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Ratio Interval-Frequency Density with Modifications to the Weighted Fuzzy Time Series

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ABSTRACT

The improvement of plantation forecasting accuracy, particularly with regard to coffee production, was an essential aspect of earth observations for the purpose of informing plantation management alternatives. These decisions included strategic and tactical decisions on supply chain operations and financial decisions. Many research initiatives have used a variety of methodologies to the forecasting of plantation areas and related industries, such as coffee production. One of these methods was known as the fuzzy time series (FTS) technique. This study combined ratio-interval and frequency density to get universe of discourse and partition followed by adopted weighted and modified that weighted. The first step was defined universe of discourse using ratio-interval algorithm. The second step was partition the universe of discourse using ratio-interval algorithm followed by frequency density partitioning. The third step was fuzzyfication. The fourth step built fuzzy logic relationship (FLR) and fuzzy logic relationship group (FLRG). The fifth step was adopted the modification weighted. The last step was defuzzyfication. The models evaluated by average forecasting error rate (AFER) and compared with existing methods. AFER value 1.24% for proposed method.

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A. INTRODUCTION

One of the most important uses of earth observations for guiding plantation management options is the enhancement of plantation forecasting accuracy, especially in the case of coffee production. These choices include financial choices as well as tactical and strategic choices pertaining to supply chain management. A range of approaches have been used by several research projects to anticipate plantation lands and associated sectors, such coffee output. The fuzzy time series (FTS) methodology is one such approach. This leads us to undertake state-of-the-art research utilising fuzzy time series to predict coffee production. Zadeh (1965) initially formulated the fundamental principles of set theory in relation to continuous feasible sets, commonly referred to as fuzzy sets. The fuzzy set, as conceptualized by Zadeh, is characterized by a membership function that represents its elements. Within the fuzzy set, each element in the domain is precisely represented by a numerical value ranging from zero to one. The field of fuzzy set theory, along with its concept of linguistic variables, and the utilization of fuzzy applications for proximate reasoning, as introduced by Zadeh (1965, 1975), has effectively permeated the domain of time series data forecasting.

The predictive model about fuzzy time series (FTS) has been applied to a variety of forecasting issues, including forecasts for Alabama university enrolment (S. M. Chen, 2002; S. M. Chen & Chung, 2006; Jeng et al., 2006; Song & Chissom, 1993; Tanuwijaya & Chen, 2009), stock price (M. Y. Chen & Chen, 2015; T. L. Chen et al., 2007; Cheng & Yang, 2018; Singh & Borah, 2014; Tavares et al., 2022), renewable energy (Çakır, 2023; Severiano et al., 2021), lumpy skin disease (Punyapornwithaya et al., 2023), temperature forecast (Lee et al., 2006), and portfolio returns forecast (Rubio et al., 2016), with the goal of lowering future uncertainty, also improving and streamlining planning.

The FTS was first presented by Song & Chissom (1993), and since then, it has seen a lot of development, including interval ratio (Huarng & Yu, 2006; Haikal et al., 2022; Vianita et al., 2023), frequency density partitioning (Hariyanto et al., 2023; Irawanto et al., 2019; Jilani et al., 2007; Mukminin et al., 2021), the weighted FTS (Jiang et al., 2017; Yu, 2005). Forecasting model uses first-order (Lu et al., 2015; Mirzaei Talarposhti et al., 2016; Singh & Borah, 2013) or high-order (Lu et al., 2015; Mirzaei Talarposhti et al., 2016; Singh & Borah, 2013) methods.

Interval ratio Huarng & Yu (2006); Haikal et al. (2022); Vianita et al. (2023) and frequency density partitioning (Hariyanto et al., 2023; Irawanto et al., 2019; Jilani et al., 2007; Mukminin et al., 2021) in fuzzy relationships that were consistently disregarded. In Yu (2005) studied that forecasting must devote responsibility for recurring fuzzy connections and provide varied weights to different fuzzy interactions. This study, focus on modifying the weighted FTS algorithm used by Yu (2005), namely by exchanging the weighted with the first weighted with the last weighted. Furthermore, the researcher used interval ratio algorithm to determine the universe of discourse and partition of universe of discourse to get automatically two variables in build universe of discourse, frequency density partitioning to get optimal interval, and apply it to forecasting coffee production in Indonesia.

B. METHODS

In this part discuss base fuzzy set S. M. Chen (1996), interval ratio algorithm Huarng & Yu (2006), frequency density partitioning Jilani et al. (2007), and weights are used to quantify the significance of an ordered FLR Yu (2005) with modification by the researcher. The method of this study as follows:

1. Defined the universe of discourse using intervals ratio algorithm automatically got universe of discourse and some partitioning intervals.
2. Partitioning intervals to get sub-interval using frequency density partitioning based on (Jilani et al., 2007) study.
3. Fuzzyfication based on base fuzzy set.
4. Build FLR and FLRG based on (S. M. Chen, 1996) study.
5. Assign modified Weighted Fuzzy Time Series (WFTS).

Weighted Fuzzy Time Series (WFTS) was an extension of FTS methodology. WFTS addresses the issue of recurring FLR by assigning weights to each fuzzy relation. These weights were used to quantify the significance of an ordered FLR (Yu, 2005). The FLR which belong to the same LHS can be organized into FLRG by establishing their RHS intact as the RHS in the

FLRG. The formula used with modification for determining the weight was defined in the following approach:

$$W(t) = [w_n \quad w_{n-1} \quad \cdots \quad w_1] = \left[\frac{j}{j+k+l+m} \quad \frac{k}{j+k+l+m} \quad \cdots \quad \frac{p}{j+k+l+m}\right]$$
$$= \left[\frac{c_n}{c_1+c_2+c_3+c_4} \quad \frac{c_{n-1}}{c_1+c_2+c_3+c_4} \quad \cdots \quad \frac{c_1}{c_1+c_2+c_3+c_4}\right] \qquad (1)$$
$$= \left[\frac{c_n}{\sum_{h=1}^{n} c_h} \quad \frac{c_{n-1}}{\sum_{h=1}^{n} c_h} \quad \cdots \quad \frac{c_1}{\sum_{h=1}^{n} c_h}\right]$$

As for the Flowcart of study, as shown in Figure 1.

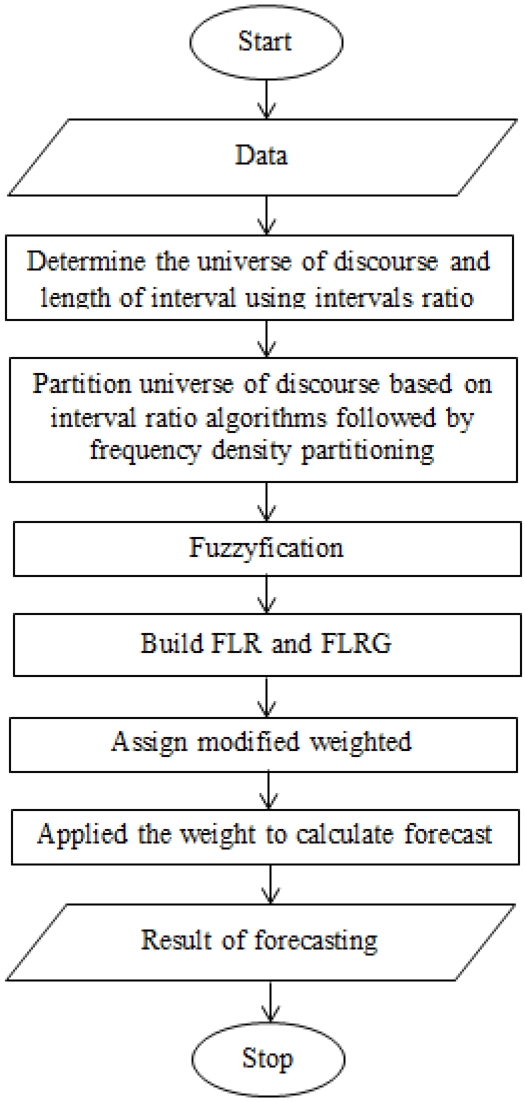

Figure 1. Flowcart of study

Model tested using The Average Forecasting Error Rate (AFER). AFER also known as the Mean Average Percentage Error (MAPE), was used in order to conduct an evaluation of the outcomes of the forecasting efforts that have been carried out. Due to the fact that it provides a fairly straightforward understanding of relative error issues, AFER was frequently utilized in the process of analyzing the error of a prediction (Tahseen Ahmed Jilani dkk., 2007).

$$AFER = \frac{\sum_{t=1}^{n} \frac{|A_t - F_t|}{A_t}}{n} \times 100\% \quad (2)$$

In the given context, A_t represents the actual value of the time series data at a specific time point "t", while "F_t" represents the predicted values of the time series data at the same time the issue. For n denotes the total number of observations in the time series data. These symbols are used according to the criteria, as shown in Table 1.

Table 1. AFER Criteria (Lewis, 1982)(H. C. Chen et al., 2013)

AFER	Criteria of Accuracy
<10%	Excellent
10%-19%	Good
20%-49%	Reasonable
>50%	Not accuracy

C. RESULT AND DISCUSSION

Forecasting with interval ratio-frequency density with modified weighted FTS as follows:

1. Step 1: Collect coffee production data period 2000-2021 in Indonesia from *Badan Pusat Statistika* website as a universe of discourse (U).
2. Step 2: Define the greatest value ($D_{max} = 769$) and smallest values ($D_{min} = 560.4$). Then, follow the interval ratio algorithm. The universe of discourse and partition values from the universe of discourse are automatically obtained by following the interval ratio algorithm. The values of interval ratio algorithm is shown in Table 2.

Table 2. Partition Interval using Interval Ratio Algorithm

	Partition Interval
u_1	[550, 567.875]
u_2	[567.875, 585.331]
⋮	⋮
u_{11}	[757.292, 781.904]

3. Step 3: Partition the universe of discourse. The value of the universe of discourse U = [550,781.904] is obtained from Table 2. Partition intervals in Table 2 are redivided using frequency density partitioning rule, the values are shown in Table 3.

Table 3. Sub-interval based on frequency density partitioning rule

Partition Interval	**Frequency**	u_i	**Sub-interval**	**Description**
[550, 567.875]	1	u_1	[550, 567.875]	consistent
[567.875, 585.331]	1	u_2	[567.875, 585.331]	consistent
[585.331, 605.387]	1	u_3	[585.331, 605.387]	consistent
[605.387, 625.062]	4	u_4	[605.387, 611.945]	Interval [605.387, 625.062] has second frequency, then it is divided to be three sub-intervals
		u_5	[611.945, 618.503]	
		u_6	[618.503, 625.062]	
[625.062, 645.376]	3	u_7	[625.062, 635.219]	Interval [625.062, 645.376] has third frequency, then it is divided to be two sub-intervals
		u_8	[635.219, 645.376]	
[645.376, 666.351]	6	u_9	[645.376, 650.620]	Interval [645.376, 666.351] has greatest frequency, then it is divided to be four sub-intervals
		u_{10}	[650.620, 655.864]	
		u_{11}	[655.864, 661.107]	
		u_{12}	[661.107, 666.351]	
[666.351, 688.007]	2	u_{13}	[666.351, 688.007]	consistent
[688.007, 710.368]	0		removed	removed
[710.368, 733.455]	1	u_{14}	[710.368, 733.455]	consistent
[733.455, 757.292]	2	u_{15}	[733.455, 757.292]	consistent
[757.292, 781.904]	1	u_{16}	[757.292, 781.904]	consistent

Then, we get the middle point (m_i) of each interval by carrying out as follow:

Table 4. Middle Value

u_i	m_i
u_1	558.938
u_2	577.103
⋮	⋮
u_{16}	769.60

4. Step 4: Fuzzyfication. Moreover, the process of defining fuzzy sets involves the creation of 16 fuzzy sets that can be derived from the given universe of discourse. The fuzzy sets that have been formed are as follows:

$$A_1 = \left\{ {}^{1}/_{u_1} + {}^{0.5}/_{u_2} + {}^{0}/_{u_3} + \cdots + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} \right\} \quad (3)$$

$$A_2 = \left\{ {}^{0.5}/_{u_1} + {}^{1}/_{u_2} + {}^{0.5}/_{u_3} + \cdots + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} \right\}$$

$$\vdots$$

$$A_{16} = \left\{ {}^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + \cdots + {}^{0.5}/_{u_{15}} + {}^{1}/_{u_{16}} \right\}$$

The following step is to execute fuzzification, and the results can be seen in Table 5 below.

Table 5. The Results of Fuzzification

Year	Actual Data (thousand ton)	Fuzzy Data
2000	585.2	A_2
2001	560.4	A_1
⋮	⋮	⋮
2021	769	A_{16}

5. Step 5: Build FLR and FLRG. The following step is establishing the FLR and FLRG, given in Table 6 and Table 7, respectively:

Table 6. FLR

Year	FLR
2000	
2001	$A_2 \rightarrow A_1$
2002	$A_1 \rightarrow A_{10}$
⋮	⋮
2021	$A_{15} \rightarrow A_{16}$

Table 7. FLRG for $\boldsymbol{k} = \mathbf{1}$ and $\boldsymbol{l} = \mathbf{1}$

Group	FLRG
1	$A_1 \rightarrow A_{10}$
2	$A_2 \rightarrow A_1$
3	$A_3 \rightarrow A_7$
4	$A_4 \rightarrow \#$
5	$A_5 \rightarrow A_5, A_{10}, A_{12}, A_{13}$
⋮	⋮
16	$A_{16} \rightarrow \#$

6. Step 6: Assign modified Weighted Fuzzy Time Series (WFTS). Set the weights, by using the modification method of the weighted FTS to find the weight matrix of the fuzzy logic relations as follows:

Group 1: with the weight matrix $W_{(t)} = [w_{10}] = [1]$
Group 2: with the weight matrix $W_{(t)} = [w_1] = [1]$

$$\vdots$$

Group 5: with the weight matrix $W_{(t)} = [w_5, w_{10}, w_{12}, w_{13}] = \left[\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}\right]$

$$\vdots$$

Group 16: with the weight matrix $W_{(t)} = [w_{\#}] = [1]$

The prediction will be given next. For example, actual data in 2004 is 618.2 with fuzzyfication A_5. FLR is used to build FLRG as $A_5 \rightarrow A_5, A_{10}, A_{12}, A_{13}$. If $F(t-1) = A_5$, the value for forecasting $F(t) = A_5, A_{10}, A_{12}, A_{13}$. Final forecasting value is calculate as follow:

$$final(t) = M(t) \times W(t)^T$$
$$final(t) = [m_5, m_{10}, m_{12}, m_{13}] \times \left[\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}\right]^T \tag{4}$$
$$final(2004) = 637.9693$$

As for the Forecasting Value, as shown in Table 8.

Table 8. Forecasting Value

Year	Actual Data (thousand ton)	$F(t)$
1	585.2	Na
2	560.4	655.8636
3	654.3	658.6218
4	645.0	618.5034
5	618.2	637.9693
⋮	⋮	⋮
16	769	769.5979

The final step is to determine the accuracy of the prediction by utilizing AFER formula. The AFER values for interval ratio-frequency density based on a modified weighted FTS, come in at 1.24%. The following graphic presents the results of forecasting utilizing interval ratio-frequency density based on a modified weighted FTS. These findings are presented in Figure 2 below.

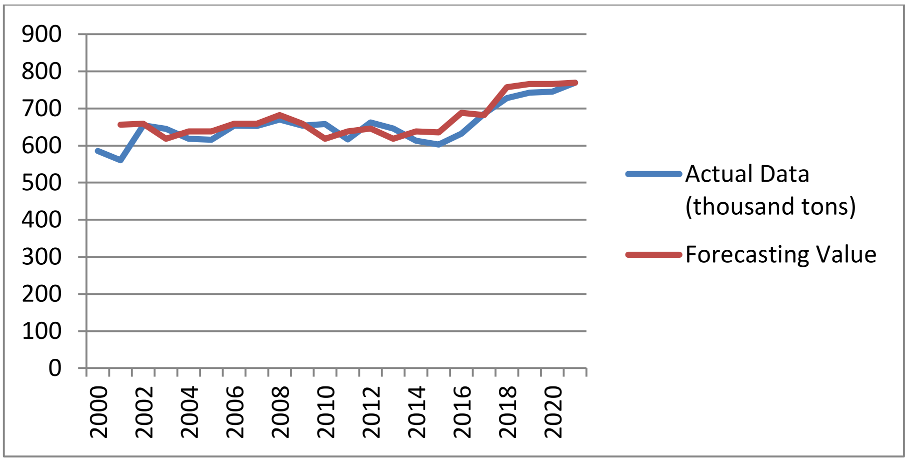

Figure 2. Graph of Forecasting Results Using Interval Ratio Based on Modified Weighted FTS

Based on Figure 2, the graph in blue showed the actual data, meanwhile the graph in red showed the forecasting results using the interval ratio followed by frequency density partitioning with modifying the weights. The graph showed a pattern that was almost the same as coffee production data, although the resulting predicted values are not the same.

Table 9. Comparision proposed method with existing method

Evaluated criteria	Huarng's method	Jilani's method	Proposed method
AFER	2.54%	1.51%	1.24%

In **Error! Reference source not found.** showed that AFER value 1.24% for proposed method has smaller error than existing method in Huarng and Jilani method with AFER value 2.54% and 1.51%, respectively.

D. CONCLUSION AND SUGGESTIONS

Forecasting coffee production in Indonesia using Interval Ratio-Frequency Density based on Modified Weighted FTS has a excellent level of accuracy, this can be seen from the AFER value obtained which is 1.24%. For the next researcher improved the accuracy and implemented in field other than coffee production.

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