 0.6506Z_{t-2} + 0.9884e_{t-1} - 0.9883e_{t-2} - 0.9999e_{t-3} \tag{5}$$

$$Z_t = -0.0912 + 0.6416Z_{t-1} + 0.1221Z_{t-2} - 0.3006Z_{t-3} + 1.9950e_{t-1} - 0.9999e_{t-2} \tag{6}$$

$$Z_t = -0.3802Z_{t-1} + 1.8254e_{t-1} + 1.6592e_{t-2} + 1.8254e_{t-3} + e_{t-4} \tag{7}$$
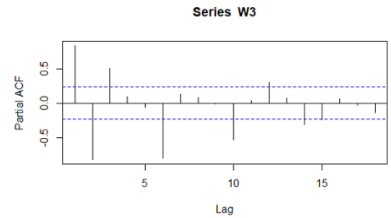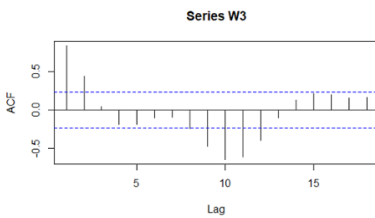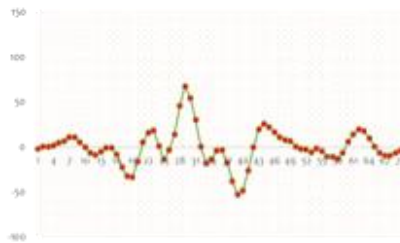
Equation (4), (5), (6) and (7) is then applied to data W_1, W_2, W_3, and V_3 to see if the estimation results match the data shown in Figure 4. Based on Figure 4, the estimated results plot and actual data coincide. Alternatively, the estimation results using Equation (4), (5), (6) and (7) match the actual data, as shown in Figure 3, Table 1 and Figure 4.
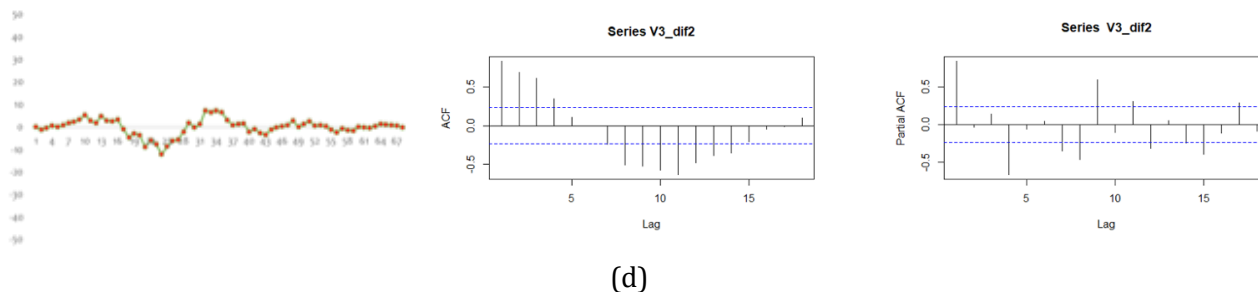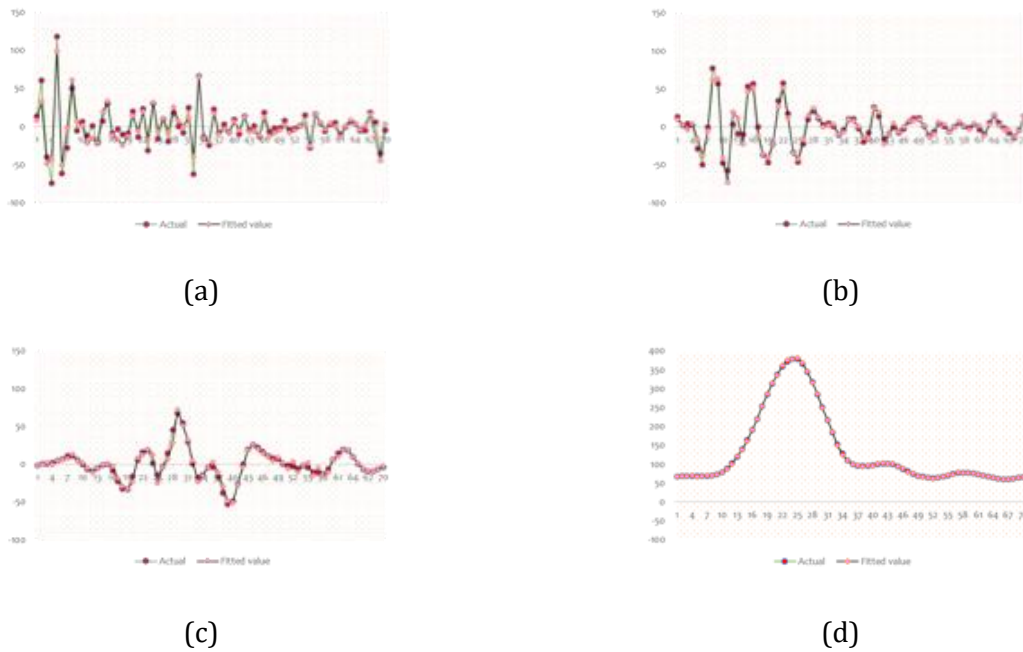
(a)

(b)

(c)

(d)
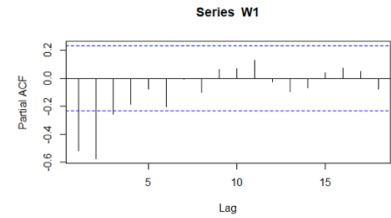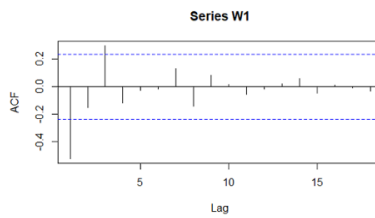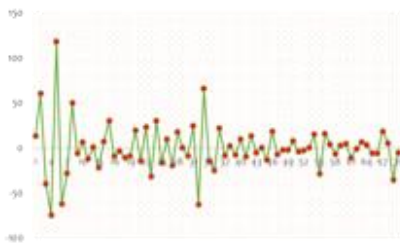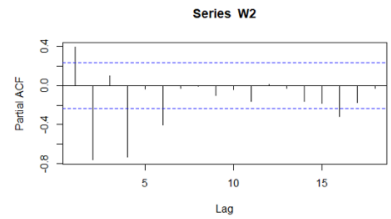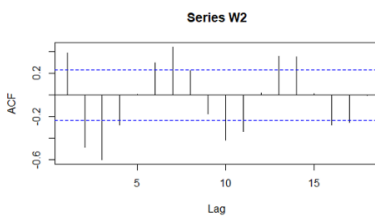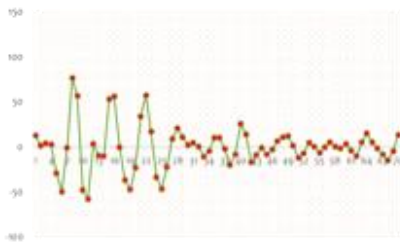
Figure 3. Wavelet coefficient data series and ACF PACF plot: (a) W_1, (b) W_2, (c) W_3 and scale coefficients: (d) second differencing of V_3

Table 1. Parameter Estimation

Coefficient	Orde (p,d,q)	Mean	$\begin{bmatrix} \phi_i \\ \theta_j \end{bmatrix} \times 10^{-2}$	Model Accuration			Residuals Plots for Best Model
				AIC	RMSE	MAE	
W_1	(3,0,0)	-0.1798	$\begin{bmatrix} -121; 114; -49 \\ 0 \end{bmatrix}$	598.26	15.88	11.34	
	(0,0,3)	-0.0045	$\begin{bmatrix} 0 \\ -257; 216; -59 \end{bmatrix}$	556.36	10.79	7.51	
	(3,0,3)	-0.0044	$\begin{bmatrix} 51; -34; 14 \\ -277; 258; -81 \end{bmatrix}$	548.41	10.08	7.24	
W_2	(2,0,0)	0.0078	$\begin{bmatrix} 69; -75 \\ 0 \end{bmatrix}$	581.85	14.40	9.47	
	(0,0,3)	0.0269	$\begin{bmatrix} 0 \\ 98; -98; -100 \end{bmatrix}$	528.59	8.97	6.02	
	(2,0,3)	0.0261	$\begin{bmatrix} 60; -65 \\ 99; -99; -100 \end{bmatrix}$	484.01	6.12	4.33	
W_3	(3,0,0)	-0.2694	$\begin{bmatrix} 190; -155; 50 \\ 0 \end{bmatrix}$	450.02	5.45	3.96	
	(0,0,2)	-0.0985	$\begin{bmatrix} 0 \\ 198; 100 \end{bmatrix}$	463.32	5.76	4.02	
	(3,0,2)	-0.0912	$\begin{bmatrix} 64; 12; -30 \\ 199; 100 \end{bmatrix}$	427.90	4.21	2.89	
V_3	(1,2,0)	0	$\begin{bmatrix} 83 \\ 0 \end{bmatrix}$	290.41	1.94	1.42	
	(0,2,4)	0	$\begin{bmatrix} 0 \\ 156; 94; 118; 81 \end{bmatrix}$	256.95	1.34	1.04	
	(1,2,4)	0	$\begin{bmatrix} -38 \\ 183; 166; 183; 1(\end{bmatrix}$	249.5	1.22	0.90	

(a)

(b)

(c)

(d)

Figure 4. Plot of actual data vs fitted values for (a)W_1, (b) W_2, (c) W_3, and (d) V_3

2. Forecasting

In the MODWT-ARIMA model, forecasting is accomplished by making predictions based on the decomposition result data, namely W_1, W_2, W_3, and V_3. Furthermore, the IMODWT procedure is carried out to obtain a predicted value of DKI Jakarta's COVID-19 cases. The results of the out-of-sample data predictions are presented below in Table 2. The most significant difference from the actual data occurred on June 18, 2022, as seen by the red text in Table 2, which illustrates that the results that were predicted were quite different from the actual data. In light of the RMSE value, the MODWT-ARIMA approach is best adapted for estimate rather than prediction. This is because the RMSE value calculated using the data from the in-sample was 178.01. This result is lower than the RMSE data out-sample, which is 373.44; therefore, this is a better option. According to the prediction findings, DKI Jakarta's COVID-19 cases will continue to be unpredictable throughout the following seven periods, as shown in Table 2.

Table 2. Prediction of Out-sample Data using the MODWT-ARIMA Method

Date	Observations	Prediction	Error
June 12, 2022	291	134	157
June 13, 2022	263	95	168
June 14, 2022	399	98	301
June 15, 2022	553	91	462
June 16, 2022	525	84	441
June 17, 2022	532	90	442
June 18, 2022	561	82	479

D. CONCLUSION

The process of modeling time series data using the MODWT-ARIMA method is carried out by determining the filter and decomposition used, namely Daubechies 4 and 3 decomposition levels. The decomposition results are in the form of wavelet coefficients and scale coefficients, namely W_1, W_2, and W_3, in the form of stationary time series. Meanwhile, V_3 is a non-stationary time series. The best model for each decomposition result is ARIMA (3,0,3) model for W_1 coefficients, ARIMA (2,0,3) for W_2 coefficients, ARIMA (3,0,2) for W_3 coefficients, and ARIMA (1,2,4) for the coefficient V_3. The accuracy of forecasting results is seen based on the RMSE value. The MODWT-ARIMA method produces an in-sample RMSE value smaller than the out-sample RMSE value. It means that the MODWT-ARIMA method is more suitable for estimation rather than prediction. The outcomes of the forecasting done with MODWT-ARIMA showed that the number of cases of COVID-19 in DKI Jakarta swung around the value of 96.29 and reached as high as 97 cases.

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