

# The Application of Frieze Groups and Crystallographic Groups in Generating Batak Toba Ornament Motifs Using a Matlab Graphical User Interface

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## ABSTRACT

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*Gorga* is a carving or sculpture typically found on the exterior of a Toba Batak traditional house. The Batak people use fractal (geometric) dimensions in Batak *gorga* carvings. In mathematics, repetitive and symmetrical patterns in planes that result from transformations are included in the plane symmetry groups. Ethnomathematics is a cultural approach to the concept of mathematics. A frieze group can be defined as a symmetrical group which arises from a unidirectional translation and subsequently generates a linear pattern that recurs exclusively in a single direction. There are seven different pattern types in the frieze groups. Meanwhile, crystallographic patterns are flat two-dimensional patterns that form a lattice. There are 17 crystallographic types of patterns with five different types of unit lattices. The purpose of this study is to generate motifs for Batak ornaments based on frieze groups and crystallographic groups using a Matlab Graphical User Interface (GUI). A total of 119 new motifs were generated based on seven types of patterns in the frieze groups, namely,  $F1, F2, F3, F4, F5, F6$ , and  $F7$ . Meanwhile, in the crystallographic groups, 153 new motifs were generated based on nine types of patterns, namely,  $p1, p2, pm, pg, cm, pmg, pmm, cmm$ , and  $pgg$ . To keep with trends, the new motifs generated can be used in everyday life as decorations or business symbols that are characteristic of the Toba Batak region.



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## A. INTRODUCTION

Indonesia is a multicultural country made up of a variety of ethnic communities, geographical regions, and languages. One of the distinct traditional cultures in Indonesia originates from North Sumatra, which has a prosperous and diversified cultural heritage. The Batak culture is one of the most ancient in North Sumatra (Purba, 2016). One characteristic of the Batak culture is apparent in its ornaments, especially in the case of the Toba Batak. The Toba Batak ethnic group in North Sumatra is one of the ethnic groups with distinctive traditional housing known as *rumah bolon* (Saragih et al., 2019). The Toba Batak traditional house is decorated with carved ornamentation that almost completely covers the entire building. The art of carving ornaments in the Toba Batak culture is commonly called *gorga*. In traditional societies, ornaments serve as a means of visual expression of feelings (Andriyanti, 2016). *Gorga* is a carving or sculpture that is commonly found on the outside of a Toba Batak traditional house (Pane & Sihotang, 2022). The Batak people also use fractal (geometric) dimensions in Batak *rumah gorga* carvings (Sihombing & Tambunan, 2021). We can see the

concept of *gorga* geometry in the use of repetitive and symmetrical forms of ornamental motifs in carving designs. Not only is it visually appealing, the geometry in Batak *gorga* carvings also has a philosophical meaning in Batak culture. Every motif and design utilized in Batak *gorga* carvings has an inferred meaning that is thought to carry a message and hope for its generations, indicating that the Toba Batak's cultural heritage value is still intact. Along with present-day advancements, culture is significant not only historically but also in other fields of learning, one of which is mathematics (Kartika & Rahmawati Suwanto, 2022).

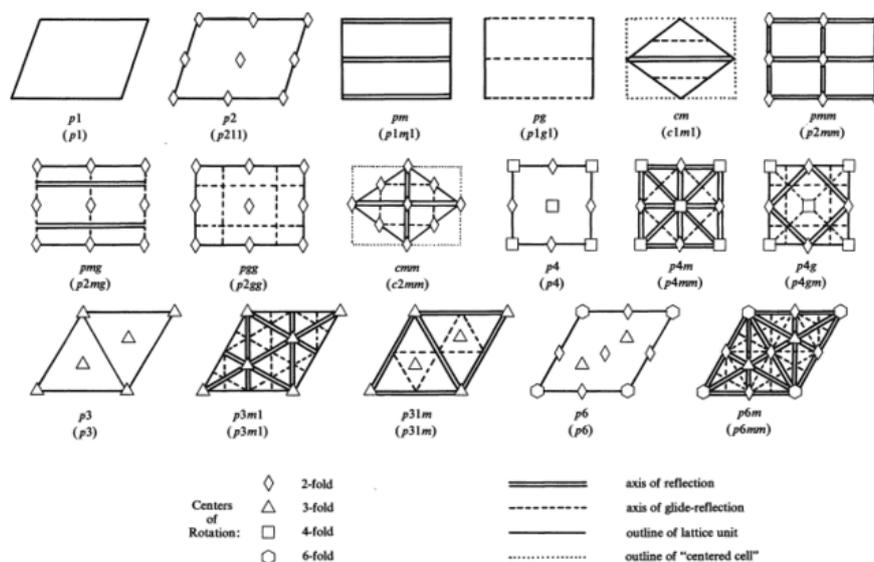
Ethnomatematics is a cultural approach to mathematical thinking regarding mathematical objects formed by a multicultural society (Sopamena et al., 2018). The Batak people have long been familiar with transformation geometry, as evidenced by the application of transformation geometry techniques in the creation of *gorga* decorations. Toba Batak decorations (*gorga*) comprise 17 distinct types of motifs, namely, *ipon-ipon*, *desa naualu*, *ogung*, *sitompi*, *simataniari*, *singa-singa*, *jorgom*, *boraspati*, *gaja dompak*, *dalihan natolu*, *singa-singa*, *sitangan*, *silintong*, *hariara sundung nilagit*, *hoda-hoda*, *mata ni ari*, and *jengger*. The motifs seen in *gorga* carvings or ornaments employ geometric transformation concepts such as reflection, shift, and rotation (Ditasona, 2018). There are also repeating and symmetrical patterns in Toba Batak ornaments. In mathematics, repetitive and symmetrical patterns in planes created by transformations are a part of the plane symmetry groups. This study employs the concept of plane symmetry, which is also known as isometry or rigid geometric transformation (Astriandini & Kristanto, 2021). Symmetry and group theory are a mathematical concept that can be applied to developing motifs or patterns (Kartika & Rahmawati Suwanto, 2022). A symmetric group is an isometric set that meets group axioms with compositional operations (Silalahi et al., 2022). There are four types of isometry in this group: Translation, which is the displacement of each point in a plane by a specific amount and in a specific direction; Reflection, which, on the other hand, is a transformation that moves one point to another as an image through a specific reflection axis that resembles a mirror (Suwanto et al., 2022); Rotation, or rotation in the plane of another point by rotating to a specific center point; (Ditasona, 2018); and Glide Reflection, which is a combination of translation and reflection, in which the reflection axis is parallel to the translation axis (Rahmawati et al., 2018). These plane symmetry groups are often referred to as either frieze groups or crystallographic groups.

A frieze group is a symmetrical group that results from a one-way translation and then creates a linear pattern that repeats in one direction (Cooper, 2013). The frieze patterns comprise seven types of recurring patterns that are not isomorphic concern to one-plane symmetry groups (Andriani & Muchyidin, 2020). According to Gallian (2021), there are seven different patterns made up of seven different types of infinite symmetry groups. The seven kinds of symmetry groups can be seen in Table 1.

**Table 1.** Seven frieze patterns and symmetry groups

Group	Generator	Formula	Pattern
<b>F1</b>	k = translation	$F_1 = \{k^a \mid a \in \mathbb{Z}\}$	$\begin{array}{cccc} k^{-1} & e & k & k^2 \\ \hline \text{R} & \text{R} & \text{R} & \text{R} \end{array}$
<b>F2</b>	k = glide reflection	$F_2 = \{k^a \mid a \in \mathbb{Z}\}$	$\begin{array}{ccc} k^{-2} & e & k^2 \\ \hline \text{R} & \text{R} & \text{R} \\ \hline \text{B} & & \text{B} \\ k^{-1} & & k \end{array}$
<b>F3</b>	k = translation l = vertical reflection	$F_3 = \{k^a l^b \mid a \in \mathbb{Z}, b = 0 \text{ or } 1\}$	$\begin{array}{ccc} k^{-1} l & k^{-1} & l \ e \\ \hline \text{R} & \text{R} & \text{R} \\ \hline \text{R} & \text{R} & \text{R} \end{array}$
<b>F4</b>	k = translation l = 180° rotation	$F_4 = \{k^a l^b \mid a \in \mathbb{Z}, b = 0 \text{ or } 1\}$	$\begin{array}{ccc} k^{-1} & e & l \\ \hline \text{R} & \text{R} & \text{R} \\ \hline \text{R} & \text{R} & \text{R} \end{array}$
<b>F5</b>	k = glide reflection l = 180° rotation	$F_5 = \{k^a l^b \mid a \in \mathbb{Z}, b = 0 \text{ or } 1\}$	$\begin{array}{ccc} k^{-1} l & e & k l \ k^2 \\ \hline \text{R} & \text{R} & \text{R} \\ \hline \text{R} & \text{R} & \text{R} \end{array}$
<b>F6</b>	k = translation l = horizontal reflection	$F_6 = \{k^a l^b \mid a \in \mathbb{Z}, b = 0 \text{ or } 1\}$	$\begin{array}{ccc} k^{-1} & e & k \\ \hline \text{R} & \text{R} & \text{R} \\ \hline \text{B} & \text{B} & \text{B} \\ k^{-1} l & l & k l \end{array}$

Another type of symmetry groups is the crystallographic groups. Crystallographic patterns are two-dimensional flat patterns which form a lattice (Panjaitan et al., 2022). The characteristics of the crystallographic groups are distinguished based on lattice and isometric types, e.g., square, parallelogrammatic, rhombic, and hexagonal (Umble & Han, 2008). There are 17 crystallographic patterns based on five existing unit lattice types. Figure 1 displays the patterns' overall shapes.



**Figure 1.** Crystallographic patterns (Schattschneider, 2018).

The frieze and crystallographic groups have been subject to numerous studies, including one entitled "Frieze Pattern on Shibori Fabric" (Puspasari et al., 2022). That study looked at the frieze patterns in the Shibori fabric motifs developed by artisans in Tulungagung and found that not all symmetrical patterns could be categorized as part of the seven frieze groups because some symmetrical patterns lack an element of translation. Ray, Steven Nataliani (2022) conducted another study using Rapid Prototyping (RP) and Reverse Engineering (RE) technology to create ceramic motifs using symmetry groups, specifically crystallographic groups. This work produced 17 patterns from 17 symmetry groups deriving from a single archetype. Meanwhile, Panjaitan et al. (2022) investigated Malay Deli *songket* motifs using frieze patterns and crystallographic patterns. It was discovered that the Malay Deli *songket* motifs contain two frieze patterns (i.e., patterns 3 and 4). There are also crystallographic patterns, specifically p1, pm, and p4m patterns. The creation of motifs based on crystallographic group patterns and frieze group patterns was also studied by Mingka et al. (2023), from which six new *songket* motifs based on frieze patterns and eleven new *songket* motifs based on crystallographic patterns were discovered.

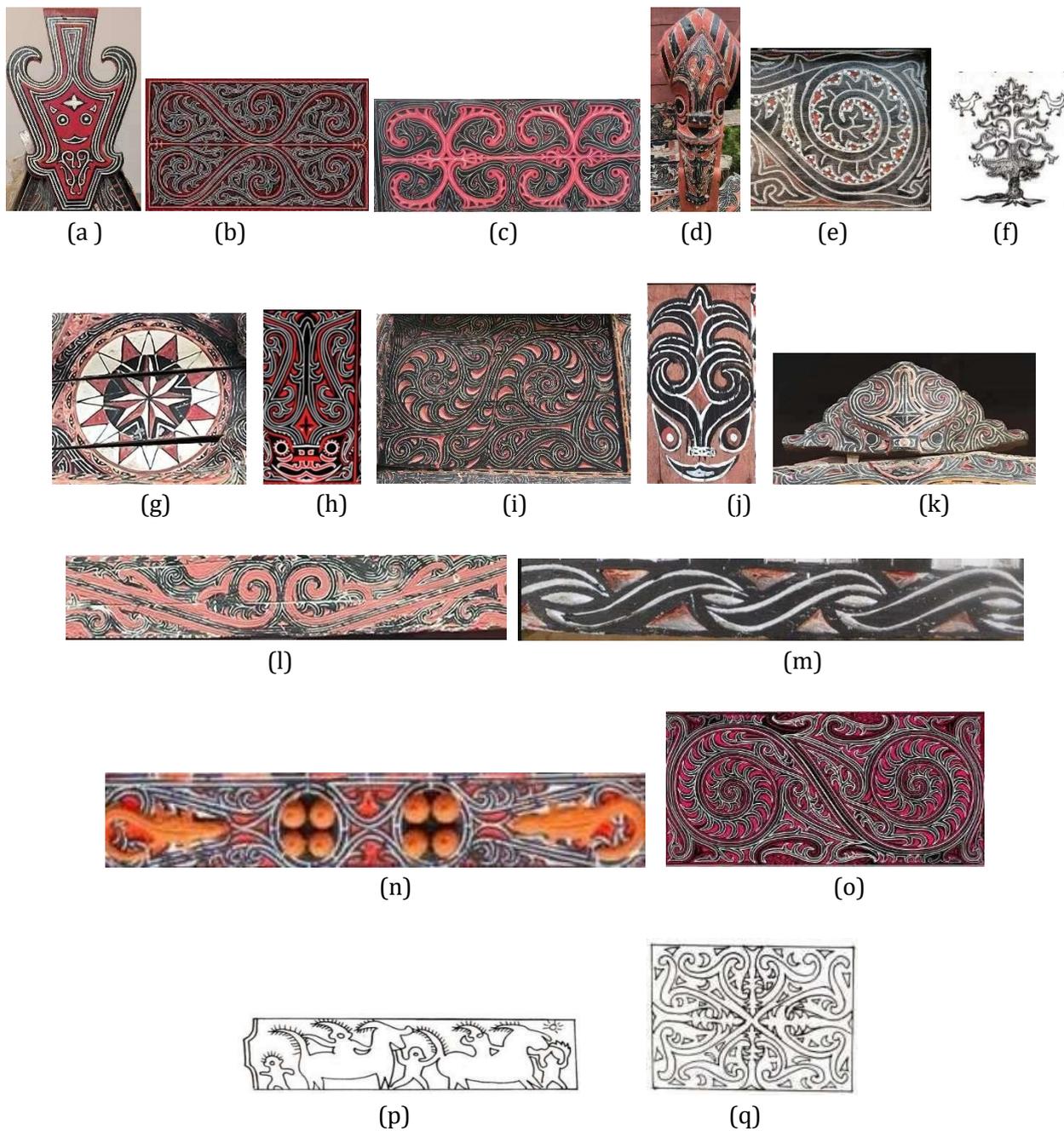
In an increasingly modern era, *gorga* ornaments are not only employed in traditional Toba Batak households; they have also begun to evolve following current trends, such as in batik clothing motifs and home wall decorations (Hermita & Sianturi, 2020). Therefore, based on previous research conducted by Panjaitan et al. and Mingka et al., who developed the Melayu Deli Songket motif using the concept of symmetrical groups from frieze group patterns and crystallographic group patterns, the researchers aimed to generate Batak gorga motifs using a Graphical User Interface (GUI) in MATLAB. old ornamental motifs are modified into new motifs while preserving the meanings of the original motifs of these ornaments with the help of the cropping technique. Cropping is, in the words of Yuhandri (2019), a technique used to precisely identify the area of the image that includes the object that has to be processed. Frieze groups and crystallographic groups are used to create cropped images, which are then further processed using a Matlab Graphical User Interface (GUI). The researchers hope that the practical implications of this study will contribute to meaningful applications in daily life, logos for SME food and beverage products that are symbolic of the Toba Batak region. In this way, Batak ornaments find wide applications and serve as a distinguishing feature. To conserve Batak gorga and prevent its extinction, the production of Batak gorga motifs indirectly represents a crucial action.

## B. METHODS

The gorga that will be generated is based on the concept of symmetry groups using the MATLAB graphical user interface. Data sources are taken from personal collections, journals, and books. The generation of the Batak gorga motif goes through the following stages:

### 1. Identifying Toba Batak *gorga* types

The Batak people use fractal (geometric) dimensions in the *gorga* carvings on the Batak Toba traditional houses (Sihombing & Tambunan, 2021). Applying the concept of geometry in *gorga*, a multitude of wonderful ornamental decorative patterns can be generated. With Matlab's cropping technique, which section of the *gorga* ornament is to be processed and elevated can be determined, as shown in Figure 2.



**Figure 2.** Seventeen types of Toba Batak *gorga*: (a) *ulu paung*; (b) *sitompi*; (c) *sitangan*; (d) *singa-singa*; (e) *silintong*; (f) *hariara sundung di langit*; (g) *mata niari*; (h) *jorgom*; (i) *ogung*; (j) *gaja dompak*; (k) *jengger*; (l) *simeol-meol*; (m) *ipon-ipon*; (n) *boraspati*; (o) *dalihan natolu*; (p) *hoda-hoda*; (q) *Desa Naualu* (Siahaan, 2019); (Saragih et al., 2019)

## 2. Designing with the Matlab GUI

At this stage, the GUI screen display is set up, including buttons for cropping, saving, browsing, and opening the frieze and crystallography application. These buttons will be used to create motifs based on the frieze groups and crystallographic groups, as shown in Figure 3.

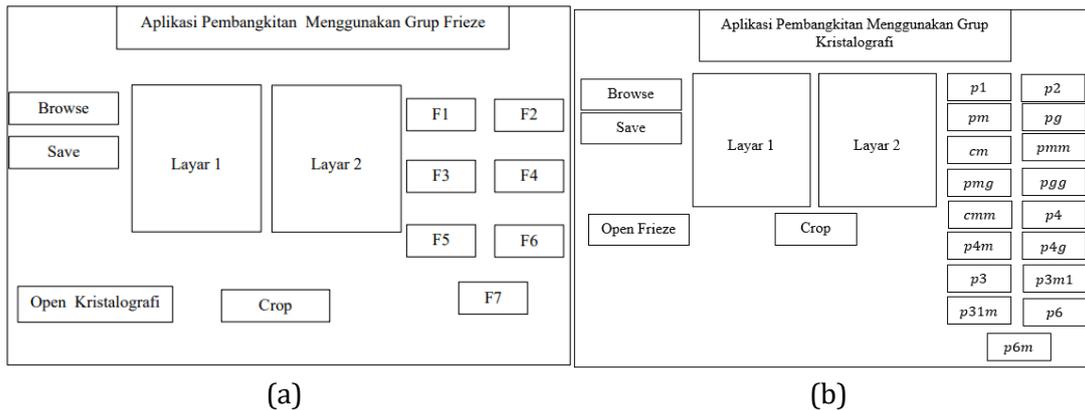


Figure 3. (a) Frieze group GUI display (b) Crystallography GUI display

### 3. Design coding with the Matlab GUI

This stage carries out coding to activate each button that will be used, so that each motif can be produced based on the function of each button that has been designed. Figure 4 illustrates the code design stages in the Matlab GUI.

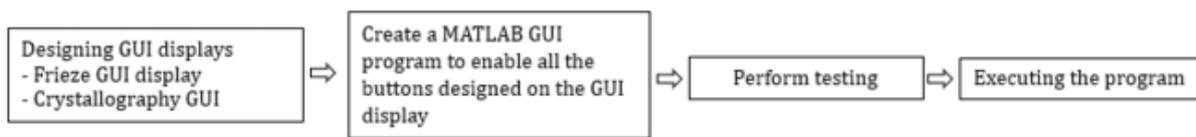


Figure 4. Matlab GUI design stages

### 4. Creating new motifs using the Matlab GUI

Here, new motifs based on the *gorga* image submitted are generated. The cropping technique is used to choose a section on which motif generation according to the user's aesthetics is based. The user could create even more motifs by cropping. The creation of ornaments (*gorga*) in this study is classified based on seven frieze groups and only nine crystallographic groups.

## C. RESULTS AND DISCUSSION

### 1. Matlab GUI Display

The Matlab GUI will be used to generate and simplify the creation of ornamental motifs (*gorga*) based on frieze groups and crystallographic groups. Figures 5 show how the GUI software displays its output.

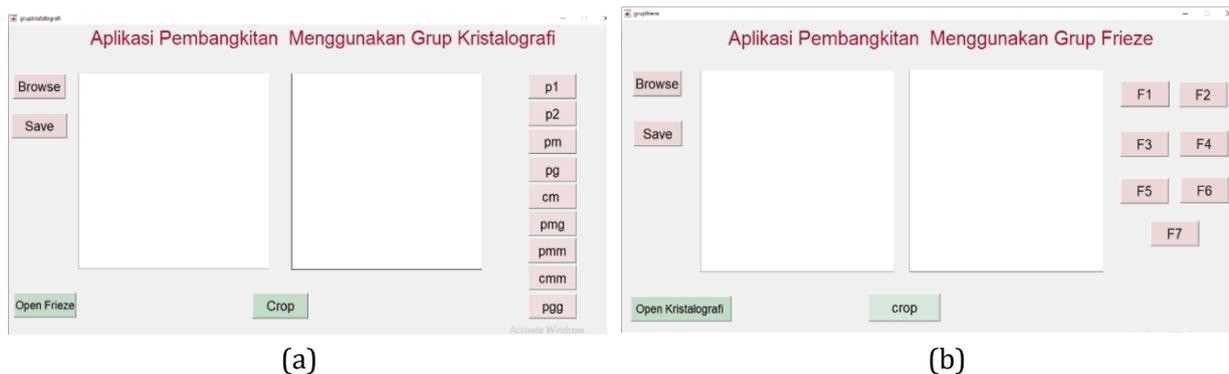


Figure 5. (a) GUI display results frieze group (b) GUI display results crystallography group

In the Matlab GUI display there are several buttons that are used in this program, namely:

Table 2. GUI button functions

Display Components	Function
<i>Browse</i>	The button used to search for and select images of gorga ornaments
<i>Save</i>	used to save the resulting image that has been generated
<i>Open frieze</i>	used to open the GUI display in the frieze group application
<i>Open crystallography</i>	button used to open the GUI display of the crystallography group application
<i>Crop</i>	used to select an image area that is suitable for generation using a square crop
<b>F1</b>	The keys used to create the resulting image are constructed by translation
<b>F2</b>	The button used to create the resulting image is created by <i>glide reflection</i>
<b>F3</b>	The button is used to produce an image built by vertical translation and reflection
<b>F4</b>	The button used to produce an image is built by translation and rotation 180°
<b>F5</b>	The button is used to produce an image built by vertical reflection and 180° rotation
<b>F6</b>	The button is used to produce an image built by horizontal translation and reflection
<b>F7</b>	The buttons are used to produce images built by translation, vertical reflection, and horizontal reflection.
<b><i>p1</i></b>	The button is used to produce an image constructed by two-way translation.
<b><i>p2</i></b>	The button is used to produce an image constructed by two-way translation and 180° rotation
<b><i>pm</i></b>	The button is used to produce an image constructed by bidirectional translation and reflection
<b><i>pg</i></b>	The button is used to produce an image constructed by two-way translation and <i>glide reflection</i>
<b><i>cm</i></b>	The button is used to produce an image constructed by two-way translation, reflection and <i>glide reflection</i>
<b><i>pmg</i></b>	The button is used to produce an image constructed by bidirectional translation, reflection, and 180° rotation.
<b><i>pmm</i></b>	The button is used to produce an image constructed by two-way translation and two vertical and horizontal reflections.
<b><i>cmm</i></b>	The button is used to produce an image constructed by two-way translation.
<b><i>pgg</i></b>	The button used to produce an image is constructed by two-way translation, glide reflection, reflection, and 180° rotation.
Screen 1	- Place the gorga image results that have been selected/searched for using the <i>Browse button</i> . - Location of image results that have been generated using the frieze group and crystallography group
Screen 2	The place used for cropping, cuts the area of the image you want to generate.

## 2. Source code in the Matlab GUI

The Matlab application is programmed to generate new motifs in the frieze groups and crystallographic groups. The following are the Matlab codes used for generating new motifs based on the  $F5$  functions.

### Code on $F5$

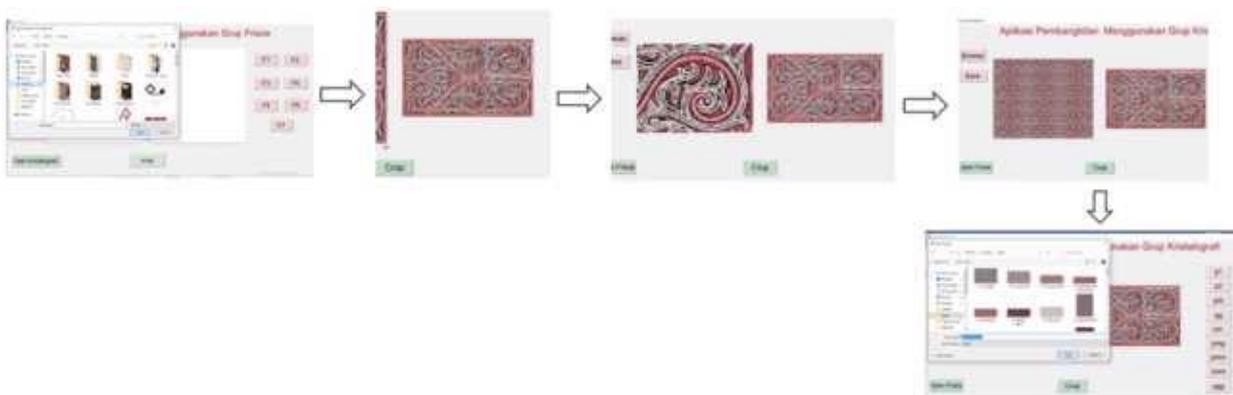
```

image = getimage(handles.axes2);
[height, width, rgb] = size(image);
S_width = floor(width / 2) - 1;
S = uint8(zeros(height, S_width, 3)) + 255;
N = uint8(image);
F_H = uint8(flip(image, 2));
R180 = uint8(imrotate(image, 180));
H_Flip = uint8(flip(image, 1));
disp(size(N));
disp(size(R180));
imshow([
    [F_H N    S    F_H N    S    F_H N    S    F_H N S];
    [S    R180 H_Flip S    R180 H_Flip S    R180 H_Flip S    R180 H_Flip];
]);

```

## 3. Ornamental motif generation results

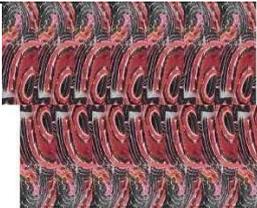
After running the Matlab GUI, the Batak gorga will be processed by inputting the gorga motif and cropping it to take pieces (snippets) of images from each type of gorga. These snippets are basic patterns for producing new motifs based on the frieze group and crystallographic group. The steps for generating motifs can be seen in Figure 6.

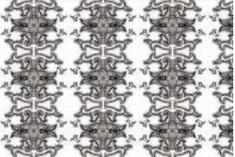
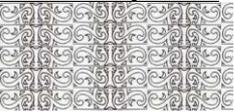


**Figure 6.** Stages of running the GUI to generate new motifs

The Matlab GUI application is programmed to generate new motifs based on frieze groups and crystallographic groups. The resulting motif is derived from a basic pattern that has been cropped. By developing motifs based on 7 types of frieze groups and 9 types of crystallographic groups, various motifs are produced, as illustrated in Table 3. The resulting motif is a combination of symmetry group members, including translation, reflection, rotation, and glide reflection.

**Table 3.** New *gorga* ornamental motifs generated

No	Types of Toba Batak Ornaments (Gorga)	Basic Patterns	Gorga Ornamental Motifs Generated	
			Frieze Group	Crystallographic Group
1.	<i>Gorga ipon-ipon</i> : It is used to polish sculptures and add a border adornment (Siahaan, 2019).		 Pattern F7	 Pattern <i>pgg</i>
2.	<i>Gorga jorgom</i> : It is usually placed above the front door of a house. It resembles both humans and animals in shape (Siahaan, 2019).		 Pattern F5	 Pattern <i>p2</i>
3.	<i>Gorga sitangan</i> : It represents a piece of advice to get rid of arrogance, especially in welcoming visitors (Siahaan, 2019).		 Pattern F2	 Pattern <i>p1</i>
4.	<i>Gorga ulu paung</i> : It symbolizes a power to protect the whole house from the disturbance of demons that enter through the door of a house (Saragih et al., 2019).		 Pattern F3	 Pattern <i>pmg</i>
5.	<i>Gorga boraspati</i> and <i>gorga adop-adop</i> : They symbolize prosperity and regeneration (Siahaan, 2019).		 Pattern F1	 Pattern <i>cm</i>
6.	<i>Gorga dalihan natolu</i> : It is created like a web of entwined plant tendrils, which stands for kinship (Rajagukguk, 2020)		 Pattern F4.	 Pattern <i>cmm</i>
7.	<i>Gorga sitompi</i> : The term <i>sitompi</i> derives from the word <i>tompi</i> , a farmer's tool used to plow fields hung around a buffalo's neck (Siahaan, 2019).		 Pattern F6	 Pattern <i>pg</i>
8.	<i>Gorga singa-singa</i> : It is nothing like a lion. Rather, it resembles someone who is squatting (Siburian, 2022).		 Pattern F1	 Pattern <i>pg</i>

<p>9. <i>Gorga silintong</i>: It represents a supernatural power that may shield people from anything bad (Siahaan, 2019).</p>		<p>Pattern <math>F1</math></p> 	<p>Pattern <math>pm</math></p> 
<p>10. <i>Gorga hariara sundung di langit</i>: It represents human creation and the significance of remembrance of the Creator of Humanity (Siahaan, 2019).</p>		<p>Pattern <math>F2</math></p> 	<p>Pattern <math>pmm</math></p> 
<p>11. <i>Gorga mata niari</i>: It is a symbol of the source of life force and a guide to deciding the direction of existence in the world, sometimes also referred to as the Ancient Man (Siahaan, 2019).</p>		<p>Pattern <math>F3</math></p> 	<p>Pattern <math>P1</math></p> 
<p>12. <i>Gorga ogung</i>: It tends to be used during celebrations, rituals, and other ceremonial activities (Siahaan, 2019).</p>		<p>Pattern <math>F4</math></p> 	<p>Pattern <math>p2</math></p> 
<p>13. <i>Gorga gaja dompak</i>: It is a sign of justice for the Batak people, representing the law that originates from Debata Mulajadi Nabolon (Siahaan, 2019).</p>		<p>Pattern <math>F5</math></p> 	<p>Pattern <math>pg</math></p> 
<p>14. <i>Gorga jengger</i>: It serves as a barrier against all types of evil, keeping the dwellers of the house safe and secure (Siahaan, 2019).</p>		<p>Pattern <math>F6</math></p> 	<p>Pattern <math>pgg</math></p> 
<p>15. <i>Gorga simeoleol</i>: It adds beauty and acts as a sign of joy (Siahaan, 2019).</p>		<p>Pattern <math>F7</math></p> 	<p>Pattern <math>pm</math></p> 
<p>16. <i>Gorga Desa Nualu</i>: It indicates the best periods in which to work, such as the times of year for cultivating paddy fields, fishing, etc. It is placed at the right and left ends of the front wall of a Toba Batak traditional house in Nualu Village (Saragih et al., 2019).</p>		<p>Pattern <math>F1</math></p> 	<p>Pattern <math>pmg</math></p> 
<p>17. <i>Gorga hoda-hoda</i>: It illustrates an animal being ridden by a person as a second person stands nearby holding the rein (Tampubolon &amp; Tampake, 2023).</p>		<p>Pattern <math>F2</math></p> 	<p>Pattern <math>cm</math></p> 

#### D. CONCLUSION AND SUGGESTIONS

In this study, a Matlab GUI was developed to produce new *gorga* batik motifs using sections of *gorga* images based on the frieze groups and crystallographic groups. As a result, 119 new motifs were generated based on the frieze groups, and 153 new motifs were generated based on the crystallographic group using the 17 different types of Toba Batak *gorga* motifs. By following modern times increasingly, the results of this new motif can be developed and implemented in everyday life as building decoration, clothing motifs, or as a characteristic business logo from the Batak region.

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