Bi-Hyperstructures in Chemical Hyperstructures of Redox Reactions with Three and Four Oxidation States

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ABSTRACT

Hyperstructures find numerous applications across various disciplines. One notable application is in chemistry, particularly in the context of chemical reactions. In 2014, Davvaz introduced the concept of bi-hyperstructures, but their application specifically in chemical reactions, has yet to be thoroughly explored in previous studies. Thus, the primary aim of this paper is to examine and analyze the different types of bi-hyperstructures present within chemical hyperstructures. The scope of this study focuses on two types of chemical hyperstructures: redox reactions and reactions in electrochemical cells. Within these chemical hyperstructures, we investigate the possibility of bi-hyperstructures among bi-semihypergroups, bi-hypergroups, bi-H_v-semigroups, and bi-H_v-groups. Next, some properties of bi-hyperstructures related to hyperstructures are also investigate.

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A. INTRODUCTION

Hyperstructures are generalizations of algebraic structures. Marty initially introduced the concept in 1934 (Marty, 1934). Later, Vougiouklis (1994) furthered generalized hyperstructures into structures known as H_v-Structures. Prominent researchers in hyperstructures, including Corsini, Davvaz, Leoreanu-Fotea, Vougiouklis, and Cristea have authored books to facilitate research in this field (Corsini and Leoreanu-Fotea, 2003; Davvaz & Cristea, 2015; Davvaz, 2016; Davvaz and Vougiouklis, 2019; Davvaz and Leoreanu-Fotea, 2022). Moreover, hyperstructures have numerous applications in various mathematical fields, such as geometry, cryptography, and coding theory (Corsini and Leoreanu-Fotea, 2003).

In addition to mathematics, hyperstructures have applications in scientific disciplines like physics, biology, and chemistry. In physics, hyperstructures are utilized for analyzing different types of hyperstructures in physical particles, particularly leptons (Nezhad et al., 2012). In biology, hyperstructures are employed to study hyperstructures related to inheritance Davvaz et al. (2013); Al Tahan & Davvaz (2017), where fuzzy sets and intuitionistic fuzzy sets play a

significant role (Al-Tahan and Davvaz, 2019; Al-Tahan & Davvaz, 2019a; Leoreanu-Fotea et al., 2023). Furthermore, hyperstructures are used in chemistry to analyze types of hyperstructures in chemical reactions. Research on hyperstructures in chemical reactions initiated by Davvaz and Dehghan-Nezhad in 2003 (Davvaz and Dehghan-Nezhad, 2013). Studies related to hyperstructures in chemical reactions encompass dismutation reactions (Davvaz et al., 2012), redox reactions (Davvaz, Dehghan Nezhad, et al., 2014), redox reactions with three oxidation numbers (S. C. Chung et al., 2014), four oxidation numbers (Al-Jinani et al., 2019; Al-Tahan and Davvaz, 2022), and examples of redox reactions with five oxidation states (Agusfrianto et al., n.d.). Additionally, hyperstructure types have been analyzed concerning reactions in electrochemical cells (M. A. Al-Tahan & Davvaz, 2018), ozone depletion reactions (Chung, 2019), ozone depletion reactions in the stratosphere (Chung and Chun, 2020), and salt formation reactions (Heidari et al., 2019).

Furthermore, in 2014, Davvaz introduced the concept of bi-hyperstructures (Davvaz et al., 2014). Since research has yet to be conducted on bi-hyperstructures in chemical hyperstructures, this paper aims to analyze the different types of bi-hyperstructures present in chemical hyperstructures. This study focuses on chemical hyperstructures within redox reactions with three and four oxidation states. Furthermore, from the results of bi-hyperstructures in redox reactions with three and four oxidation states, theorems regarding the relationship between bi-hyperstructures and hyperstructures that have not been studied in previous studies are also obtained.

B. METHODS

The following theories are sourced from Davvaz et al. (2014), Davvaz and Vougiouklis (2019), (Leoreanu-Fotea, 2022). Let $\mathfrak{L}$ be a nonempty set and define a mapping $\ominus: \mathfrak{L} \times \mathfrak{L} \to P(\mathfrak{L})\backslash\{\emptyset\}$ where $P(\mathfrak{L})$ represent all subsets of $\mathfrak{L}$. Then, the mapping " $\ominus$ " is referred to as a hyperoperation on $\mathfrak{L}$ and the mathematical system $(\mathfrak{L},\ominus)$ is called a hypergroupoid. If " $\ominus$ " is associative, i.e., for every $\mathfrak{l}, \mathfrak{m}, \mathfrak{n} \in \mathfrak{L}$, $\mathfrak{l} \ominus (\mathfrak{m} \ominus \mathfrak{n}) = (\mathfrak{l} \ominus \mathfrak{m}) \ominus \mathfrak{n}$, then $(\mathfrak{L},\ominus)$ is called a semihypergroup. If $(\mathfrak{L},\ominus)$ satisfies reproduction axiom, which states that for every $\mathfrak{u} \in \mathfrak{L}$, $\mathfrak{u} \ominus \mathfrak{L} = \mathfrak{L} \ominus \mathfrak{u} = \mathfrak{L}$, then $(\mathfrak{L},\ominus)$ is called a hypergroup. Furthermore, a hyperoperation " $\ominus$ " is considered weakly associative if for every $\mathfrak{l}, \mathfrak{m}, \mathfrak{n} \in \mathfrak{L}, \mathfrak{l} \ominus (\mathfrak{m} \ominus \mathfrak{n}) \cap (\mathfrak{l} \ominus \mathfrak{m}) \ominus \mathfrak{n} \neq \emptyset$. $(\mathfrak{L},\ominus)$ is called a H_v-semigroup if it is weakly associative and a H_v-semigroup $(\mathfrak{L},\ominus)$ is called a H_v-group if it satisfies the reproduction axiom.

On the other hand, let $\mathfrak{L}$ be a semihypergroup (hypergroup, H_v-semigroup, or H_v-group), with " $\ominus$ " and " $\odot$ " representing hyperoperations on $\mathfrak{L}$. Then, $\mathfrak{L}$ is called bi-semihypergroup (hypergroup, H_v-semigroup, or H_v-group) if there exist proper subsets $\mathfrak{M}$ and $\mathfrak{N}$ of $\mathfrak{L}$ satisfying the following condition: (1) $\mathfrak{L} = \mathfrak{M} \cup \mathfrak{N}$, (2) $(\mathfrak{M},\ominus)$ is a semihypergroup (hypergroup, H_v-semigroup, or H_v-group), and (3) $(\mathfrak{N},\odot)$ is a semihypergroup (hypergroup, H_v-semigroup, or H_v-group). Next, the research methods that used in this study is literature studies. The research process in this paper is given as follows.

1. Take a chemical reaction that wants to analyze the type of bi-hyperstructures. These are chemical reactions with three and four oxidation states.
2. Define a hyperoperation table whose elements are the reactants and products of the chemical reaction.

3. Analyze the different types of bi-hyperstructures.
4. Based on the bi-hyperstructures analysis results obtained, connect the concept of bi-hyperstructures to hyperstructures. Write the result in the form of a theorem. For a more concise process, consider the research process diagram in Figure 1.
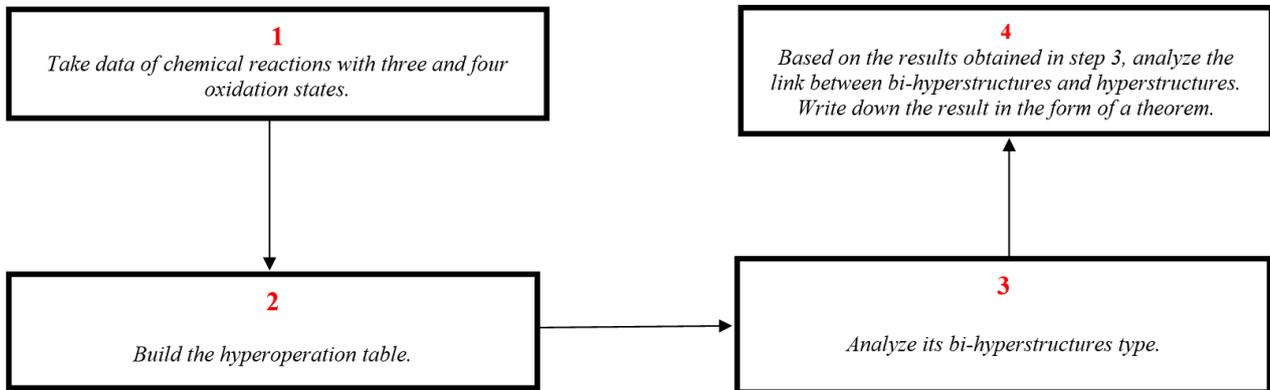

Figure 1. Research Process Diagram

C. RESULTS AND DISCUSSION

In this section, we present the research results pertaining to the analysis of bi-hyperstructures in redox reactions with three and four oxidation states.

1. Bi-Hyperstructures in Chemical Redox Reactions

a. Bi-Hyperstructures in Chemical Redox Reactions with Three Oxidation States. If $\mathfrak{P}, \mathfrak{Q}$, and $\mathfrak{R}$ are elements, then we have following Latimer diagram.

$$\mathfrak{P} \rightarrow_\alpha \mathfrak{Q} \rightarrow_\beta \mathfrak{R}$$
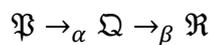
where α and β are represent the potential in volts. Furthermore, if p, q, r are oxidation states of $\mathfrak{P}, \mathfrak{Q}$, and $\mathfrak{R}$ respectively, then we have the relation $p > q > r$. Now, let $Z = \{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\}$ and define a hyperoperation " $\cdot$ " as the product of the reaction that occurs between two elements in Z. Based on Chun, et al., (2014), there are two conditions.

First Condition: If $\alpha > \beta$, then we have the following Table 1 (Chun, et al., 2014).

Table 1. $(Z, \cdot)$

$\cdot$	$\mathfrak{P}$	$\mathfrak{Q}$	$\mathfrak{R}$
$\mathfrak{P}$	$\mathfrak{P}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\mathfrak{Q}$
$\mathfrak{Q}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$
$\mathfrak{R}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$

Accordingly, we have the following Theorem 3.1.1 related to type of bi-hyperstructure of mathematical system $(Z, \cdot)$.

Theorem 3.1.1 $(Z, \cdot)$ is a bi-hypegroup.
Proof. Since $Z = \{\mathfrak{P}, \mathfrak{Q}\} \cup \{\mathfrak{Q}, \mathfrak{R}\}$, $(\{\mathfrak{P}, \mathfrak{Q}\}, \cdot)$ and $(\{\mathfrak{Q}, \mathfrak{R}\}, \cdot)$ are hypergroups, therefore $(Z, \cdot)$ is a bi-hypergroup.

Second Condition: If $\beta > \alpha$, then we have the following Table 2 (Chun, et al., 2014).

Table 2. $(Z, \boxdot)$

$\boxdot$	$\mathfrak{P}$	$\mathfrak{Q}$	$\mathfrak{R}$
$\mathfrak{P}$	$\mathfrak{P}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$
$\mathfrak{Q}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$
$\mathfrak{R}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\mathfrak{R}$

Then, based on Table 2, the following Remark 3.1.2 is obtained.

Remark 3.1.2 $(Z, \boxdot)$ does not form a bi-hyperstructure. Only $(\{\mathfrak{P}, \mathfrak{R}\}, \boxdot)$ forms a hyperstructure, which is hypergroup. However, $(\{\mathfrak{P}, \mathfrak{Q}\}, \boxdot)$ and $(\{\mathfrak{Q}, \mathfrak{R}\}, \boxdot)$ does not form bi-hyperstructures. Now, let's consider the following example.

Example 3.1.3 Based on Douglas et al. (1992), the Latimer diagram of Niobium (Nb) and Silicon (Si) are given as follows:

$$Nb_2O_5 \rightarrow_{-0.1} Nb^{3+} \rightarrow_{-1.1} Nb$$
$$SiO_3^{2-} \rightarrow_{-1.69} Si \rightarrow_{-0.93} SiH_4$$
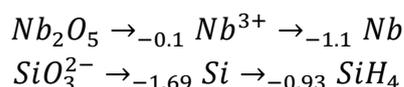
Let $Z = \{Nb_2O_5, Nb^{3+}, Nb\}$ and $\mathcal{Z} = \{SiO_3^{2-}, Si, SiH_4\}$ and define the hyperoperation " $\boxplus$ " and " $\boxminus$ " as the product of reactions that occur between two elements in Z and $\mathcal{Z}$ respectively. Then, $(Z, \boxplus)$ forms a bi-hypergroup and $(\mathcal{Z}, \boxminus)$ does not form a bi-hyperstructures.

b. Bi-Hyperstructures in Chemical Redox Reactions with Four Oxidation States. If $\mathfrak{P}, \mathfrak{Q}$, and $\mathfrak{R}$ are elements, then we have following Latimer diagram.
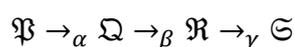
$$\mathfrak{P} \rightarrow_\alpha \mathfrak{Q} \rightarrow_\beta \mathfrak{R} \rightarrow_\gamma \mathfrak{S}$$

where α, β, and γ are the potential in volts. Furthermore, if p, q, r, s are oxidation states of $\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}$ and $\mathfrak{S}$ respectively, we have the relation $p > q > r > s$. Now, let $W = \{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}$ and define a hyperoperation "$\uplus$" as product of reaction that occurs between two elements in W. Based on Jinani, et al., (2019) and Al-Tahan and Davvaz, (2022), we have all following conditions.

First Condition: If $\alpha > \beta > \gamma$, then we have the following Table 3 (Al-Tahan and Davvaz, 2022).

Table 3. $(W, \uplus)$ for $\alpha > \beta > \gamma$

$\uplus$	$\mathfrak{P}$	$\mathfrak{Q}$	$\mathfrak{R}$	$\mathfrak{S}$
$\mathfrak{P}$	$\mathfrak{P}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$
$\mathfrak{Q}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$
$\mathfrak{R}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$
$\mathfrak{S}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$	$\mathfrak{S}$

Then, we the have following Theorem 3.1.4 related to type of bi-hyperstructure of the mathematical system $(W, \uplus)$ when $\alpha > \beta > \gamma$.

Theorem 3.1.4 $(W, \uplus)$ is a bi-H_v-semigroup.
Proof. Since $W = \{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\} \cup \{\mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}$, $(\{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\}, \uplus)$ is a H_v -semigroup, and $(\{\mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}, \uplus)$ is a H_v-semigroup, then $(W, \uplus)$ is a bi-H_v-semigroup.

Corollary 3.1.5 $(\{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\}, \uplus)$ and $(\{\mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}, \uplus)$ are bi-hypergroups.

Second Condition: If $\alpha > \beta = \gamma$, then we have the following Table 4 (Al-Jinani, et al., 2019).

Table 4. $(W, \uplus)$ for $\alpha > \beta = \gamma$

$\uplus$	$\mathfrak{P}$	$\mathfrak{Q}$	$\mathfrak{R}$	$\mathfrak{S}$
$\mathfrak{P}$	$\mathfrak{P}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{Q}, \mathfrak{R}\}$
$\mathfrak{Q}$	$\{\mathfrak{P}, \mathfrak{Q}\}$	$\mathfrak{Q}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$
$\mathfrak{R}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$
$\mathfrak{S}$	$\{\mathfrak{Q}, \mathfrak{R}\}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$	$\mathfrak{S}$

Then, we the have following Theorem 3.1.6 related to type of bi-hyperstructure of the mathematical system $(W, \uplus)$ when $\alpha > \beta = \gamma$.

Theorem 3.1.6 $(W, \uplus)$ is a bi-hypergroup. Proof. It follows from Theorem 3.1.1

Corollary 3.1.7 $(\{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\}, \uplus)$ and $(\{\mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}, \uplus)$ are bi-hypergroups.

Third Condition: If $\alpha = \gamma > \beta$, then we have following Table 5 (Al-Jinani, et al., 2019).

Table 5. $(W, \uplus)$ for $\alpha > \gamma > \beta$

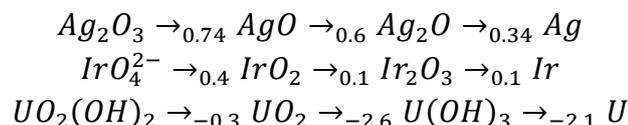| $\uplus$	$\mathfrak{P}$	$\mathfrak{Q}$	$\mathfrak{R}$	$\mathfrak{S}$
$\mathfrak{P}$	$\mathfrak{P}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\mathfrak{R}$
$\mathfrak{Q}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\mathfrak{R}$
$\mathfrak{R}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\{\mathfrak{P}, \mathfrak{R}\}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$
$\mathfrak{S}$	$\mathfrak{R}$	$\mathfrak{R}$	$\{\mathfrak{R}, \mathfrak{S}\}$	$\mathfrak{S}$

Then, we the have following Remark 3.1.8 related to type of bi-hyperstructure of the mathematical system $(W, \uplus)$ when $\alpha > \gamma > \beta$.

Remark 3.1.8 $(W, \uplus)$ does not form a bi-hyperstructure.

Remark 3.1.9 $(\{\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}\}, \uplus)$, $(\{\mathfrak{P}, \mathfrak{Q}, \mathfrak{S}\}, \uplus)$, and $(\{\mathfrak{Q}, \mathfrak{R}, \mathfrak{S}\}, \uplus)$ do not form bi-hyperstructures.
Now, let's consider the following example.

Example 3.1.10 Based on Douglas et al. (1992), the Latimer diagram of silver (Ag), Iridium (Ir), and Uranium (U) are given as follows.

$$Ag_2O_3 \to_{0.74} AgO \to_{0.6} Ag_2O \to_{0.34} Ag$$
$$IrO_4^{2-} \to_{0.4} IrO_2 \to_{0.1} Ir_2O_3 \to_{0.1} Ir$$
$$UO_2(OH)_2 \to_{-0.3} UO_2 \to_{-2.6} U(OH)_3 \to_{-2.1} U$$

Let $\Gamma_1 = \{Ag_2O_3, AgO, Ag_2O, Ag\}$, $\Gamma_2 = \{IrO_4^{2-}, IrO_2, Ir_2O_3, Ir\}$, and $\Gamma_3 = \{UO_2(OH)_2, UO_2, U(OH)_3, U\}$. Next, define hyperoperations "$\boxdot_1$", "$\boxdot_2$", and "$\boxdot_3$" as the product of the most positive spontaneous reactions that occur in Γ_1, Γ_2, and Γ_3 respectively. Thus, $(\Gamma_1, \boxdot_1)$ forms a bi-H_v-semigroup, $(\Gamma_2, \boxdot_2)$ forms a bi-hypergroup, and $(\Gamma_3, \boxdot_3)$ does not form a hyperstructure.

2. Some Relations Between Hyperstructures and Bi-Hyperstructures

Based on the results obtained in Section 1, the following properties hold. First, we have properties about relation between bi-hyperstructures and hyperstructures.

Theorem 3.2.1 Let $(\mathfrak{L}, \ominus)$ be a commutative hypergroupoid and let $\mathfrak{M}$ and $\mathfrak{N}$ be proper subsets of $\mathfrak{L}$ with the same hyperoperation as $\mathfrak{L}$. If $(\mathfrak{L}, \ominus)$ is a bi-semihypergroup (hypergroup, bi-H_v-semigroup, or bi-H_v-group), then $(\mathfrak{L}, \ominus)$ is a commutative semihypergroup (hypergroup, H_v-semigroup, or H_v-group).

Proof. If $(\mathfrak{L}, \ominus)$ is a commutative bi-hypergroup, then $\mathfrak{L} = \mathfrak{M} \cup \mathfrak{N}$, and both $(\mathfrak{M}, \ominus)$ and $(\mathfrak{N}, \ominus)$ is are commutative hypergroups. Since both $(\mathfrak{M}, \ominus)$ and $(\mathfrak{N}, \ominus)$ are hypergroups, it follows that $(\mathfrak{L}, \ominus)$ is a hypergroup since $\mathfrak{M}$ and $\mathfrak{N}$ are subsets of $\mathfrak{L}$, i.e., associativity and reproduction axiom also prevailing in $\mathfrak{L}$. The proof for other conditions is similar.

Remark 3.2.2 The converse of Theorem 3.2.1 is not necessarily true. For instance, based on Table 5, $(W, \uplus)$ is a H_v-semigroup, but does not form a bi-hyperstructure.

Furthermore, we have the following Theorem 3.2.3 about type of bi-hyperstructure when the hyperstructure is isomorphic.

Theorem 3.2.3 Let $(\mathfrak{L}_1, \ominus_1)$ and $(\mathfrak{L}_2, \ominus_2)$ be commutative semihypergroups (hypergroups, H_v-semigroups, or H_v-groups). If $(\mathfrak{L}_1, \ominus_1) \cong (\mathfrak{L}_2, \ominus_2)$, then $(\mathfrak{L}_1, \ominus_1)$ have same bi-hyperstructures with $(\mathfrak{L}_2, \ominus_2)$.

Proof. The proof of this theorem is obvious.

Remark 3.2.4 The converse of Theorem 3.2.3 is not necessarily true. Let $\mathfrak{L}_1 = \{\mathfrak{p}_1, \mathfrak{q}_1, \mathfrak{r}_1\}$ and $\mathfrak{L}_2 = \{\mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{r}_2\}$. Define hyperoperations "$\ominus_1$" and "$\ominus_2$" as shown in Table 6 and Table 7.

Table 6. $(\mathfrak{L}_1, \ominus_1)$

$\ominus_1$	p_1	q_1	r_1
p_1	p_1	$\{p_1,q_1\}$	$\{p_1,r_1\}$
q_1	$\{p_1,q_1\}$	q_1	$\{q_1,r_1\}$
r_1	$\{p_1,r_1\}$	$\{q_1,r_1\}$	r_1

Table 7. $(\mathfrak{L}_2, \ominus_2)$

$\ominus_2$	p_1	q_1	r_1
p_1	p_1	$\{p_1,q_1\}$	H
q_1	$\{p_1,q_1\}$	q_1	$\{q_1,r_1\}$
r_1	H	$\{q_1,r_1\}$	r_1

Based on Table 6 and Table 7, $(\mathfrak{L}_1,\ominus_1)$ and $(\mathfrak{L}_2,\ominus_2)$ are commutative hypergroups and they have the same bi-hyperstructures, that is bi-hypergroups. However, it easy to see that $(\mathfrak{L}_1,\ominus_1)$ not isomorphic to $(\mathfrak{L}_2,\ominus_2)$.

D. CONCLUSION AND SUGGESTIONS

Based on the explanation above, bi-hyperstructures have been obtained in hyperstructures of redox reactions with three and four oxidation states. The types of bi-hyperstructures that obtained are bi-hypergroup and bi-H_v-semigroup. Additionally, several related properties of bi-hyperstructures have been obtained. These properties include the fact that if a hypergroupoid is a bi-hyperstructure, then it is also a hyperstructure, and if two hyperstructures are isomorphic, then they share the same bi-hyperstructures. However, it should be noted that the converse of these two traits is not necessarily true. For future research, it would be beneficial to analyze the types of bi-hyperstructures in chemical reaction hyperstructures that have not yet been explored in the context of bi-hyperstructures.

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