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Implementation of Inquiry Learning Model in Collaboration with PBL to Improve Student Understanding in Number Theory Course

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ABSTRACT

The purpose of this study was to describe the presence or absence of the influence of the infusion learning model collaboration with Problem-Based Learning (PBL) to develop students' understanding of proof and mathematical argumentation in number theory courses. This research is an experimental study with a randomized control group pretest-posttest design, two groups namely the experimental group and the control group. The experimental group is the group that uses the infusion learning model in collaboration with PBL, while the control group is the group that uses conventional learning. The subjects of this study consisted of 40 students at a university in Jombang, Indonesia. Data collection techniques through observation sheets, proof understanding tests and observation sheets of students' mathematical argumentation abilities. The results of the research show that the significant difference between the average proof of understanding of students in the experimental group and the control group. The difference between the average proof of understanding in the experimental group and the average proof of understanding of students in the control group was 21.75. Furthermore, the significant difference between the average argumentation ability of students in the experimental group and the control group. The difference between the average argumentation ability of students in the experimental group and the average argumentation ability of students in the control group is 5.25. Therefore, the implementasion of infusion learning in collaboration with PBL is more effective than conventional learning models for developing students' ability to understand mathematical proof and argumentation. This learning model promotes the development of critical thinking skills, problem-solving, conceptual and different understanding needed to construct a formal proof, and strong and valid arguments.

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A. INTRODUCTION

The concepts of argumentation and proof are closely related, considering both of them helps to draw attention to a wider range of important processes related to proof than when considering them separately (Stylianides et al., 2016). The term argumentation is used to describe the discourse or rhetorical means (not necessarily mathematical) used by individuals or groups to convince others that a statement is true or not (Boero et al., 1996; Duval, 1989; Krummheuer, 1995; L. B. Tristanti et al., 2017). Argumentation focuses on the epistemic value of the statements given and can embody the relationship between the process of ascertaining (a process used to dispel one's own or others' doubts about the truth or falsity of a statement) and the process of proof (Stylianides et al., 2016). Argumentation is also called the persuasion process (a process used to dispel other people's doubts about the truth or falsity of a statement) (Harel & Sowder, 2007).

Proof is a deductive argument that expresses reasons why a statement is true, by making use of other mathematical results and/or understanding of the mathematical structure involved in the statement (Knuth, 2002). Another definition of mathematical proof describes it as a series of formal and logical reasoning that starts from axioms and goes through logical steps to conclude (Griffiths, 2000). By referring to these views, it can be concluded that mathematicians associate proof with logical deduction and the application of structured arguments to show the truth of a statement in the field of mathematics.

Proof methods in mathematics include formal and informal proofs (Leitgeb, 2009). Formal proof has a formal syntax, a clear logical sequence, formulas or terms and logical arguments arranged syntactically. In contrast, informal proofs do not use certain rules such as logical sequences, logical axioms, and formulas. Informal proof may be experienced by high school students as using specific examples to prove odd and even number problems (Edwards, 1998) although some undergraduate students still use it for generalization (Sari et al., 2018; L. B. Tristanti et al., 2015, 2016). Panza (2003) suggests an informal proof is one of tangible (mathematical) proof in which students generate their arguments.

Proof and argumentation are important process standards in the teaching and learning of Mathematics (Campbell et al., 2020). However, students experience many difficulties in learning mathematics, especially proof, and mathematics educators think that one of the main difficulties students face is in constructing mathematical proofs (Douek, 1999). Argumentation and understanding of proof are important abilities that must be possessed by students in solving problems. The ability to prove mathematics is currently not visible in students when studying Number Theory Courses. They have not been able to optimize all their mathematical abilities in learning so they tend to give up on assignments when experiencing difficulties. Through this research, it is hoped that it can become a reference and discourse for mathematics education practitioners to improve understanding of mathematical proof and mathematical argumentation abilities through appropriate learning.

Based on this description it appears that students have difficulty understanding proof and mathematical argumentation skills. To solve these problems, infusion learning and PBL learning models can be applied. This infusion learning model has an instructional impact and an accompanying impact (Tristanti & Nusantara, 2021, 2022, 2023). The instructional impact is the increase in students' argumentation skills. The accompanying impact is that students become more fluent in solving various proof problems, even though complex problems. Problem-Based Learning (PBL) is a learning model that exposes students to authentic problems so students are expected to construct knowledge and understanding independently (Afifah et al., 2019; Tristanti et al., 2017). PBL provides students with many opportunities for mathematical activities in making arguments (Soekisno, 2015). The Infusion Learning model that is collaborated with PBL has a characteristic where knowledge is constructed by students from problems. They actively cooperate in discussions to find solutions to problems and build arguments to convince themselves and the audience through infusion learning.

Many studies related to learning models to develop an understanding of proof and students' mathematical argumentation abilities. Rahman et al. (2020) analyzed the learning of peer tutors in identifying gaps and improving student performance in learning proof and understanding of mathematics. Maya & Sumarmo (2011) applied a modified Moore learning approach to improve students' mathematical understanding and proof abilities. Tristanti & Nusantara (2022) apply an infusion learning strategy to improve students' mathematical argumentation skills. Tristanti & Nusantara (2022) applies a problem-based and CIRC type cooperative learning model to improve students' mathematical argumentation skills. Indrawatiningsih et al. (2020) analyzed the mapping of arguments in learning mathematics on students' mathematical argumentation abilities. It appears that no previous research has developed an understanding of proof and mathematical argumentation simultaneously, and no one has applied the infusion learning model collaborative with PBL to students' understanding of proof and mathematical argumentation in number theory courses. The purpose of this study was to describe the presence or absence of the influence of the infusion learning model collaboration with PBL to develop students' understanding of proof and mathematical argumentation in number theory courses.

The infusion learning model collaborates with PBL in the number theory course developed based on the needs of students and lecturers in developing an understanding of proof and mathematical argumentation skills. This development aims to get multiple benefits from the two learning models. Problem-based learning trains students to be able to solve problems, participate in discussions and presentations. PBL facilitates students in utilizing their critical thinking to solve problems by compiling facts or finding data, analyzing information, and compiling alternative solutions (Gunawan, 2019; Santyasa et al., 2020; Utami & Giarti, 2020; Vahlia et al., 2001).

The Insusion learning model requires students to work scientifically and trains them to develop valid and convincing arguments for themselves and others (Tristanti & Nusantara, 2021, 2022, 2023). Each model has its characteristics and advantages. When collaborate, students achieve maximum benefits. The problem-based learning syntax includes problem orientation, organizing students to learn, facilitating students to study either in groups or individually, developing and presenting results, analyzing and evaluating problem-solving processes (Arends, 2012). Meanwhile, the syntax of the infusion learning model includes an introduction, presentation of teaching material, reasoning, arguments not in dialogue, presenting arguments in small dialogues, presenting arguments in class dialogues, assessing student arguments, and conclusions (Tristanti & Nusantara, 2023). Therefore, the stages of the infusion learning model that is collaborated with PBL include (1) problem-oriented students with the theme of proof; (2) students studying the material; (3) individually developing arguments to convince themselves (arguments not in dialogue); (4) presenting the results of the arguments he compiled to convince others; dan (5) analyzing and evaluating the results of the problem-solving process of student proof and arguments. Development of an infusion learning model in collaboration with PBL to improve understanding of proof and students' mathematical argumentation skills. Understanding of proof refers to the theory of Mejia-Ramos et al. (2012) because this theory is specifically aimed at undergraduate-level students, as listed in Table 1.

Table 1. Components of Understanding Mathematical Proof

Type of Assessment	Aspects of Proof	Indicator
Assess local understanding of proof	Define terms and statements	Identifying the terms in the proof identifying the key statements in the proof
	Logical Status of Statement and Proof Framework	Using logical statements in the proof logical relationship between the statements being proved
	Justification of claims	Making warrants in proof identifying specific data that supports the claim identifying specific claims that are supported by specific statements
Assess the holistic understanding of proof	Summarize high-level ideas	Identifying main strategies/ ideas from the proof compiled Identifying the approach from which the proof is compiled
	Identify the modular structure.	Inheriting the proof in the component Identifying the purpose of the evidentiary component Identifying logical relationships between components of proof.
	Transferring general ideas or methods to other contexts	adapting ideas to solve other evidentiary tasks. adapting evidentiary procedures to accomplish other evidentiary tasks.
	Illustration with example	Illustrating the sequence of conclusions with specific examples Interpreting statements or proof in diagrammatic form

While students' abilities in mathematical argumentation refer to the theoretical opinions of Toulmin (2003) and Tristanti & Nusantara (2022a).

B. METHODS

This research is experimental research with a randomized control group pretest-posttest design. In this design there are two groups, namely the experimental group and the control group. The experimental group is the group that uses the infusion learning model in collaboration with PBL, while the control group is the group that uses conventional learning. This research was conducted at a university in Jombang, East Java, Indonesia in the Mathematics Education Study Program. The population is semester 1 students with a total of 100 students. The sample consisted of 40 students. Sampling using stratified random sampling. The experimental and control groups were 20 students each.

Data collection techniques through observation sheets, proof understanding tests and observation sheets of students' mathematical argumentation abilities. This observation sheet is used to test the practicality of the infusion learning model in collaboration with PBL. Before being used, this questionnaire was validated by an expert validator. This observation sheet is filled in by the observer. This observation sheet consists of 5 statements. Each statement is assessed with a scale of 1-4. then matched to class intervals and classification of effectiveness criteria as Table 2.

Table 2. Practicality Criteria for the Infusion Learning Model that is Collaborated with PBL

Practical Percentage (PP)	Practicality Criteria	Information
PP > 80	Very practical	No Revision Needed
60 < PP ≤ 80	Practical	No Revision Needed
40 < PP ≤ 60	Quite Practical	Minor Revision
20 < PP ≤ 40	Less Practical	Revision
PP ≤ 20	Impractical	Revision

The proof understanding test consists of 1 description question. Here's a matter of proof:

Prove "if a is an even number and b is an odd number then $a + b$ is an odd number"!

This problem was chosen because it has several solutions. The proof comprehension test has gone through a process of validity, reliability, and measuring the level of difficulty. To analyze students' understanding of proof, an assessment is carried out with the scoring rubric that has been prepared in Table 3. The rubric used can determine whether students have met the indicators of understanding mathematical proof (Table 1) given or not.

Table 3. Assessment Rubric

Score	Description
2	The students shows an indicators correctly
1	The student shows an indicator but there is an error
0	The Students do not show an indicators

The observation sheet is used when the sample expresses student arguments in learning during the discussion process. This observation sheet refers to the theories of (Toulmin, 2003) (Tristanti & Nusantara, 2022a). Before being used this observation sheet was validated by an expert validator. To analyze students' mathematical argument skills, an assessment is carried out on a scale of 1-4. Table 4 below is the observation sheet used in this study.

Table 4. Observation Sheet of Students' Mathematical Argumentation Ability

Argumentation Ability	Observed Aspects	Scoring scale			
		1	2	3	4
Completeness of mathematical argumentation	Disclosing data and claims				
	Disclosing warrants				
	Disclosing trusted backing				
	Drawing conclusions				
The quality of mathematical argumentation	using deductive arguments correctly				
	convincing the audience of the truth of the argument				
	the audience accepts and believes in the proposed argument, which is marked by the absence of a rebuttal				

The draft of the infusion learning model in collaboration with PBL that was developed was validated by learning model experts and education experts to get suggestions. These suggestions are used to revise the draft learning model. In addition, Forum Group Discussions are conducted to find out the strengths and weaknesses of the learning model, as well as to get suggestions from lecturers and stakeholders. These suggestions are used to refine the learning model before it is implemented to determine effectiveness. To investigate the effectiveness of the learning model, the research sample was divided into two groups, namely the control group and the experimental group. Both groups were given a pretest and posttest to measure students' understanding of proof and mathematical argumentation abilities. The control group was given treatment with a learning model that used the lecture method. The experimental class was given treatment using an infusion learning model in collaboration with PBL. Lessons in the control and experimental classes were given in two meetings. At the end of the meeting, the two groups were given a questionnaire used to find out the responses or suggestions from lecturers and students to the learning model applied, and to measure the ability to understand proof and mathematical argumentation.

The data analysis technique uses a mixed-method design (quantitative and qualitative research methods), namely analyzing quantitative and qualitative data simultaneously (Sugiyono, 2011). The quantitative test was carried out using an independent sample t-test with the help of the SPSS version 20 program to achieve accurate data calculations, but previously the data had been tested for normality and homogeneity. Qualitative descriptive analysis was carried out on validation sheets and observation sheets on the application of the infusion learning model in collaboration with PBL, describing the results of understanding the proof and students' mathematical argumentation abilities. Triangulation analysis was carried out by analyzing both data (qualitative and quantitative) and comparing the results, then interpreting whether the two data support each other or not. The following is the research hypothesis:

H_{01} : There is no difference in understanding the proof of students who use infusion learning models that collaborated with PBL and conventional.
H_{11} : There are differences in understanding the proof of students who use infusion learning models that collaborated with PBL and conventional.
H_{02} : There is no difference in the ability of students' mathematical argumentation using infusion learning models that are collaborated with PBL and conventional.
H_{12} : There are differences in the ability of students' mathematical argumentation using infusion learning models that are collaborated with PBL and conventional.

C. RESULT AND DISCUSSION

1. Observation Results

This learning model experiment was carried out from June to July 2022 in a number theory courses with 6 meetings. Figure 1 shows when students discuss solving problems.

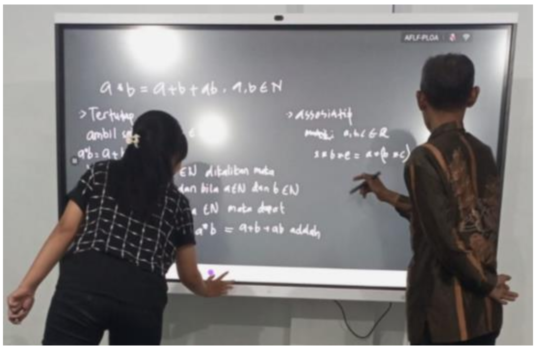

Figure 1. The Student Discussion in Solving Problems

During the experiment, the learning process was observed by the observer to find out the implementation of the infusion learning model in collaboration with PBL. The results of observations that have been filled in by the observer can be seen in Table 5.

Table 5. Observations on the Implementation of the Infusion Learning Model in Collaboration with PBL

No	Observed Aspects	Score	
		Observer 1	Observer 2
1	Orientation of the problem of proof	3	4
2	conduct questions and answers to explore material related to the material to be studied	3	3
3	Facilitating individual students to solve proof problems and develop self-convincing arguments	3	3
4	Facilitating students presenting solutions to problems proving and arguing with others	3	3
5	analyze and evaluate the results of the problem-solving process of student proof and argumentation	3	4
Total Score		32	
Practical Percentage (PP)		80%	
Practicality category		Practical	
Information		No Revision Needed	

The results of observing the implementation of the infusion learning model in collaboration with PBL show a feasibility of 80% in the practical category. So that the learning model is feasible to use without revision to develop students' understanding of proof and mathematical argumentation abilities.

2. The understanding of proof

Before experimenting, the experimental group and the control group were given the same test (pre-test). After the experiment, the samples were given the same post-test. The pretest and posttest use proof comprehension test instruments. The results of the pretest and posttest understanding of proof are calculated for normality as shown in Table 6.

Table 6. Normality of Pretest and Posttest Understanding of Proof

	N	Normal [a]Parameters[b]		Most Extreme Differences			Kolmogorov -Smirnov Z	Asymp. Sig. (2-tailed)
		Mean	**Std. Deviation**	**Absolute**	**Positive**	**Negative**		
Pretest_Understanding_Proof_Experiment	20	49.75	8.503	.232	.212	-.232	1.035	.234
Posttest_Understanding_Proof_Experiment	20	74.00	4.168	.245	.205	-.245	1.095	.182
Pretest_Understanding_Proof_Control	20	44.00	6.100	.207	.146	-.207	.925	.359
Posttest_Understanding_Proof_Control	20	52.25	5.955	.247	.247	-.203	1.106	.173

a. Test distribution is Normal

b. Calculated from data

The output from SPSS shows that the Asymp. The sig of each data for the experimental and control groups is > 0,05. This indicates that each data is normally distributed.

Table 7. Independent Samples Test of Data Understanding Proof

			Understanding Proof	
			Equal variances assumed	Equal variances not assumed
Levene's Test for Equality of Variances	F		2.164	
	Sig.		.149	
t-test for Equality of Means	t		13.383	13.383
	df		38	34.011
	Sig. (2-tailed)		.000	.000
	Mean Difference		21.75000	21.75000
	Std. Error Difference		1.62525	1.62525
	95% Confidence Interval of the Difference	Lower	18.45985	18.44713
		Upper	25.04015	25.05287

Based on the output of SPP Table 7, it is known that the significant value is 0,149 > 0,05, so it can be interpreted that the variance of the proof understanding data between the experimental group and the control group is homogeneous. So that the interpretation of Table 7 of the sample independent output is guided by the values contained in the assumed equal variances. The significant value of assumed equal variances is 0,000 < 0,05, so according to the basis of decision making in the independent test sample t-test, it can be concluded that H0 is rejected and H1 is accepted, thus it can be concluded that there is a significant difference between the average proof understanding of students in the group experimental and control groups. While the mean difference is 21,75. This value indicates the difference between the average understanding of proof in the experimental group and the average understanding of proof of students in the control group is 74,00 – 52,25 = 21,75 and the difference in the difference is 18,45985 to 25,04015.

Following are the results of constructing proof from one of the students in the experimental and control classes. Based on the picture, it appears that students in the experimental class build formal proof because the group is emphasized to compile and understand formal proof and understand the reasons behind each step of the proof. Whereas in the control class, more students construct a non-formal proof, namely using specific examples in proving.

The Infusion learning model in collaboration with PBL was developed to increase students' understanding of proof understanding and argumentation skills as well as number theory concepts in a more in-depth and sustainable way. There are several reasons why this model can have a positive effect on students' understanding of proof in this course, namely, a problem-based approach, this model involves solving real problems that require a strong understanding of number theory concepts. In this way, students must apply their understanding of proof to solve problems, which naturally increases their understanding. Both collaborative activities, this model encourage students to work together in groups, which can help them understand proof more profoundly. In teaching number theory, proofs are sometimes complex and abstract, and discussing with their peers can help students see different points of view and different approaches to proof. The effect of problem-based learning (PBL) on student understanding is because the teacher does not dominate learning activities, the teacher provides the widest opportunity for students to be actively involved and provides many opportunities for students to develop concepts individually or in groups (Tristanti, 2017). Students learn by actively discussing and working together, finding principles in solving problems. In addition, students are trained to be able to solve the problems they face in real situations, for example in the form of simulations and problems that do exist in the real world.

The third is critical thinking skills, because students are asked to construct and understand proofs in the context of number theory, this promotes their critical thinking skills. They must analyze arguments, evaluate the truth of a statement, and understand the reasons behind each piece of proof. Fourth, in personal teaching, lecturers can provide more personal guidance to students at the stage where students study the material and individually compile arguments to convince themselves (arguments not in dialogue). This can help students understand the proof and overcome any difficulties they may experience. Duch et al. (2001) stated that problem-based learning provides opportunities for students in terms of a strong understanding of basic, factual and applied knowledge, demonstrating effective and accurate communication skills both orally and in writing, working cooperatively in small groups.

3. Students' mathematical argumentation abilities

Before experimenting, the experimental group and the control group were asked to express their arguments after completing the pretest and posttest understanding of the proof. This was done to determine students' mathematical argumentation skills by using the observation sheet instrument for students' mathematical argumentation abilities. The results of the pretest and posttest of mathematical argumentation abilities were calculated for normality as shown in Table 8.

Table 8. Normality of Pretest and Posttest Results of Mathematical Argumentation Ability

	N	Normal ^{a}Parameters^b		Most Extreme Differences			Kolmogorov -Smirnov Z	Asymp. Sig. (2-tailed)
		Mean	Std. Deviation	Absolute	Positive	Negative		
Pretest_Argumentation_Experiment	20	37.25	7.518	.218	.218	-.167	.973	.300
Posttest_Argumentation_Experiment	20	54.75	7.340	.141	.141	-.119	.632	.820
Pretest_Argumentation_Control	20	39.00	6.609	.277	.277	-.173	1.241	.092
Posttest_Argumentation_Control	20	49.50	8.413	.204	.204	-.193	.911	.378

a. Test distribution is Normal
b. Calculated from data.

In Table 8 shows that the Asymp. The sig of each data for the experimental and control groups is > 0,05, this indicates that each data is normally distributed.

Table 9. Independent Samples Test of Argumentation Ability Data

			Argumentation_Ability	
			Equal variances assumed	Equal variances not assumed
Levene's Test for Equality of Variances	F		1.923	
	Sig.		.174	
t-test for Equality of Means	t		2.103	2.103
	Df		38	37.314
	Sig. (2-tailed)		.042	.042
	Mean Difference		5.25000	5.25000
	Std. Error Difference		2.49671	2.49671
	95% Confidence Interval of the Difference	Lower	.19568	.19262
		Upper	10.30432	10.30738

Based on the output of SPSS Table 9, it is known that the significant value is 0.174 > 0.05, so it can be interpreted that the variance of the argumentation ability data between the experimental group and the control group is homogeneous. So that the interpretation of Table 8 of the sample independent output is guided by the values contained in the assumed equal variances. The significant value of assumed equal variances is 0.042 <0.05, so as a basis for decision-making in the independent sample t-test it can be concluded that H0 is rejected and H1 is accepted. Thus it can be concluded that there is a significant difference between the average student argumentation ability in the group experimental and control groups. While the mean difference is 5.25. This value indicates the difference between the average argumentation ability of students in the experimental group and the average argumentation ability of students in the control group is 54.75 – 49.50 = 5.25 and the difference between these differences is 0.19568 to 10.30432.

The results showed that the PBL collaboration infusion learning model effected students' mathematical argumentation abilities in the Number Theory course because this approach promoted the development of critical thinking skills, problem-solving, and conceptual understanding needed to construct strong and valid arguments. There are several reasons why this model can have a positive effect on students' mathematical argumentation skills in this course, the first is active problem-solving, students are faced with proving problems that require solving. They must find reasonable solutions and formulate arguments in favor of those solutions. This helps them practice in constructing and supporting their arguments. Mathematical argumentation ability is a long process that requires repeated experience and practice (Osborne, 2005).

Second, collaborative activities, this model encourage collaboration between students. In discussing and working together to solve problems, they must convey and defend their views. This forces them to formulate clear arguments and communicate them effectively to their peers, so that their peers are convinced and not contradicting their arguments. Thirdly criticism and evaluation, in this model, students are taught to evaluate other people's arguments. This involves critical thinking and identifying weaknesses in arguments. By practicing this ability, students become better at constructing strong and valid arguments because they are more sensitive to aspects that need attention. In implementing problem-based learning, it is expected that students can think critically and creatively (Kurniasih & Sani, 2016; Tristanti, 2017), so that students can develop mathematical arguments.

Based on the research results, the application of infusion learning in collaboration with PBL is more effective than conventional learning models to develop students' proof understanding and mathematical argumentation skills. The results of this study support the results of Gunawan (2019); Palupi et al. (2020) which show that PBL is more effective in improving students' academic abilities than traditional learning. The research of Tristanti & Nusantara (2022a) stated PBL is more effective in improving students' mathematical argumentation abilities compared to traditional learning. Tristanti & Nusantara (2021, 2023) implement infusion learning in developing students' mathematical argumentation skills.

It is important to remember that the implementation of this model also plays an important role in its effectiveness. Lecturers must have a good understanding of how to properly apply this model in Number Theory subjects. In addition, each student has a different level of readiness, so this approach may require adjustments to suit individual needs. One learning model can’t be superior for all learning objectives (Arends, 2012). Therefore, the selection of learning models is based on the characteristics of learning materials, learning objectives, and skills that suit student learning needs (Darmuki et al., 2017). In reality, each learning model is suitable for a specific type of learning, but can be combined to make it easier for students to achieve learning goals (Affandi et al., 2022). No learning model is consistently better than another.

D. CONCLUSION AND SUGGESTIONS

The results of this study are very important in learning number theory courses. The infusion learning model that is collaborated with PBL has a positive influence on student success and the effectiveness of learning in class, especially on understanding proof and students' mathematical argumentation skills. It can be concluded that the infusion learning model that is collaborated with PBL is more effective than conventional learning because it is can increase student understanding in number theory courses. This is inseparable from the role of lecturers, students, appropriate learning models or methods in producing good learning outcomes, and other factors. Further research requires learning media or supporting technology or worksheets in applying the infusion learning model in collaboration with PBL to improve learning outcomes, proof understanding, or students' mathematical argumentation skills. The implication of this research is to provide understanding to lecturers to improve their understanding of proof, or students' mathematical argumentation abilities by implementing infusion learning models with PBL and emphasizing problems through activities that are suitable for students.

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