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Matric Flux Potential in Time Independent Infiltration Problems from a Single Triangular and a Trapezoidal Irrigation Channel

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ABSTRACT

In this paper, steady infiltration problems into a homogeneous soil from a single triangular and trapezoidal irrigation channel are considered. The governing equation is Richard's equation that represents the movement of water in unsaturated soil. It is a non-linear equation and can be solved by linearizing to become a modified Helmholtz equation. Dual Reciprocity Boundary Element Methods (DRBEM) are used in this study to numerically solve the modified Helmholtz equation. Therefore, by using a provided solution, the numerical Matric Flux Potential (MFP) is calculated. This method was applied to the homogeneous soil problem of stationer infiltration from triangular and trapezoidal single irrigation. Both numerical solutions were compared. The result show that the MFP value from the triangular irrigation is higher than the trapezoidal irrigation. This indicates that content water from the triangular irrigation channel is higher than the trapezoidal irrigation channel.

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A. INTRODUCTION

The irrigation system is seen as necessary to overcome the problem of agricultural land with less water intensity. Irrigation as an effort to provide, regulate, and dispose of irrigation water to support agriculture, whose types include surface irrigation, swamp irrigation, underground water irrigation, pump irrigation, and pond irrigation (Nuchsin, 2014a). According to Nuchsin (2014b), surface water sources that can be used as irrigation water sources are former mining or subdued water, waterfalls, river flows, and springs. Moreover, this study is concerned with surface water irrigation that use water sources in the form of river flows. To Munadi et al. (2020), a single irrigation canal is an artificial channel or river of large and small rivers that divide agricultural or plantation areas. This type of irrigation canal is still found in many villages whose irrigation canals must be modernized or concreted. Figure 1 illustrates a single irrigation extracted from Diskominfo Kabupaten Madiun (2020).

Figure 1. Illustration of a single irrigation system.

The governing equation for the infiltration problem is the Richards equation (Chávez-Negrete et al., 2018; List & Radu, 2016; Sun et al., 2013; Tartakovsky et al., 2020; Zha et al., 2019). Zha et al. (2019) reviewed the numerical solution of Richardson–Richards equation for variably saturated flow in soils. Tartakovsky et al. (2020) examined physics-informed deep neural networks for learning parameters and constitutive relationships in subsurface flow problems. Chávez-Negrete et al. (2018) researched the numerical solution of Richards' water flow equation by generalized finite differences. List & Radu (2016) studied iterative methods for solving Richards' equation. Meanwhile, Sun et al. (2013) proposed a fractal Richards' equation to capture the non-Boltzmann scaling of water transport in unsaturated media.

Align with the growth of science, and many studies have been conducted related to infiltration in irrigation canals on homogeneous soils (Azis et al., 2003; Batu, 1978; Clements & Lobo, 2010; Solekhudin & Ang, 2013; Munadi et al., 2019). Azis et al. (2003) examined stationary infiltration from periodic irrigation canals using Boundary Element Methods (BEM). The results show that the matric flux potential associated with the flat strip and semicircular channel are similar; whereas for the particular rectangular channel considered the matric flux potential is subtantially increased in the region adjacent to the channel (Azis et al., 2003). Batu (1978) discussed the timeless infiltration of single and periodic channels. The author obtained results for infinite sets of equally-spaced parallel strip sources by applying superposition to the solution for single strip source. Meanwhile, Clements and Lobo (2010) used BEM to research the problem of time-dependent infiltration in irrigation canals. The results indicate how the distance from the channel influences the speed with which the matric flux potential reaches its steady state value.

Unlike previous researchers, Solekhudin and Ang (2013), Solekhudin (2016, 2018) applied Dual Reciprocity Boundary Element Methods (DRBEM) to the problem of infiltration from periodic irrigation canals. Solekhudin and Ang (2013) proposed DRBEM is tested on problems involving infiltration from different types of periodic channels in a homogeneous soil. Solekhudin (2016) described DRBEM on problem of water infiltration from periodic trapezoidal channels with different types of root-water uptake. Solekhudin (2018) researched a numerical method for time-dependent infiltration from periodic trapezoidal channels with different types of root-water uptake. The results indicate the influence of the distance of points from the channels to reach their steady state values of MFP and suction potential. Meanwhile, Munadi et al. (2019) researched infiltration problems from single channel with various geometrical shapes. The results indicate that up to a certain point of depth, the matrix flux potential or water content of flows on all types differ significantly. After that point, the matrix flux potential or water content of all types are same. Munadi et al. (2020) researched steady infiltration problems into a homogeneous soil froma single trapezoidal channel. The results indicate that the closer location to the impermeable layer, the higher values of suction potential

In other fields, DRBEM is also used as research material. Yu et al. (2021) studied three-dimensional transient heat conduction problems in FGMs via IG-DRBEM, Fendoğlu et al. (2018) researched DBEM and DRBEM solutions to 2D transient convection-diffusion-reaction type equations, The results are presented by equivelocity and current lines for various values of problem parameters, which show the well-known characteristics of the MHD flow. Han Aydin and Tezer-Sezgin (2014) examined about a DRBEM solution for MHD pipe flow in a conducting medium, and Gümgüm and Tezer-Sezgin (2014) studied about DRBEM solution of mixed convection flow of nanofluids in enclosures with moving walls. It is disclosed that the average Nusselt number increases with the increase in volume fraction, and decreases with an increase in both the Richardson number and heat source length.

MFP has become an important research topic (Bigah et al. 2019; Li et al. 2021; Pinheiro et al. 2017). Pinheiro et al. (2017) studied about a matric flux potential approach to assess plant water availability in two climate zones in Brazil. Bigah et al. (2019) examined about development of a steady-state model to predict daily water table depth and root zone soil matric potential of a cranberry field with a subirrigation system. Li et al. (2021) researched about emissions of nitrous oxide, dinitrogen and carbon dioxide from three soils amended with carbon substrates under varying soil matric potentials.

Research on triangular irrigation canals has indeed been carried out by Lobo (2008) but with a different method. This research used BEM (Boundary Element Method), while the research we conducted used DRBEM (Dual Reciprocity Boundary Element Method). This study aimed to determine water content distribution in the soil around triangular and trapezoidal single irrigation. Water content distribution is described by the distribution of potential values of the matric flux potential (MFP). According to Lobo (2008), MFP is defined as the driving force of water flow and is directly proportional to the level of water content. This study will compare the distribution of MFP values of triangular and trapezoidal single irrigation.

B. METHODS

This research is simulation research based on literature studies and theoretical studies. The research steps are as follows.

1. Conduct a study on the concept of irrigation and the infiltration process of a single irrigation.
2. Construct a mathematical model of the problem of water infiltration in a single irrigation.
3. Apply DRBEM to solve the formed governing equation.
   DRBEM is the development of the standard Boundary Element Method (BEM) to solve Helmholtz equation numerically. The initial procedure of DRBEM is to transform the governing equations in the form of a partial differential equation (PDP) into an integral equation contains the fundamental solution of the PDP. The integral equation formed in the form of boundary integral and domain integral equations. The boundary integral equation is then discretized into an integral equation over a number of elements in the form of segments line or segment.

4. Forming the model domain and boundary terms of single triangular and trapezoidal irrigation.
5. Using the Matlab program to help solve problems in this research.
6. Interpret the results obtained from matlab programming.

In general, the research flow chart can be seen in Figure 2.

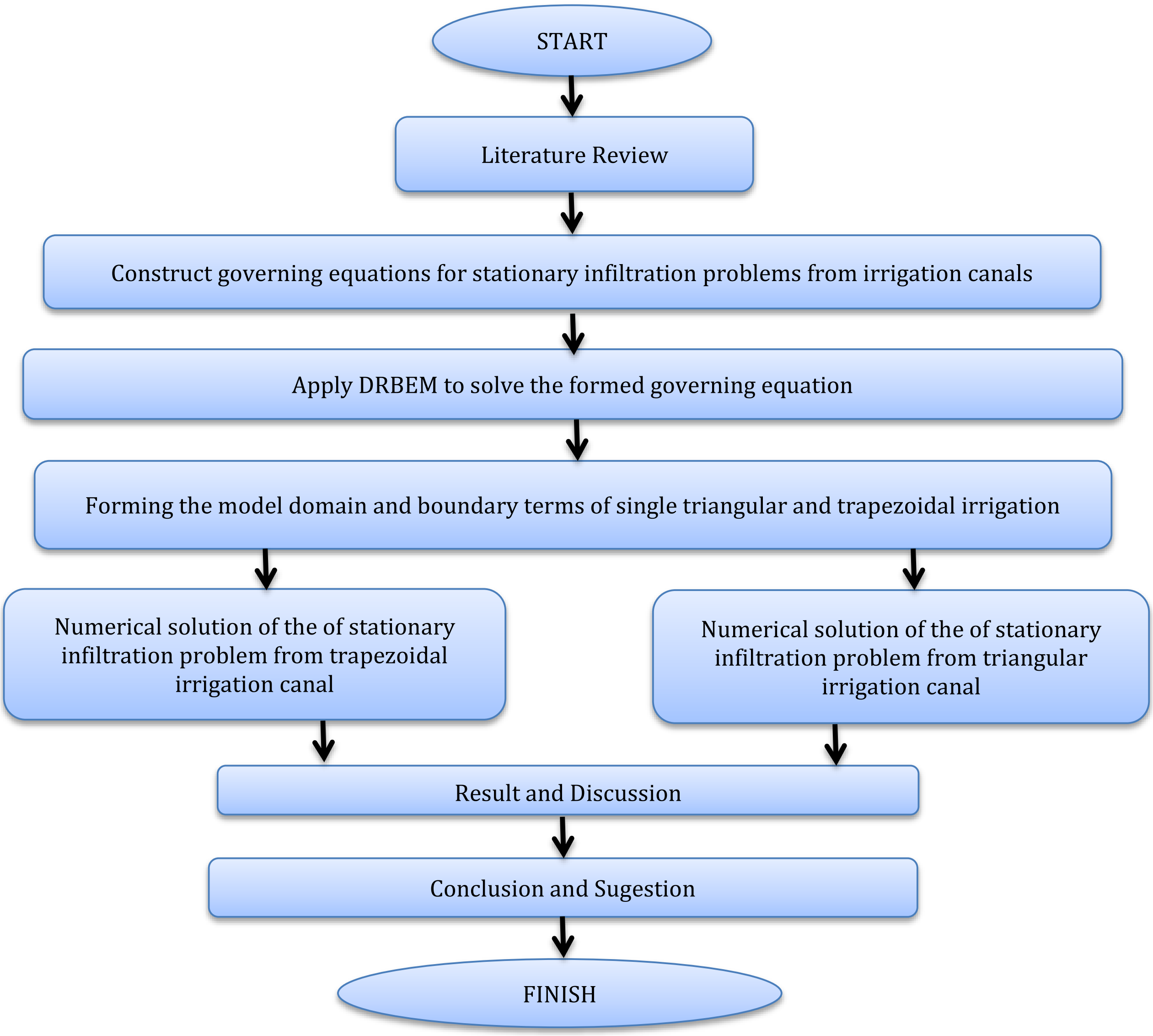

Figure 2. Research flow chart

This study begins with setting the domain and limitation for the infiltration problem of irrigation were compiled based on the following assumptions, namely the cross-sectional length of the channel entered by flux (wet cross-section) is $2L$. Using the 2L value, the width of the two channels is determined to be $\frac{4\pi}{L}$ each. The maximum depth is $\frac{3L}{2\pi}$ and $\sqrt{L^2-\left(\frac{2\pi}{L}\right)^2}$ for trapezoidal and triangular irrigation channels, respectively. Since the channel has an indefinite length, the irrigation's length can be disregarded. Thus, the width-to-length ratio is minimal, the irrigation canal is always filled with water, and there are no additional effects such as root water uptake and pressure saltiness of irrigation water. Except for irrigation canals, there are no other sources of water flow (infiltration), and the flux entering the surface of the canals is constant, or v_0. Defined stationary infiltration problem domains with $-\infty < X < \infty$ and $Z \geq 0$ (positive Z axis pointing down) expressed with R as in Figure 3.

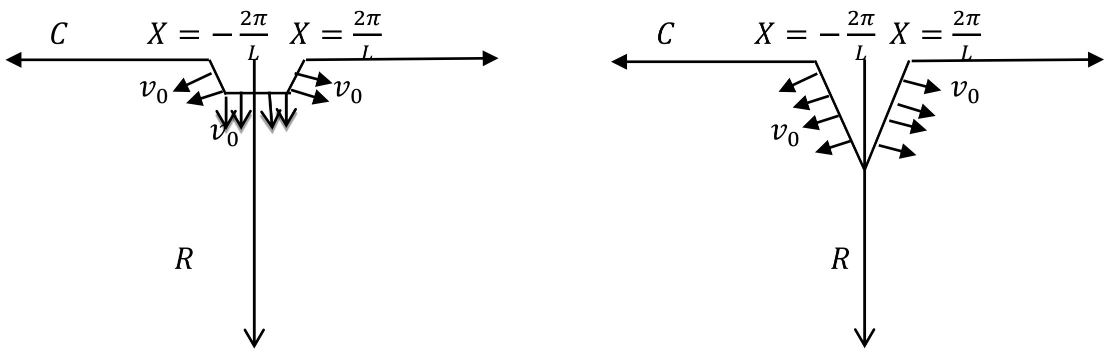

Figure 3. Trapezoidal Single Irrigation (Left) & Triangular Single Irrigation (Right)

C. RESULT AND DISCUSSION

In this study, infiltration is independent of time. A mathematical model that can be used is differential partial non-linear.

$$\frac{\partial}{\partial X}\left(K(\psi)\frac{\partial \psi}{\partial X}\right) + \frac{\partial}{\partial Z}\left(K(\psi)\frac{\partial \psi}{\partial Z}\right) - \frac{\partial K(\psi)}{\partial Z} = 0 \tag{1}$$

where K is the conductivity of hydrology, and ψ is the suction potential. Equation (1) is Richard's equation describing water's movement in two-dimensional unsaturated soil. Next, the solution to equation (1) will be determined. Using the transformations rendered by Kirchhoff (Raats, 1970), MFP is defined as

$$\Theta = \int_{-\infty}^{\psi} K(s)ds \tag{2}$$

where K is represented in exponential relations (Gardner, 1958)

$$K(\psi) = K_0 e^{\alpha\psi}, \quad \alpha > 0 \tag{3}$$

with α is an empirical parameter that describes the degree of roughness of the soil and K_0 is the hydraulic conductivity of saturated soil. Equations (2) and (3) transform equations (1) into the following equations

$$\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Z^2} = \alpha \frac{\partial \Theta}{\partial Z} \tag{4}$$

The horizontal and vertical components of successive fluxes are written as

$$U = -\frac{\partial \Theta}{\partial X} \tag{5}$$

And

$$V = \alpha\Theta - \frac{\partial\Theta}{\partial Z} \tag{6}$$

Normal flux on a surface with a normal vector unit **n** = (n_X, n_Z) is given as

$$F = -\frac{\partial\Theta}{\partial X}n_X + \left(\alpha\Theta - \frac{\partial\Theta}{\partial Z}\right)n_Z \tag{7}$$

The magnitude of the flux on the ground outside the channel is 0, while the quantity of flux coming through the channel is v_0. According to Batu (1978), $X = -\infty$, $X = \infty$ dan $Z = \infty$ in assumption $\Theta \to 0, \frac{\partial\Theta}{\partial X} \to 0$, and $\frac{\partial\Theta}{\partial Z} \to 0$. Then the result to solve the equation are:

$F = -v_0$, on each channel surface , (8)
$F = 0$, on the ground surface outside each channel, (9)
$\Theta = \frac{\partial\Theta}{\partial X} = \frac{\partial\Theta}{\partial Z} = 0,\ X = -\infty$ and $Z \geq 0$,
(10)
$\Theta = \frac{\partial\Theta}{\partial X} = \frac{\partial\Theta}{\partial Z} = 0,\ X = \infty$ and ≥ 0 , (11)
and
$\Theta = \frac{\partial\Theta}{\partial X} = \frac{\partial\Theta}{\partial Z} = 0,\ Z = \infty$ dan $-\infty \leq X \leq \infty$. (12)

Using the following dimensionless variables,

$$x = \frac{\alpha}{2}X; \quad z = \frac{\alpha}{2}Z; \quad \Phi = \frac{\pi\Theta}{v_0 L}\ ; \quad u = \frac{2\pi}{v_0\alpha L}U; \quad v = \frac{2\pi}{v_0\alpha L}V; \quad f = \frac{2\pi}{v_0\alpha L}F\ , \tag{13}$$

and implement transformations

$$\Phi = \phi e^z\ . \tag{14}$$

In equation (4), the following linear partial differential equation is obtained,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \phi\ . \tag{15}$$

Equation (15) is referred to as the modified Helmholtz equation. As a result, the following limit conditions are obtained,

$\frac{\partial\phi}{\partial n} = \frac{2\pi}{\alpha L}e^{-z} + n_2\phi$, on each channel surface, (16)
$\frac{\partial\phi}{\partial n} = -\phi$, on the surface of the soil outside each channel, (17)
$\phi = 0; \frac{\partial\phi}{\partial n} = 0,\ x = -\infty$ and $z \geq 0$, (18)
$\phi = 0; \frac{\partial\phi}{\partial n} = 0,\ x = \infty$ and $z \geq 0$, (19)

and

$$\phi = 0; \frac{\partial \phi}{\partial n} = 0,\ z = \infty \text{ dan } -\infty \leq x \leq \infty\,, \tag{20}$$

with $\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial x} n_x + \frac{\partial \phi}{\partial z} n_z$ Is derivative normal from ϕ.

According to Ang (2007), The Integral equation from (16)-(20) is

$$\lambda(\xi,\eta)\phi(\xi,\eta) = \int_C \left\{\phi(x,z)\frac{\partial}{\partial n}[\varphi(x,z;\xi,\eta)] - \varphi(x,z;\xi,\eta)\frac{\partial}{\partial n}[\phi(x,z)]\right\} ds(x,z)$$

$$+ \iint_R \varphi(x,z;\xi,\eta)\phi(x,z)dx\,dz, \tag{21}$$

with

$$\lambda(\xi,\eta) = \begin{cases} \frac{1}{2}, & \text{if } (\xi,\eta) \text{ placed in part of smooth C} \\ 1, & \text{jika} \quad (\xi,\eta) \in R \end{cases} \tag{22}$$

and

$$\varphi(x,z;\xi,\eta) = \frac{1}{4\pi}\ln((x-\xi)^2 + (z-\eta)^2)\,. \tag{23}$$

Equation (23) is the fundamental solution of Laplace's equation. Equation (21) cannot be solved analytically. Therefore, a numerical method is needed to solve the integral equation. DRBEM was used in this study to overcome the integral equation (Ang, 2007). A simple closed curve must bind the domain to implement DRBEM. Therefore, imposed boundaries will be imposed to replace $-\infty \leq x \leq \infty$ and $z = \infty$ become $-b \leq x \leq b$ dan $z = c$ the value of b and c was chosen to be ten each in this study. As with previous research by Munadi (2021), the type of soil that is the object of this study is Pima Clay Loam (PCL) which has a level of soil roughness $\alpha = 0{,}014\ \text{cm}^{-1}$ (Amoozegar-Fard et al., 1984). This type of soil was chosen because it is widely found in agricultural areas of Indonesia.

For comparison purposes, the cross-sectional length entered by the flux of a single irrigation canal of a trapezoidal and triangular shape is equal, i.e. $2L$ = 200 cm. Using the $2L$ value, it is determined that the width of both channels is $\frac{400}{\pi}$cm. The depth of the trapezoidal shape channel is $\frac{150}{\pi}$ cm and the maximum depth of the triangular shape channel is 77.11 cm. N = 800 and M = 625 are assigned for trapezoidal shape channels; for triangular shape channels, N and M are 800 and 624, respectively. Small differences in M arise due to differences in depth of each channel.

The variation of MFP value toward the dimensionless MFP to depth (Z) can be seen in Figure 4a – 4d for depths of 0 to 500 cm. Figure 4a describes the distribution of MFP values at the subsurface points of the channel, it was taken X = 30 cm. The value of the dimensionless MFP starts from a depth of 50 cm. It can be seen in the figure that the dimensionless MFP value decreases with the increasing depth of points in the soil. This suggests that the water content at shallow soil levels is higher than in deeper parts.

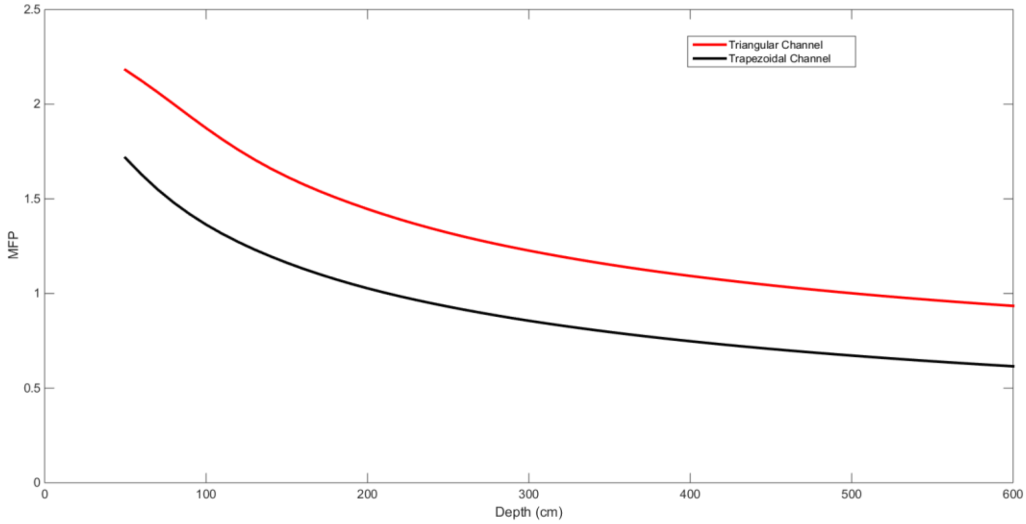

Figure 4a. X = 30cm

Figure 4b shows the distribution of MFP values at X = 70 cm, it was the points near both channels. While Figure 4c shows the distribution of MFP values at X = 110 cm, it was points quite far from the two channels. Both images illustrate the same thing. At first, the MFP value will increase as Z increases, then the value will decrease. No water flow passes through the ground surface, so it is the initial condition. MFP values at the vertical points next to the triangular shape channel are higher than those beside the trapezoidal shape channel. It is easy to understand because the cross-sectional wall of the triangular-shaped channel is longer than the trapezoid-shaped channel's cross-section so that the water flow source is more.

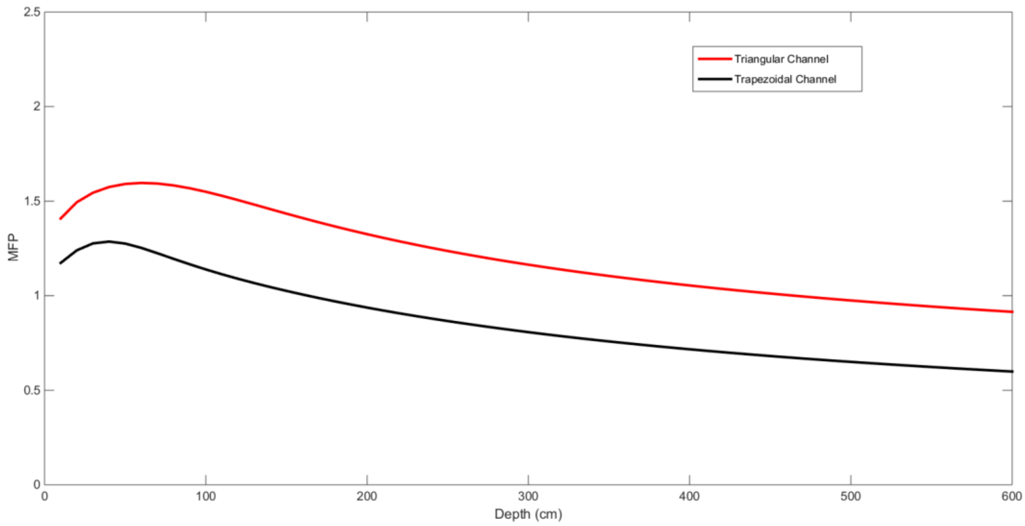

Figure 4b. X = 70cm

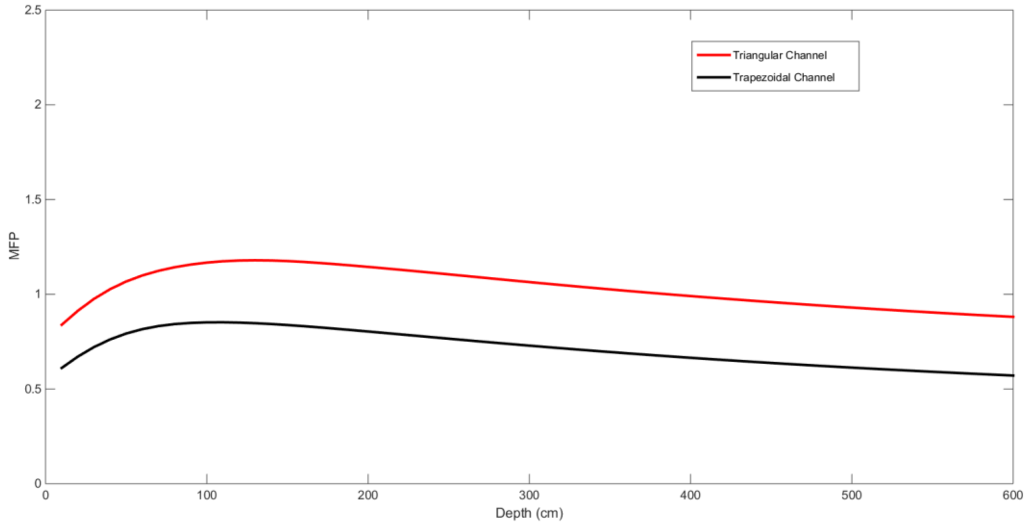

Figure 4c. X = 110cm

However, The 4d figure shows the distribution of MFP values at X = 500 cm, it was points far away from both channels. It is noticed that quite small MFP values rise to a depth of 500 cm. The reason MFP values at vertical points next to triangular channels are higher than those next to trapezoidal channels is the same as for Figures 4b and 4c, namely the presence of more flow sources in triangular channels.

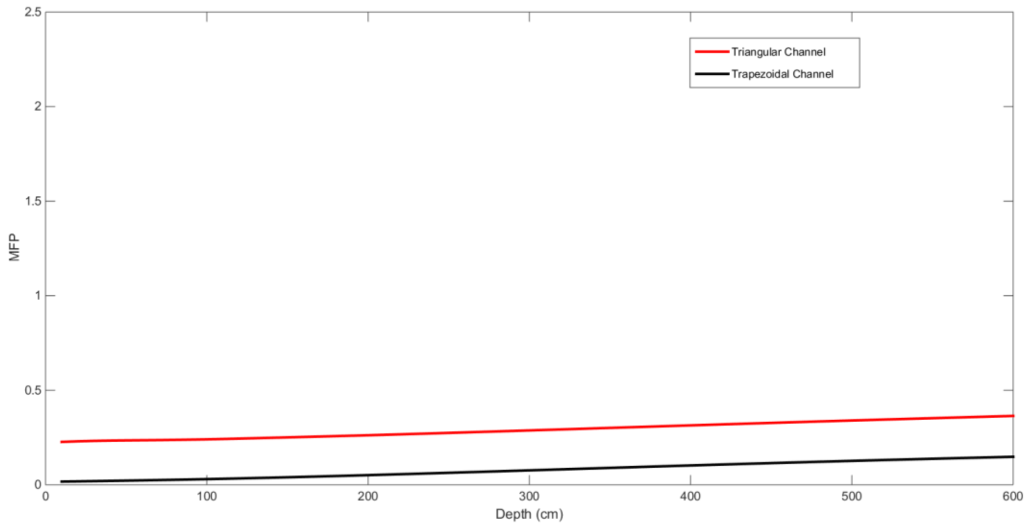

Figure 4d. X = 500cm

Furthermore, it is discussed about the distribution of MFP values by taking points in the ground along the horizontal direction up to 500 cm. Variations in dimensionless MFP values at a depth of 25 cm can be seen in Figure 5a. From the figure, it can be seen that the MFP value of the points at that depth decreases as they increase as they move further away from the channel wall. This suggests that the water content at the soil level close to the channel wall is higher than that of the more distant part. The MFP value of a triangular channel is higher because it has a wider channel wall than a trapezoidal channel, so there are more water flow sources.

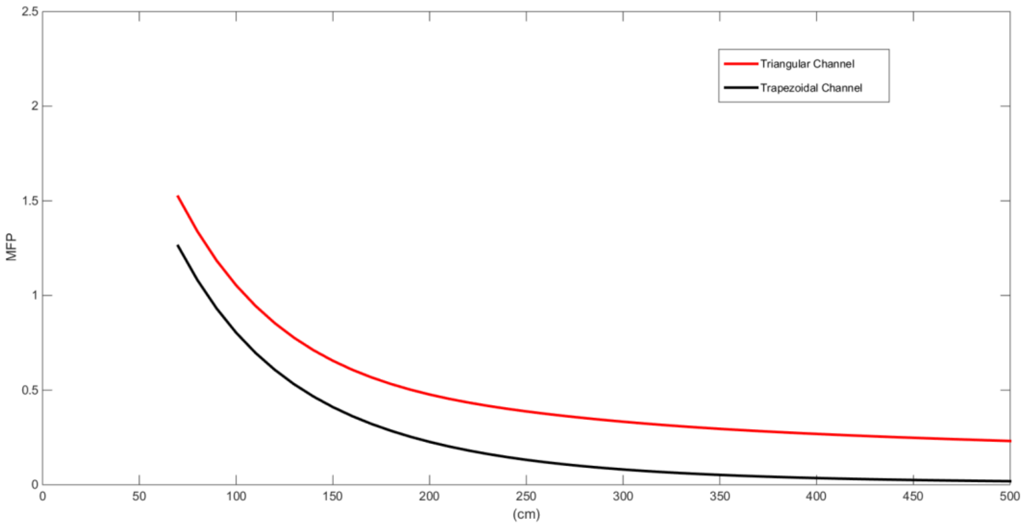

Figure 5a. Z = 25 cm

Figure 5b shows the dots at a depth of 75 cm. As discussed the distribution of MFP values at a depth of 25 cm, it can be seen that the MFP values of points at that depth decrease as they increase further away from the channel wall. This suggests that the water content at the soil level close to the channel wall is higher than in the more distant part. The MFP value of the triangular channel is higher because it has a deeper water flow source (flux) than the trapezoidal channel.

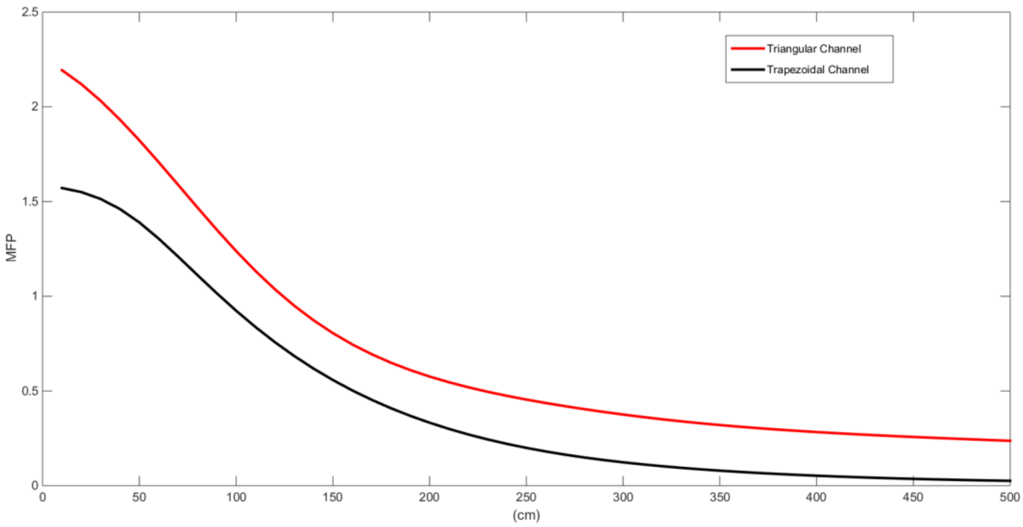

Figure 5b. Z = 75 cm

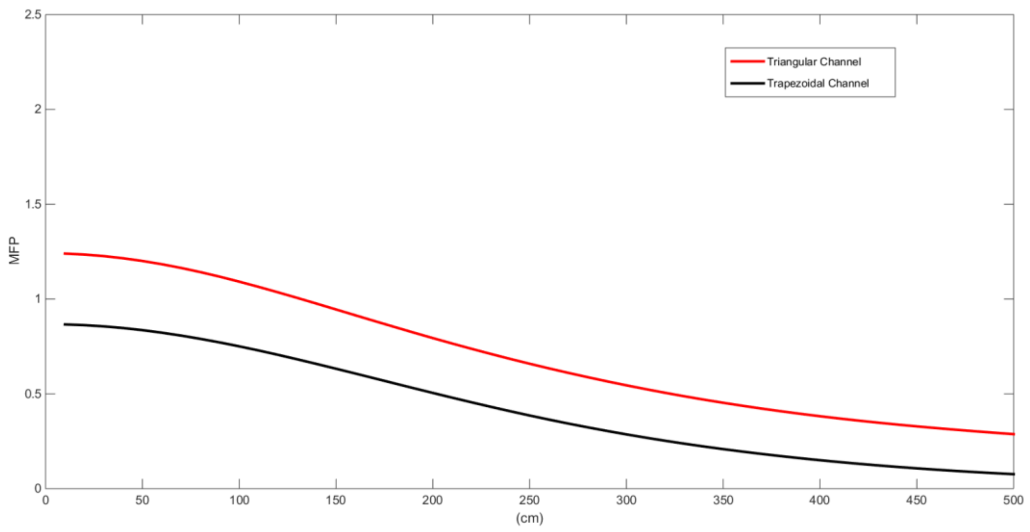

Figure 5c. Z = 300 cm

Figure 5c shows the points at a depth of 300 cm. As in the previous discussion, it can be seen that the MFP value of points at that depth decreases as they increase further away from the water flow source (flux). This suggests that the water content at ground levels close to the source of the water flow (flux) is higher than in the more distant parts. The MFP value of the triangular shape channel is higher than the trapezoidal shape channel.

D. CONCLUSION AND SUGGESTIONS

This study aims to discuss about infiltration stationer from triangular and trapezoidal irrigation. The research results show that for each point taken, it is seen that the water content from the triangular shape irrigation is higher than that from the trapezoidal shape. Based on the results of this research, it is recommended that triangular irrigation channels be implemented in Indonesia.

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