

Exploring Multivariate Copula Models and Fuzzy Interest Rates in Assessing Family Annuity Products

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ABSTRACT

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This research explores the development of a reversionary annuity product transformed into a family annuity covering three individuals: husband, wife, and children. The innovative design of this product considers the sequencing of annuity payments post-participant's demise, aiming to mitigate the risk of parents' death impacting their children. Recognizing the inadequacy of assuming independence among individuals in premium calculations, the study employs a multivariate Archimedean Copula model to account for interdependence. The primary objective is to compute the survival single-life function for each individual taken from TMI IV 2009. Then the copula model is implemented with Clayton and Frank copulas at varying Kendall's tau values (0.25, 0.5, and 0.75). Meanwhile, the interest rates are modelled using the BI-7-day (reverse) rate with a Triangular Fuzzy α -cut. The findings reveal that increasing Kendall's tau values lead to higher pure premiums, and notably, the Frank Copula model yields higher premium values than the Clayton Copula model. This research contributes valuable insights into the actuarial assessment of family annuity products, shedding light on the significance of considering dependencies among individuals for more accurate premium calculations.



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A. INTRODUCTION

In the realm of life insurance, the concept of annuities plays a crucial role, offering a series of payments to participants periodically (Baláž, 2023). A reversionary annuity, a specific type of life annuity, initiates payments after the demise of one of the insured individuals specified in the contract (Godfrey et al., 2022). While traditional models focus on pairs like husband and wife, the evolving market dynamics necessitate the development of innovative annuity products. Introducing a family annuity, encompassing not just spouses but also children, presents a novel approach to cater to changing market demands and ensure business sustainability (Sari et al., 2023). This shift towards more inclusive annuity models reflects the industry's adaptation to diverse customer needs and preferences, marking a significant evolution in the annuity landscape.

Preliminary studies on married couples indicate that "broken heart syndrome" or "widowhood effect" can increase the risk of mortality seems to be highest immediately after the loss of spouse, but studies have also found a persistent "widowhood effect" even after 25 years

(Blanner et al., 2020). Research has shown that widowhood is associated with a higher risk of mortality, with the effect being particularly pronounced in the first few years after the loss. A meta-analysis of the literature on marital status and mortality among individuals 65 years of age and older found that widowhood has a harmful effect on mortality in almost all age groups, with the magnitude of the effect decreasing with age (Roelfs et al., 2012). Broken heart syndrome is an acute, reversible, transient left ventricular dysfunction that can be triggered by emotional or physical stress (Vakamudi, 2016). However, patients with broken heart syndrome related to physical stress may present a worse short- and long-term prognosis in terms of mortality (Uribarri et al., 2019).

In addition, previous studies were conducted using joint life data from a large Canadian insurance company in joint life dependency analysis (Spreeuw & Owadally, 2013; Dufresne et al., 2018; Arias & Cirillo, 2021). Multiple life annuity data from census data in the Netherlands, insurance companies in France, and genealogical data in France about married couples (Sanders & Melenberg, 2016; Lu, 2017; Cabrignac et al., 2020). Furthermore, studies on joint life annuities and last survivors from survey data in Ghana and insurance companies in Kenya (Henshaw et al., 2020; Walter et al., 2021). Based on existing research, the last two studies examined inter-annual dependency in a socioeconomic context. Most research is related to dependency between husband and wife, but only a few have examined the dependence of parents and children. This is because in real life there is a dependence between parents and children's life span (Wilhelmsen et al., 2011). The death of a parent during childhood or adolescence is also associated with an increase in all causes of death for the child (Elwert & Christakis, 2008; Li et al., 2014). The results of these studies indicate that the co-distribution model must be able to capture the dependence between the insured parties, especially in calculating the pure premium value at once.

In premium calculations assuming dependence between the insured parties, the copula model is the most widely used. Firstly, the use of the Gumbel-Hougaard copula with the Weibull distribution function in determining the price of joint life insurance (Youn & Shemyakin, 1999). Determining the price of a reversionary annuity uses the Archimedean copula model with 1 and 2 parameters (Luciano et al., 2016). Differences in the age of policyholders in the Archimedean copula-based study, with the margin inference function and the Pseudo-maximum likelihood approach used for parameter estimation (Dufresne et al., 2018). Furthermore, the determination of pure premiums at once in the reversionary annuity for married couples uses Frank's copula model (Godfrey et al., 2022). From the results of this study, the calculation of the premium for an insurance product for multiple life with the assumption of mutual independence and the assumption of dependence between the insured parties will be different.

One of the factors that determine the contribution price is the actuarial margin level which is taken from the interest rate or yield. Actuarial margin levels, influenced by interest rates, impact contribution prices. Inaccurate rates can lead to losses for both insurers and the insured due to funding ratio and liability changes (Chen & Matkin, 2017). As for actuarial science, Fuzzy set theory has been used to model problems related to subjective judgments and situations when the information available is imprecise and incomplete. This can be found in articles on general actuarial issues that use Fuzzy sets on life and non-life insurance. Life insurance issues such as calculating the price of life insurance policies, life insurance portfolios, life

contingencies, life actuarial obligations, and life annuities can use Fuzzy sets (De Andreas-Sanchez & Puchades, 2017). Determination of annuity premiums can also use Fuzzy interest rates (Aalaei, 2022). The use of Fuzzy interest rates causes investment gains and surplus processes in the form of intervals.

Based on this background, this study will apply the copula model to model the joint distribution of the insured parties which is constructed from the marginal distribution of future lifetimes. Three insured parties were chosen from each party, namely husband, wife, and children. In this study, the marginal distribution of each insured party was constructed using the 2019 Indonesian Mortality Table (TMI IV) with the probability of dying using the assumption of a uniform distribution of death (UDD). Furthermore, the actuarial margin rate uses BI-7-day data which is estimated using the Fuzzy interest rate. This is expected to be able to answer the obstacles that occur due to limited data regarding the lifetime of the insured parties at certain age combinations. Then, the copula model is used when the husband, wife, and children are complete. This can describe the reality that exists because parties will depend on each other in their lives. The purpose of this study is to provide an alternative to determining the joint distribution of the insured parties based solely on the mortality table. Furthermore, based on the joint distribution, it can also be calculated the value of the pure premium at once from an insurance product, namely a family annuity.

B. METHODS

The research outlined in this paper follows a systematic approach illustrated in **Figure 1**. Initially, mortality rate data from TMI IV is employed, comprising the probability of death values for the next year, essential for constructing the survival single-life distribution. Subsequently, two copula models, Clayton and Frank, are applied to determine parameter fits for both bivariate copula models involving two and three individuals, respectively. The joint distribution functions are formulated for each case. Moreover, by integrating fuzzy interest rates, the family annuity premium can be accurately calculated. This comprehensive methodology provides a robust framework for analyzing mortality risk and determining annuity premiums within a multivariate context.

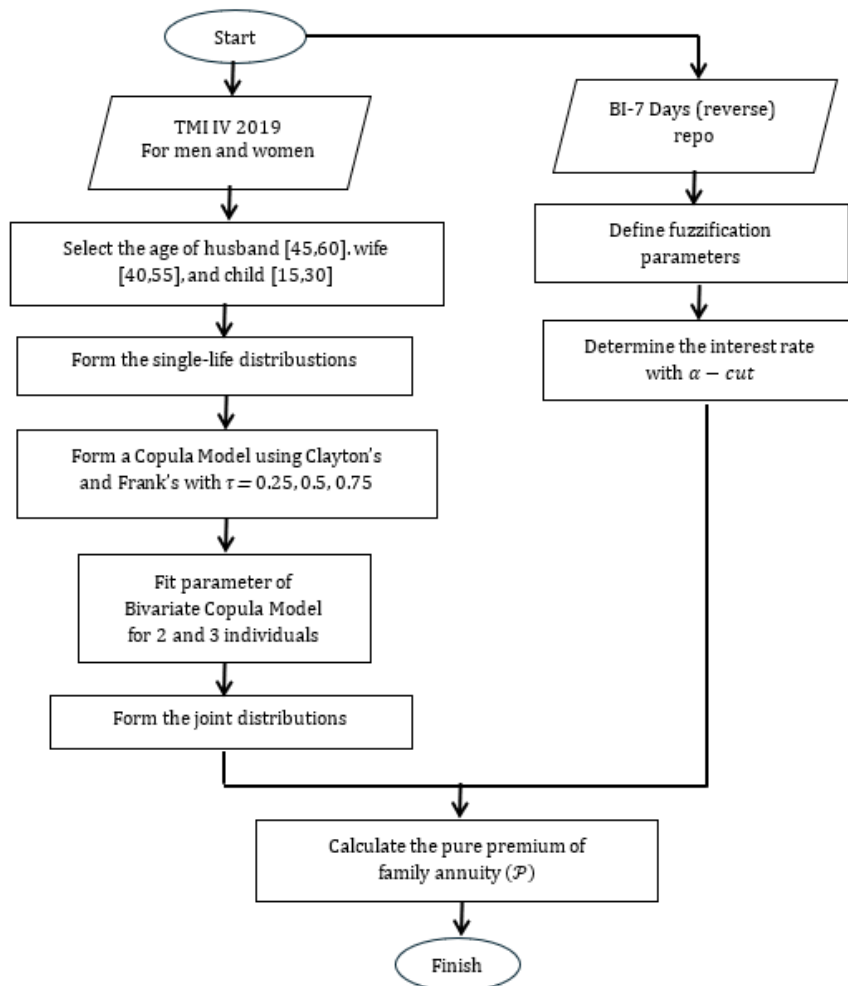


Figure 1. Flowchart of the Methodology

1. Survival Single-Life Distribution with UDD Assumption

The survival distribution for the insured parties follows the probability of death at TMI IV with non-integer ages using the UDD assumption. Suppose $T(x)$ is a continuous random variable that represents future lifetime with x indicating an individual's age $[0,111]$. The distribution of marginal survival for individuals aged x can be expressed as the probability that individuals aged x will survive until they reach age $x + t$, written ${}_t p_x$ which is formulated as follows:

$${}_t p_x = \begin{cases} 1 - tq_x, & 0 \leq t \leq 1 \\ p_x(1 - (t - 1)q_{x+1}), & 1 < t \leq 2 \\ {}_2p_x(1 - (t - 2)q_{x+2}), & 2 < t \leq 3 \\ \vdots \\ {}_{\omega-x-1}p_x(1 - (t - (\omega - x - 1))q_{x+(\omega-x-1)}), & \omega - x - 1 < t \leq \omega - x \end{cases}$$

The above form can also be written in the following form,

$${}_t p_x = \left[\prod_{i=0}^{\lceil t \rceil - 1} (1 - q_{x+i}) \right] [1 - (t - \lceil t \rceil)q_{x+\lceil t \rceil}] \tag{1}$$

(Godfrey et al., 2022).

2. Joint Distribution with Copula's Model

The joint distribution of the insured parties will be modelled with the assumption of dependency using a copula. This joint distribution model uses three values of Kendall's tau (τ) as follows:

$$\tau = \begin{cases} 0.25; & \text{weak} \\ 0.50; & \text{moderate} \\ 0.75; & \text{strong} \end{cases}$$

The joint distributions $T(x_1), T(x_2), \dots, T(x_n)$ can be expressed as the probability that $(x_1), (x_2), \dots, (x_n)$ will survive at least until they reach the ages $x_1 + t_1, x_2 + t_2, \dots, x_n + t_n$ defined as follows,

$$S_{T(x_1), T(x_2), \dots, T(x_n)}(t_1, t_2, \dots, t_n) = P(\{T(x_1) > t_1\} \cap \{T(x_2) > t_2\} \cap \dots \cap \{T(x_n) > t_n\}) \quad (2)$$

for $0 \leq s \leq \omega - x_1, 0 \leq t \leq \omega - x_2$, and $0 \leq u \leq \omega - x_n$.

The copula model is a mathematical method for correlating the marginal distributions of several random variables and producing a joint distribution. One class of copula that is easily constructed is the multivariate Archimedean copula family. The multivariate Archimedean copula family used is Clayton and Frank. Clayton's and Frank's copula study has the following generating functions and multivariate copula functions respectively.

$$\psi_{\theta_c}(t) = (\max\{1 + \theta_c t, 0\})^{-\frac{1}{\theta_c}} \quad (3)$$

$$C(u_1, u_2, \dots, u_d; \theta_c) = \max \left\{ \left(\left(\sum_{i=1}^d u_i^{-\theta_c} \right) - 1 \right)^{-\frac{1}{\theta_c}}, 0 \right\}, \theta_c \geq -\frac{1}{d-1}, \theta_c \neq 0 \quad (4)$$

with θ_c is the parameter of Clayton's copula.

$$\psi_{\theta_F}(t) = -\frac{1}{\theta_F} \ln(1 - (1 - e^{-\theta_F t})e^{-t}) \quad (5)$$

$$C(u_1, u_2, \dots, u_d; \theta_F) = -\frac{1}{\theta_F} \ln \left(1 + \frac{\prod_{i=1}^d (e^{-\theta_F u_i} - 1)}{(e^{-\theta_F} - 1)^{d-1}} \right), \theta_F > 0 \quad (6)$$

with θ_F is the parameter of Frank's copula (Nelsen, 2006).

3. Fuzzy Interest Rates

Fuzzification is the process of converting crisp values to fuzzy values. Values in the field are expressed in the form of Fuzzy data which has two. The fuzzy triangular membership function

of the Fuzzy number X in U in the interval $[a, c]$ which is determined by three values (a, b, c) is defined as follows:

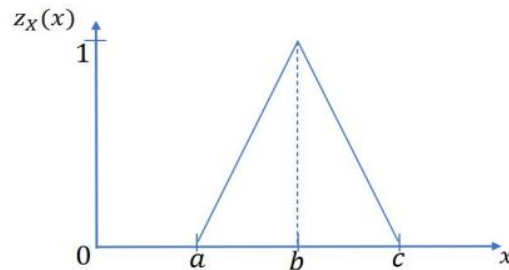


Figure 2. Triangular Fuzzy Number Graph

where (a, b, c) are the lowest value, the trusted value, and the highest value respectively from the data used. In triangular fuzzy numbers, the b value used is usually the average data (Aggarwal & Sharma, 2018). Then, given that α – cut is the threshold level that changes the Fuzzy set to crisp. The process of converting Fuzzy sets to crisp is called defuzzification. The α – cut of the Fuzzy set X is defined as follows:

$${}^\alpha X = \{x \in U: z_X(x) \geq \alpha\}, \quad \alpha \in [0,1]$$

with ${}^\alpha X$ denotes an interval that contains all x values that have a membership level greater than or equal to α . On Figure 2, intervals are in $[a', c']$ with their areas in the shaded part of the curve (Noor et al., 2013).

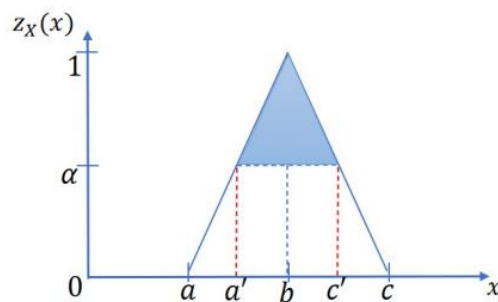


Figure 3. Triangular Fuzzy Number Graph with α – cut = α

Let \tilde{i} represents Fuzzy interest rate with,

$$\tilde{i} = (a, b, c), \quad a < b < c$$

Fuzzy number can be defined uniquely with an interval by determining the α – cut value of \tilde{i} , namely:

$${}^\alpha \tilde{i} = [a + (b - a)\alpha, c - (c - b)\alpha] = [\underline{i}(\alpha), \bar{i}(\alpha)], \quad \alpha \in [0,1] \tag{7}$$

with $\underline{i}(\alpha)$ and $\bar{i}(\alpha)$ respectively are the lower and upper limits of ${}^{\alpha}\tilde{i}$ at a certain value of α (Mircea & Covrig, 2015).

4. Pure Premium of Family Annuity

A life annuity is an annuity that is paid for life or for a certain period. As in this study, the annuity used is for life and payments during life annuity. The annuity paid at the beginning and at the end of the period paid in 1 (one) unit is stated by,

$$a_{x_1} = \sum_{t=1}^{\omega-x_1} v^t {}_t p_{x_1} \tag{8}$$

So, for cases with two or more individuals, the annuity can be expressed as follows (Dickson et al., 2009):

$$a_{x_1 x_2 \dots x_n} = \sum_{t=1}^{\omega-\max\{x_1, x_2, \dots, x_n\}} v^t {}_t p_{x_1 x_2 \dots x_n} \tag{9}$$

Furthermore, from the join distribution equation with Clayton’s and Frank’s copula applied to **Equation 9**.

The family annuity referred to here is a lifetime annuity where payments are made if the insured is still alive, payments can be made at the beginning or end of the policy period. This annuity is intended for families consisting of husband, wife, and children. Suppose a family annuity contract with the insured parties being husband (x_1), wife (x_2), and child (x_3), then the family annuity premium (\mathcal{P}) for the contract is as follows:

$$\begin{aligned} \mathcal{P} &= a_{x_1 x_2} + p_1(a_{x_1|x_2 x_3} + a_{x_2|x_1 x_3}) + p_2 p_1 a_{\overline{x_1 x_2}|x_3} \\ &= a_{x_1 x_2} + p_1(a_{x_2 x_3} - a_{x_1} + a_{x_1 x_3} - a_{x_2}) + p_2 p_1(a_{x_3} - a_{\overline{x_1 x_2}:x_3}) \\ &= a_{x_1 x_2} + p_1(a_{x_2 x_3} - a_{x_1} + a_{x_1 x_3} - a_{x_2}) + p_2 p_1(a_{x_3} - (a_{x_1 x_3} + a_{x_2 x_3} - a_{x_1 x_2 x_3})) \end{aligned}$$

with p_1 is the benefit paid to the widower and p_2 is the benefit paid to the child.

C. RESULT AND DISCUSSION

1. Determination of Survival Multiple Life Distribution

This study begins by forming a single life distribution constructed directly using TMI IV with the probability of death using the uniform distribution of death (UDD) assumption. The dependence assumption with the multivariate copula model is a relevant assumption to describe the relationship between husband, wife, and child in real life. The parameter θ on **Equation 4** and **Equation 6** can be estimated using Kendall's tau correlation which can be obtained by the following equation:

$$\tau = 1 + 4 \int_0^1 \frac{\psi_{\theta}(t)}{\frac{d}{dt} \psi_{\theta}(t)} dt \tag{10}$$

obtained an estimate of $\hat{\theta}_C$ for the copula Clayton, namely:

$$\hat{\theta}_C = \frac{2\tau}{1-\tau}$$

Whereas $\hat{\theta}_F$ for Frank's copula cannot be generated analytically (Nelsen, 2006). Using RStudio, we can determine the parameter θ through **Equation 13**. From Table 1, it can be concluded that the stronger the measure of dependence on Kendall's tau, the greater the parameters for Clayton's and Frank's copulas.

Table 1. Parameter $\hat{\theta}_C$ and $\hat{\theta}_F$

Dependability Measures	τ	$\hat{\theta}_C$	$\hat{\theta}_F$
Weak	0.25	0.66667	2.37757
Moderate	0.50	2	5.74756
Strong	0.75	6	14.14028

Paired data were simulated 105 times for each value of τ . In Figure 4 with t_x and t_y in months expressing the estimated length of life, in the scatter plot between husband and wife, the greater the value of τ the shape of the diagram decreases in the interval 200-500 months and increases after. This indicates that the dependency is strong in that interval. As for Figure 5, Figure 4 and Figure 6, the scatter plot of parents and child has almost the same shape of both father and mother with child, namely, the greater the value of τ , the smaller the shape of the scatter plot.

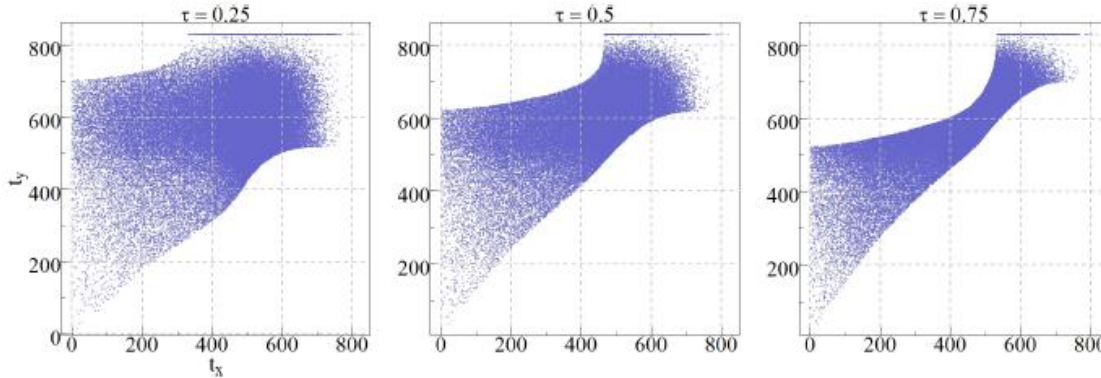


Figure 4. Scatter Plot between Husband and Wife with different Kendall'

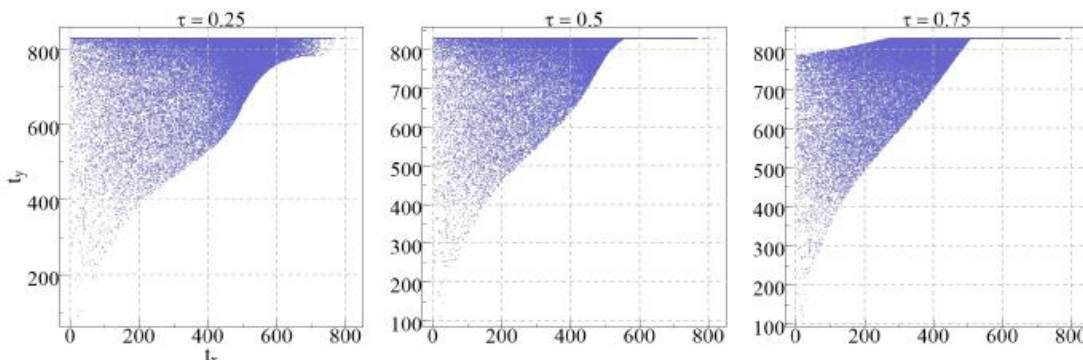


Figure 5. Scatter Plot between Husband and Child

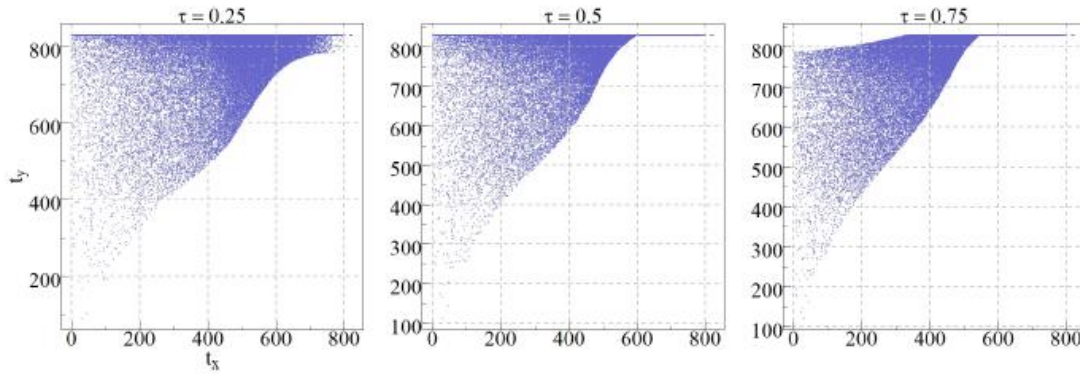


Figure 6. Scatter Plot between Wife and Child

Figure 7 shows that the graphs of husband-wife and husband-wife-child survival probabilities have almost the same characteristics with strong dependency having a larger probability value than weaker dependency. Meanwhile, the husband-child and wife-child survival probabilities show that the difference in dependency only causes a small difference in the probability value. Figure 8 shows that the characteristics of each graph are like the characteristics of the graph in Figure 7. However, the odds values in Figure 8 are slightly different from the odds values in Figure 7 for each multiple-life pair formed.

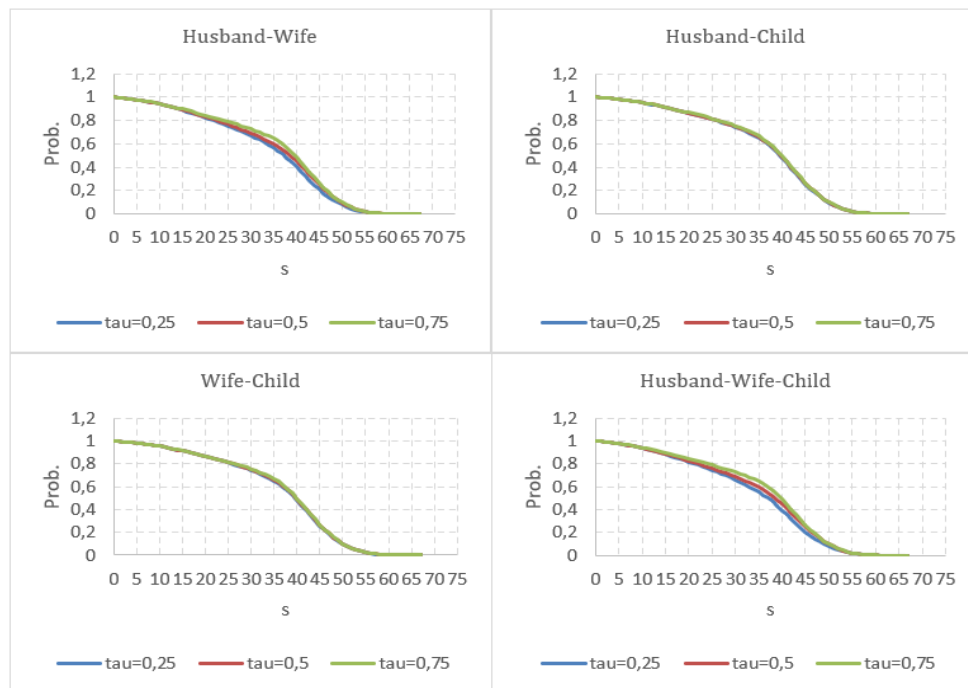


Figure 7. Survival Multiple Life Distribution using Clayton's Copula ($x_1 = 45, x_2 = 40, x_3 = 15$)

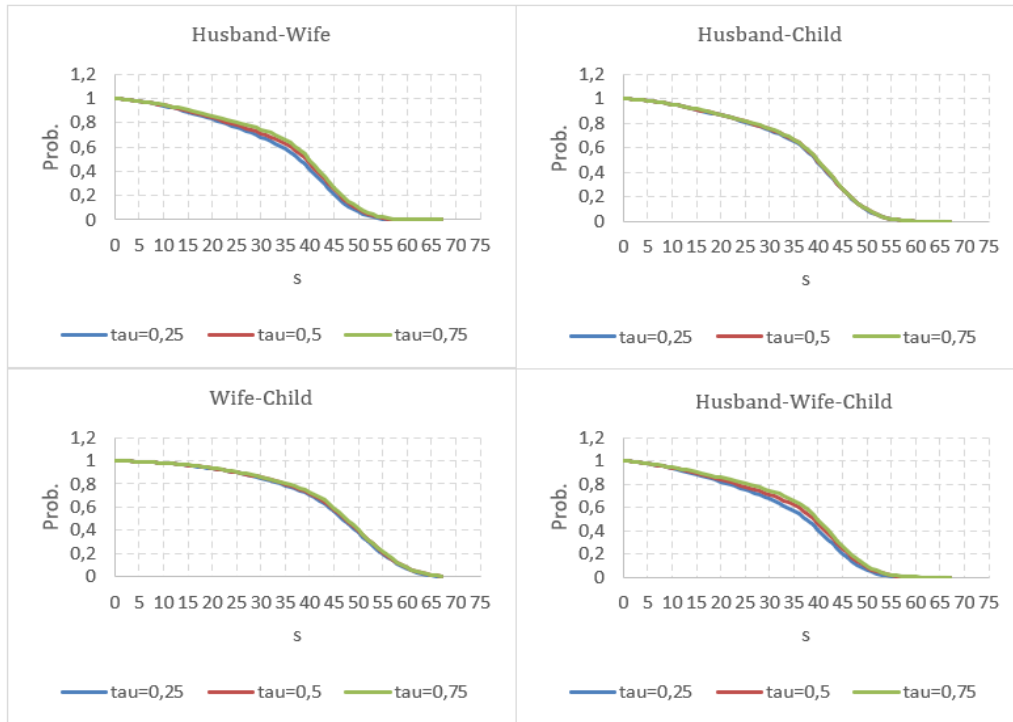


Figure 8. Survival Multiple Life Distribution using Frank's Copula ($x_1 = 45, x_2 = 40, x_3 = 15$).

2. Determination of Fuzzy Interest Rate

In estimating the actuarial margin rate, BI data per year for the period April 2016 to December 2022. The descriptive statistics of this data are shown,

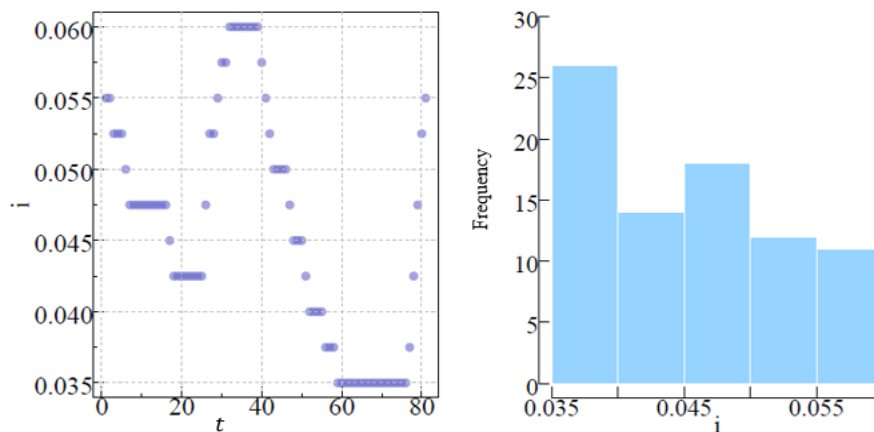


Figure 9. Plot and Histogram of BI Data

From this data, the value $\alpha = 0.437$ is obtained, which is the average of fuzzy membership values using the median as a trusted value. It is also known that $a = 0.035$ (minimum), $b = 0.060$ (maximum), and $c = 0.048$ (median). By using **Equation 10**, the actuarial margin level is obtained,

$$\begin{aligned} {}^{0.437}\tilde{i} &= [0.035 + ((0.048 - 0.035) \times 0.437), 0.060 - ((0.060 - 0.048) \times 0.437)] \\ &= [0.0404625, 0.0545375] \end{aligned}$$

In the calculation of the family annuity pure premium, the maximum value of the fuzzy interest rate is selected as $\delta = 0.054375$.

3. Determination of Family Annuity Pure Premium

The simulation results of survival single-life and joint distributions are used to calculate single-life and multiple-life annuity values. These annuity values are the components to simulate the family annuity pure premium calculation. The family annuity pure premium calculation uses fuzzy interest rate of $\delta = 0.0545375$ and benefits are paid continuously to the beneficiary with a payment rate of 1 unit each year. Annuity purchases can be made with the assumption that the husband is $45 + s$ years old, the wife is $40 + s$ years old, and the child is $15 + s$ years old. This research calculates the family annuity premium until the husband is 60 years old so that the possible s values are $s = 0, 1, 2, \dots, 15$.

Based on the simulation results in Table 2, it can be concluded that the pure premium value of the Frank copula model is greater than the Clayton copula model for each Kendall's tau value. The pure premium value at $\tau = 0.25$ for the Frank copula is 0.101 until 0.133 greater than that of the Clayton copula, at $\tau = 0.50$ for the Frank copula is greater than 0.194 until 0.259, and at $\tau = 0.75$ the Frank copula is greater than 0.199 until 0.267. Furthermore, the stronger the level of dependency, the greater the value of the annuity pure premium, but the value of the family annuity pure premium will get smaller as the age of the husband, wife, and child increases for both copula models.

Table 2. Family Annuity Pure Premium

Family Annuity ($\mathcal{P}_{x_1, x_2, x_3}$)	Clayton			Frank		
	τ			τ		
	0.25	0.5	0.75	0.25	0.5	0.75
$\mathcal{P}_{45,40,15}$	14.744	15.040	15.418	14.845	15.234	15.617
$\mathcal{P}_{46,41,16}$	14.626	14.935	15.327	14.731	15.135	15.532
$\mathcal{P}_{47,42,17}$	14.506	14.828	15.234	14.615	15.035	15.446
$\mathcal{P}_{48,43,18}$	14.386	14.721	15.141	14.498	14.934	15.359
$\mathcal{P}_{49,44,19}$	14.266	14.613	15.047	14.381	14.833	15.271
$\mathcal{P}_{50,45,20}$	14.145	14.505	14.952	14.263	14.730	15.182
$\mathcal{P}_{51,46,21}$	14.024	14.396	14.857	14.145	14.627	15.093
$\mathcal{P}_{52,47,22}$	13.901	14.286	14.760	14.025	14.523	15.001
$\mathcal{P}_{53,48,23}$	13.779	14.176	14.662	13.905	14.418	14.908
$\mathcal{P}_{54,49,24}$	13.656	14.065	14.563	13.784	14.311	14.814
$\mathcal{P}_{55,50,25}$	13.533	13.954	14.462	13.663	14.204	14.718
$\mathcal{P}_{56,51,26}$	13.411	13.844	14.362	13.543	14.097	14.621
$\mathcal{P}_{57,52,27}$	13.291	13.734	14.260	13.424	13.990	14.523
$\mathcal{P}_{58,53,28}$	13.172	13.624	14.158	13.305	13.882	14.423
$\mathcal{P}_{59,54,29}$	13.052	13.514	14.054	13.185	13.772	14.320
$\mathcal{P}_{60,55,30}$	12.930	13.401	13.945	13.063	13.659	14.212

This result constitutes a pioneering effort in the realm of family annuity products by introducing a unique paradigm involving three individuals, a distinctive departure from previous studies predominantly focused on scenarios involving two participants. In doing so, it not only advances the current understanding of actuarial assessments in this domain but also

underscores the pivotal importance of accounting for interdependencies among multiple individuals for more accurate premium calculations.

D. CONCLUSION AND SUGGESTIONS

Survival single life distribution for husband aged [45.60], wife aged [40.55], and child aged [15.30] can be constructed from TMI IV with UDD assumption. Furthermore, the survival multiple life distribution for each pair of husband-wife, husband-child, wife-child and husband-wife-child relationships can also be constructed from the survival single-life distribution. The survival multiple-life distributions are constructed using the Clayton and Frank copula model dependence assumptions for several values of Kendall's tau. Based on the simulation results, the characteristics of each graph of the Clayton model survival multiple life distribution have similarities with the Frank model, although the values of the odds between the two are slightly different. Then, the pure premium of the family annuity of the Frank copula model obtained is greater than the Clayton copula model for each value of τ . The greater the value of τ , the greater the value of the pure premium of each age. However, inversely proportional to the increase in age, the value of the pure premium will be smaller with an average decrease of about 0.99%. To enhance the versatility of premium offerings, it is recommended to explore alternative copula models beyond Clayton and Frank, considering their distinct characteristics, to provide a more comprehensive range of premium options in the analysis of family annuity products.

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