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Algorithm for Constructing Total Graph of Commutative Ring

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ABSTRACT

Let R be a commutative ring. The total graph of R, denoted by $T\Gamma(R)$ is a graph whose vertices are all elements of the ring R and every $i, j \in R$ with $i \neq j$, then i and j vertices are connected by edges if and only if $i + j \in Z(R)$, where $Z(R)$ is the set of zero-divisors in R with $0 \in Z(R)$. Python programming is code that is easy to learn, read, understand, and helpful in explaining problems regarding graphs and algebra. In this paper, we determine an algorithm to construct the total graph of ring $\mathbb{Z}_n$ using Python. The research methods in this paper is a literature studies. The results generated by the algorithm can be utilized to observe the characteristic patterns displayed by the graph. Based on the algorithm's constructed graph pattern, several properties of $T\Gamma(\mathbb{Z}_n)$ can be inferred. For instance, if n is a prime number, then $T\Gamma(\mathbb{Z}_n)$ is a disconnected graph. On the other hand, if n is a prime number and $n \geq 3$, then $T\Gamma(\mathbb{Z}_{2n})$ and $T\Gamma(\mathbb{Z}_{4n})$ is a connected graph, regular graph, Hamiltonian graph, and has a girth $gr\left(T\Gamma(\mathbb{Z}_n)\right) = 3$. In this paper we creating an algorithm to construct total graphs from commutative rings streamlines the construction process, enhances accessibility and utilization of total graphs, and supports parameter variation exploration and application in problem-solving.

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A. INTRODUCTION

Mathematics is a science that is widely studied and applied in various fields. Mathematics is very closely related to the way of thinking of someone who can influence life. The rapid development of information and communication technology is based on the development of mathematics in analysis, number theory, probability theory, algebra, and discrete mathematics. Discrete mathematics is a branch of mathematics that studies all discrete things. One branch of the discussion of discrete mathematics namely graph theory. Graph theory has a long history in mathematics, originating from the Seven Konigsberg Bridges problem, which was proven by Leonhard Euler in 1736 (Mir, 2014). Graph G is a pair (V, E) where V is a finite set and non-empty of objects called vertices, and E is a set (can be empty) unordered pairs of distinct vertices V, which are called edges (Akbari et al., 2009). The set of vertices and edges in a sequential graph G can be denoted by $V\ (G)$ and $E(G)$. Graph G which is not having edges is said to be an empty graph. Graph G is called a connected graph if a path is found for each pair of different vertices in G. The girth of G, denoted by $gr(G)$, is shortest cycle length in G.

Graph theory has become an essential mathematical tool across various subjects recently. Graphs can also be obtained from algebraic structures in groups and rings. Beck (1988) was the first to introduce graph theory in relation to the ring. Several graphs were obtained of ring algebraic structures, including zero-divisor graphs over a commutative ring (Anderson & Lewis, 2016), the unit graph of a non-commutative ring (Akbari et al., 2015), the generalized total graph of a commutative ring (Anderson & Badawi, 2013), the annihilator graph of a commutative ring (Nikmehr et al., 2017), and the total zero-divisor graph of commutative rings (Đurić et al., 2018). Ring R is a non-empty set R which contains two binary operations viz addition and multiplication that satisfy the abelian group properties for operations addition, closed for multiplication, associative for multiplication, and the multiplication operation is distributive to the addition operation. Let R be a commutative ring. In (2008), Anderson and Badawi introduced the total graph of R, denoted by $T(\Gamma(R))$ is a graph whose set of vertices is all members of the ring R and for each different $i, j \in R$ will be adjacent if and only if $i + j \in Z(R)$ where $Z(R)$ is the set of zero-divisors in R and in line with (Akbari et al., 2009) in this paper we assume that $0 \in Z(R)$.

In this paper, the ring used is a ring of integer modulo n. A ring of integer modulo n is denoted by $\mathbb{Z}_n$. Some researchers have also studied a graph with ring of integer modulo n, as in (Basak et al., 2019), (Mishra & Patra, 2020), and (Ju et al., 2014). In (2021), Any and Hidayah have researched the girth of the total graph of $\mathbb{Z}_{n,}$ denoted by $gr\left(T_\Gamma(\mathbb{Z}_n)\right)$. Kalita et al. (2014) introduce the prime graph of the commutative Ring $\mathbb{Z}_{n,}$. Prime graph is a graph associated with a ring denoted $PG(R)$. $PG(R)$ is defined as the graph whose vertices are elements of a ring R and any two vertices a and b of R are adjacent if and only if $xRy = 0$ or $yRx = 0$. The vertex degree of vertex v on graph G is denoted by $deg(v)$ is the number of vertices adjacent to v in graph G (Akbari et al., 2009). Several researchers have researched total graphs, including (Anderson & Badawi, 2008), (Akbari et al., 2009), and (Mondal et al., 2023). Anderson and Weber (2018) investigated zero-divisor graphs of R, where R contains no identities. By modifying the definition of zero-divisor graphs, relationships between zero-divisor graphs and commutative rings has increased greatly interest among scholars (Aalipour & Akbari, 2016).

Python is a versatile widely used programming language renowned for its simplicity and readability. Python is well renowned for being straightforward and adaptable, making it a popular choice for a variety of applications (Tangirov & Rakhimov, 2023). The Python programming language was created with the aim of making it easier to read code from productivity and writing syntax in higher level development. Python was created by Guido van Rossum in the late 1980s and early 1990s at the National Research Institute for Mathematics and Computer Science in Netherlands (Saabith et al., 2019). In the graphic concept of ring, several researchers have used python algorithms such as in (Tapanyo et al., 2022), (Kansal et al., 2022), (Hartati & Kurniawan, 2023), and (Tian & Li, 2022). In previous research, no algorithm was given to construct the total graph. This paper aims to determine an algorithm for constructing the total graph of ring $\mathbb{Z}_n$ using Python and determine the properties of the graph. In this paper we creating an algorithm to construct total graphs from commutative rings streamlines the construction process, enhances accessibility and utilization of total graphs, and supports parameter variation exploration and application in problem-solving. The first section of this paper is the introduction. Then the algorithm constructs a graph. An example of how the algorithm works is given, and the last section is the conclusion of this discussion.

B. METHODS

This method used in this research is a literature studies by collecting and reviewing various references related to be research topics such as graph theory, structure of algebra, Python programming, and the total graph of a commutative ring. The steps taken in this research are as follows.

a. Learning basic definitions and theorems related to graph theory and stucture of algebra.
b. Create an algorithm for constructing the total graph of the ring of integers modulo n with Python.
c. Investigating the properties of the total graph of the ring of integers modulo n.
d. Making conclusions.

In general, the research procedure is presented in Figure 1.

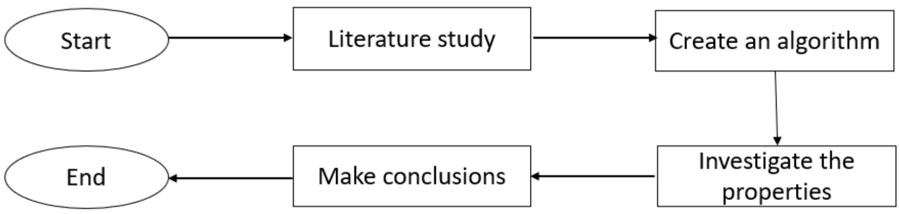


Figure 1. Research procedure chart

The following explains several definitions that underlie this research, include the basic concepts of graphs, rings, dan the total graph of a commutative ring.

1. Basic Concepts of Graphs

The definitions of basic graph concepts discussed below refer to Chartrand et al. (2016).

Definition 2.1. A graph G is a finite non-empty set V containing objects called vertices (the singular from is vertex) and a possibly empty set E which consists of two element subsets of V called edges. The order of a graph G is represented by the symbols $|V(G)|$ for the number of vertices and $|E(G)|$ for the number of edges. As an example, below is an illustration of graph G_1.

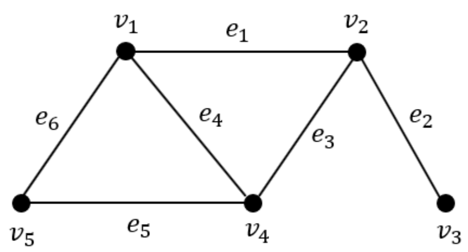


Figure 2. Graph G_1

Figure 2 is an illustration of graph G_1 which has $|V(G_1)| = 5$, and $|E(G_1)| = 6$, with a vertex set $V(G_1) = \{v_1, v_2, v_3, v_4, v_5\}$ and an edge set $E(G_1) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Two vertices are said to be adjacent if there is an edge connecting them. For example, graph G_1 in Figure 2, vertices v_1

and v_2 are adjacent because there is $e_1$which connects them, while v_3 and v_4 are not adjacent. Then edge e_1 is called incident with vertices v_1 and v_2.

Definition 2.2. The degree of a vertex v in a graph G, denoted $deg(v)$, is the number of proper edges incident on v plus twice the number of self-loops.

Definition 2.3. A $u - v$ walk in a graph G is a finite sequence of vertices starting with vertex u and ends at vertex v, where the vertices in the sequence are adjacent.

Definition 2.4. A $u - v$ walk on a graph G that does not contain repeated vertices is called a $u - v$ path. For example, consider the graph G_1 in Figure 2 sequence $v_1 - e_1 - v_2 - e_3 - v_4 - e_4 - v_1 - e_6 - v_5$ is a walk, while the sequence $v_5 - e_5 - v_4 - e_3 - v_2 - e_2 - v_3$ is a path because it doesn't repeat any vertex.

Definition 2.5. A graph G is connected if every two vertices u and v of G, there is a $u - v$ path.

Definition 2.6. A empty graph is a graph whose size 0.

Definition 2.7. A complete graph is a simple graph where any two distinct vertices are adjacent.

Definition 2.8. A regular graph G is a graph whose vertices in G have the same degree.

Definition 2.9. A cycle in G that passes through all vertices in G exactly once is called a Hamiltonian cycle of G.

Definition 2.10. A Hamiltonian graph of G is a connected graph that contains the Hamiltonian cycle.

Definition 2.11. The girth of a graph G is the length of the shortest cycle in graph G.

2. Basic Concepts of Ring

The definitions of basic ring concepts discussed below refer to Adhikari & Adhikari (2014).

Definition 2.12. A ring is an ordered triple $(R, +, \cdot)$ composed up of a non-empty set R and two binary operations '$+$' (also known as addition) and '$\cdot$' (also known as multiplication) where

a. $(R, +)$ is an abelian group;
b. $(R, \cdot)$ is a semigroup, i.e., $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in R$; and
c. the operation '$\cdot$' is distributive (on both sides) over the operation '$+$', i.e., $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b, c \in R$ (distributive laws).

Definition 2.13. The non-zero element a in the ring R is said to be a left (right) zero-divisor iff $\exists$ a non-zero element $b \in R$ such that $ab = 0$ ($ba = 0$). The zero-divisor is an element of R which is the left and a right zero-divisor i.e., the zero-divisor is the element a which divides 0.

3. The Total Graph of Commutative Ring

The definitions of total graph of commutative ring discussed below refer to Anderson & Badawi (2008).

Definition 2.14. Let R be a commutative ring. The total graph of R, denoted by $T(\Gamma(R))$ is a graph whose set of vertices is all members of the ring R and for each different $i, j \in R$ will be adjacent if and only if $i + j \in Z(R)$ where $Z(R)$ is the set of zero-divisors in R. Example 1. The following is an example is an example of $T(\Gamma(\mathbb{Z}_6))$ where $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$.

Table 1. Cayley Table of $\mathbb{Z}_6$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

By definition, the vertex set of $T(\Gamma(\mathbb{Z}_6))$ is all members of the ring $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$. For all $\mathbb{Z}_6$ elements, check if there is a positive integer b such that $ab = 0$. Based of Table 1, we have a zero divisor elemen in ring $Z(\mathbb{Z}_6) = \{0,2,3,4\}$. Then, for each different $i, j \in \mathbb{Z}_6$, will be adjacent if and only if $i + j \in Z(\mathbb{Z}_6)$. The total graph of $\mathbb{Z}_6$ is shown in Figure 3.

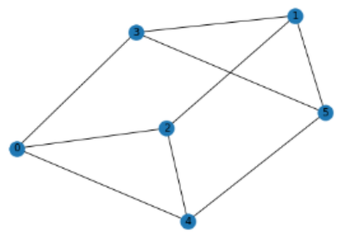


Figure 3. $T(\Gamma(\mathbb{Z}_6))$

Figure 3 is the total graph of $\mathbb{Z}_6$ which has 6 vertices and 9 edges.

C. RESULT AND DISCUSSION

1. Algorithm for Constructing $T\Gamma(\mathbb{Z}_n)$

The first algorithm to construct a total graph on a $\mathbb{Z}_n$ integer ring is parameterized by a zero-divisor. The recursive algorithm is called TotalGraph. Then import the library for drawing graphs in Python. The following defines a total graph.

GraphTotal (var $T(\Gamma(\mathbb{Z}_n))$: graph)

```
Import library networkx and matplotlib.pyplot
Step 1:
    for a, b ∈ ℤ_n
        if a * b = 0
            Add a to Z(ℤ_n)
        else
            goto Step 1;
    end for;
Step 2:
    for i ∈ ℤ_n
        add i to V(T(Γ(ℤ_n)))
        for j ∈ ℤ_n do
            if i ≠ j
                if i + j ∈ Z(ℤ_n)
                    Add i~j to E(T(Γ(ℤ_n)))
                else
                    goto Step 2;
        else
                goto Step 2;
    end for;
Step 3:
    draw T(Γ(ℤ_n))
end;
```

Import library networkx and mathplotlib.pyplot first. In step 1, the algorithm looks for the zero-divisor elements of $\mathbb{Z}_n$. For all $\mathbb{Z}_n$ elements, check if there is a positive integer b such that $ab = 0$. If there is, then a is a zero-divisor element of $\mathbb{Z}_n$, add a to $Z(\mathbb{Z}_n)$. However, if no b satisfies the condition, it returns to the looping process in step 1. Next, during step 2, determine the set of vertices $V\big(T\Gamma(\mathbb{Z}_n)\big)$. For all $i \in \mathbb{Z}_n$, add i to $V\big(T\Gamma(\mathbb{Z}_n)\big)$ as a vertex. Afterward, the algorithm verifies the adjacency between vertices. For $i, j \in V\big(T\Gamma(\mathbb{Z}_n)\big)$, if $i \neq j$, then continue. Otherwise, it reverts to the looping process in step 2. If $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. Otherwise, it reverts to the looping process in step 2. At this point, we have acquired the set of vertices $V\big(T\Gamma(\mathbb{Z}_n)\big)$ and the set of edge $E\big(T\Gamma(\mathbb{Z}_n)\big)$. Lastly, in step 3, draw the total graph of $\mathbb{Z}_n$.

a. Example 3.1.

Given $\mathbb{Z}_n$ for $n = 1, 2$.

For $n = 1$, we have $\mathbb{Z}_1 = \{0\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_1$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_1)$. At this point, we have $Z(\mathbb{Z}_1) = \{\ \}$. Then, in step 2, determines $V\big(T\Gamma(\mathbb{Z}_1)\big)$. For all $i \in \mathbb{Z}_1$, add i to $V\big(T\Gamma(\mathbb{Z}_1)\big)$ as a vertex, so that $V\big(T\Gamma(\mathbb{Z}_1)\big) = \{0\}$. Then the algorithm checks adjacency between vertices. For $i, j \in V\big(T\Gamma(\mathbb{Z}_1)\big)$, if $i \neq j$, then continue.

However, in this case, there is only one vertex, the condition can't be fulfilled. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph.

For $n = 2$, we have $\mathbb{Z}_1 = \{0,1\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_2$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_2)$. At this point, we have $Z(\mathbb{Z}_2) = \{0\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_2))$. For all $i \in \mathbb{Z}_2$, add i to $V(T\Gamma(\mathbb{Z}_2))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_2)) = \{0,1\}$. Then the algorithm checks adjacency between vertices. If there are $i, j \in V(T\Gamma(\mathbb{Z}_2))$ with $i \neq j$, then the vertices i and j are connected by edges if and only if $i + j \in Z(\mathbb{Z}_n)$. But in this case, no edges connecting between vertices are found. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph.

(a) $T\Gamma(\mathbb{Z}_1)$ (b) $T\Gamma(\mathbb{Z}_2)$

Figure 4. The Total Graph of $\mathbb{Z}_n$ for (a) $n = 1$, (b) $n = 2$

Figure 4 is the result of the construction of the total graph of $\mathbb{Z}_n$, for $n = 1,2$. The total graph of $\mathbb{Z}_1$ consists of 1 vertex and has no edges, while the total graph of $\mathbb{Z}_2$ consists of 2 vertices and has no edges. Because the graphs has no edges, then for $n = 1,2$, $T\Gamma(\mathbb{Z}_n)$ is an empty graph.

b. Example 3.2.

Construct the total graph of $\mathbb{Z}_n$, for n is prime and $n \geq 3$. For $n = 3$, we have $\mathbb{Z}_3 = \{0,1,2\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_3$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_3)$. At this point, we have $Z(\mathbb{Z}_3) = \{0\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_3))$. For all $i \in \mathbb{Z}_3$, add i to $V(T\Gamma(\mathbb{Z}_3))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_3)) = \{0,1,2\}$. Then the algorithm checks adjacency between vertices. For $i, j \in V(T\Gamma(\mathbb{Z}_3))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 1, j = 2$, so that $i + j = 0 \in Z(\mathbb{Z}_3)$. Because the conditions are met, vertices 1 and 2 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_3))$, the program will loop back to the process. For $i = 0$, there is no $j \in V(T\Gamma(\mathbb{Z}_3))$, so vertex 0 is not adjacent to any vertex. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_3$.

For $n = 5$, we have $\mathbb{Z}_5 = \{0,1,2,3,4\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_5$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_5)$. At this point, we have $Z(\mathbb{Z}_5) = \{0\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_5))$. For all $i \in \mathbb{Z}_5$, add i to $V(T\Gamma(\mathbb{Z}_5))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_5)) =$

$\{0,1,2,3,4\}$. Then the algorithm checks adjacency between vertices. For $i, j \in V(T\Gamma(\mathbb{Z}_5))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 1$, $j = 4$, so that $i + j = 0 \in Z(\mathbb{Z}_5)$. Because the conditions are met, vertices 1 and 4 are adjacent. Upon resuming the looping process, it is discovered that $i = 2, j = 3$, so that $i + j = 0 \in Z(\mathbb{Z}_5)$. Because the conditions are met, vertices 2 and 3 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_5))$, the program will loop back to the process. For $i = 0$, there is no $j \in V(T\Gamma(\mathbb{Z}_5))$, so vertex 0 is not adjacent to any vertex. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_5$.

(a) $T\Gamma(\mathbb{Z}_3)$ (b) $T\Gamma(\mathbb{Z}_5)$

Figure 5. The Total Graph of $\mathbb{Z}_n$ for (a) $n = 3$, (b) $n = 5$

Figure 5 is the result of the construction of the total graph of $\mathbb{Z}_n$, for $n = 3,5$. The total graph of $\mathbb{Z}_3$ consists of 3 vertices and 1 edge, while the total graph of $\mathbb{Z}_5$ consists of 5 vertices and 2 edges. Because a path is not always found at every vertex in the graph, then for $n = 3,5$, $T\Gamma(\mathbb{Z}_n)$ is disconnected graph.

c. Example 3.3.

Construct the total graph of integer ring mudulo $2n$, for n is prime and $n \geq 3$. For $n = 3$, we have $\mathbb{Z}_6 = \{0,1,2,3,4,5\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_6$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_6)$. At this point, we have $Z(\mathbb{Z}_6) = \{0,2,3,4\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_6))$. For all $i \in \mathbb{Z}_6$, add i to $V(T\Gamma(\mathbb{Z}_6))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_6)) = \{0,1,2,3,4,5\}$. Then the algorithm checks adjacency between vertices. For $i, j \in V(T\Gamma(\mathbb{Z}_6))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 1, j = 3$, so that $i + j = 4 \in Z(\mathbb{Z}_6)$. Because the conditions are met, vertices 1 and 3 are adjacent. Upon resuming the looping process, it is discovered that $i = 1, j = 5$, so that $i + j = 0 \in Z(\mathbb{Z}_6)$. Because the conditions are met, vertices 1 and 5 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_6))$, the program will loop back to the process. Through the looping process, adjacencies between different vertices are identified. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_6$.

For $n = 5$, we have $\mathbb{Z}_{10} = \{0,1,2,3,4,5,6,7,8,9\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_{10}$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_{10})$. At this point, we have $Z(\mathbb{Z}_{10}) = \{0,2,4,5,6,8\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_{10}))$. For all $i \in \mathbb{Z}_{10}$, add i to $V(T\Gamma(\mathbb{Z}_{10}))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_{10})) = \{0,1,2,3,4,5,6,7,8,9\}$. Then the algorithm checks adjacency between vertices. For $i,j \in V(T\Gamma(\mathbb{Z}_{10}))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 1, j = 3$, so that $i + j = 4 \in Z(\mathbb{Z}_6)$. Because the conditions are met, vertices 1 and 3 are adjacent. Upon resuming the looping process, it is discovered that $i = 1, j = 5$, so that $i + j = 6 \in Z(\mathbb{Z}_{10})$. Because the conditions are met, vertices 1 and 5 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_{10}))$, the program will loop back to the process. Through the looping process, adjacencies between different vertices are identified. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_{10}$.

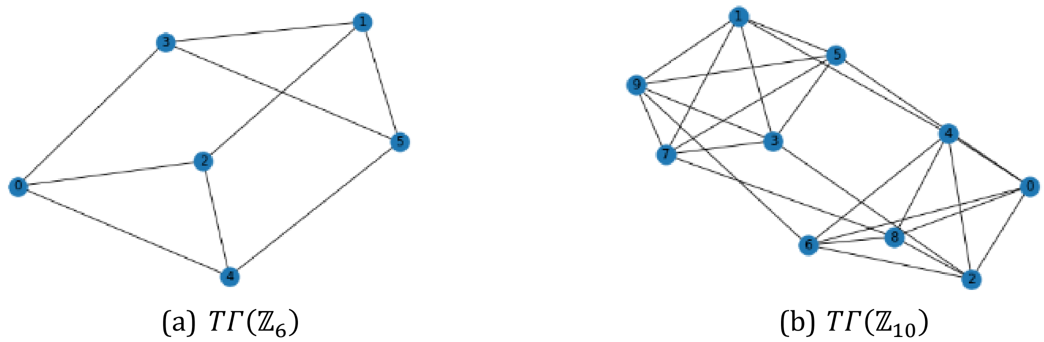
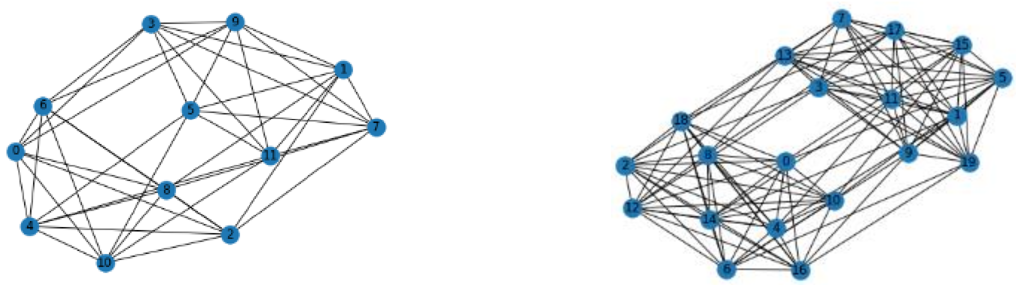


Figure 6. The Total Graph of $\mathbb{Z}_{2n}$ for (a) $n = 3$, (b) $n = 5$

Figure 6 is the result of the construction of the total graph of $\mathbb{Z}_{2n}$, for $n = 3{,}5$. The total graph of $\mathbb{Z}_6$ consists of 6 vertices and 9 edges, while the total graph of $\mathbb{Z}_{10}$ consists of 10 vertices and 25 edges. Because if every two vertices of the graph there is a path and every vertex has the same degree, then for $n = 3{,}5$, $T\Gamma(\mathbb{Z}_{2n})$ is connected graph and regular graph.

d. Example 3.4.

Construct the total graph of integer ring mudulo $4n$, for n is prime and $n \geq 3$. For $n = 3$, we have $\mathbb{Z}_{12} = \{0,1,2,\ldots,10,11\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_{12}$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_{12})$. At this point, we have $Z(\mathbb{Z}_{12}) = \{0,2,3,4,6,8,9,10\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_{12}))$. For all $i \in \mathbb{Z}_{12}$, add i to $V(T\Gamma(\mathbb{Z}_{12}))$ as a vertex, so that $V(T\Gamma(\mathbb{Z}_{12})) = \{0,1,2,\ldots,10,11\}$. Then the algorithm checks adjacency between vertices. For $i,j \in V(T\Gamma(\mathbb{Z}_{12}))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 2$, $j = 4$, so that $i + j = 6 \in Z(\mathbb{Z}_{12})$. Because the conditions are met, vertices 2 and 4 are adjacent. Upon resuming the looping process, it is discovered that $i = 2$, $j = 6$, so that $i + j = 8 \in Z(\mathbb{Z}_{12})$. Because the conditions are met, vertices 2 and 6 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_{12}))$, the program will loop back to the process. Through the looping process, adjacencies between different vertices are identified. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_{12}$. For $n = 5$, we have $\mathbb{Z}_{20} = \{0,1,2,\ldots,18,19\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_{20}$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_{20})$. At this point, we have $Z(\mathbb{Z}_{20}) = \{0,2,4,5,6,8,10,12,14,15,16,18\}$. Then, in step 2, determines $V(T\Gamma(\mathbb{Z}_{20}))$. For all $i \in \mathbb{Z}_{20}$, add i to $V\big(T\Gamma(\mathbb{Z}_{20})\big)$ as a vertex, so that $V\big(T\Gamma(\mathbb{Z}_{20})\big) = \{0,1,2,\dots,18,19\}$. Then the algorithm checks adjacency between vertices. For $i,j \in V\big(T\Gamma(\mathbb{Z}_{20})\big)$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 2, j = 4$, so that $i + j = 6 \in Z(\mathbb{Z}_{20})$. Because the conditions are met, vertices 2 and 4 are adjacent. Upon resuming the looping process, it is discovered that $i = 2, j = 6$, so that $i + j = 8 \in Z(\mathbb{Z}_{20})$. Because the conditions are met, vertices 2 and 6 are adjacent. As there remains an unexplored vertex $V\big(T\Gamma(\mathbb{Z}_{20})\big)$, the program will loop back to the process. Through the looping process, adjacencies between different vertices are identified. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_{20}$.



(a) $T\Gamma(\mathbb{Z}_{12})$ (b) $T\Gamma(\mathbb{Z}_{20})$

Figure 7. The Total Graph of $\mathbb{Z}_{4n}$ for (a) $n = 3$, (b) $n = 5$

Figure 7 is the result of the construction of the total graph of $\mathbb{Z}_{4n}$, for $n = 3,5$. The total graph of $\mathbb{Z}_{12}$ consists of 12 vertices and 42 edges, while the total graph of $\mathbb{Z}_{20}$ consists of 20 vertices and 110 edges. Because if every two vertices of the graph there is a path and every vertex has the same degree, then for $n = 3,5$, $T\Gamma(\mathbb{Z}_{4n})$ is connected graph and regular graph.

e. Example 3.5.

Construct the total graph of integer ring mudulo 2^n, for $n \geq 2$. For $n = 2$, we have $\mathbb{Z}_4 = \{0,1,2,3\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_4$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_4)$. At this point, we have $Z(\mathbb{Z}_4) = \{0,2\}$. Then, in step 2, determines $V\big(T\Gamma(\mathbb{Z}_4)\big)$. For all $i \in \mathbb{Z}_4$, add i to $V\big(T\Gamma(\mathbb{Z}_4)\big)$ as a vertex, so that $V\big(T\Gamma(\mathbb{Z}_4)\big) = \{0,1,2,3\}$. Then the algorithm checks adjacency between vertices. For $i,j \in V\big(T\Gamma(\mathbb{Z}_4)\big)$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 0, j = 2$, so that $i + j = 2 \in Z(\mathbb{Z}_4)$. Because the conditions are met, vertices 0 and 2 are adjacent. Upon resuming the looping process, it is discovered that $i = 1, j = 3$, so that $i + j = 0 \in Z(\mathbb{Z}_4)$. Because the conditions are met, vertices 1 and 3 are adjacent. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_4$.

For $n = 3$, we have $\mathbb{Z}_8 = \{0,1,2,3,4,5,6,7\}$. In step 1, the algorithm checks for zero-divisor elements, that is, if $a, b \in \mathbb{Z}_8$, if there exists a positive integer b in such way that $ab = 0$, then add a to $Z(\mathbb{Z}_8)$. At this point, we have $Z(\mathbb{Z}_8) = \{0,2,4,6\}$. Then, in step 2, determines $V\big(T\Gamma(\mathbb{Z}_8)\big)$. For all $i \in \mathbb{Z}_8$, add i to $V\big(T\Gamma(\mathbb{Z}_8)\big)$ as a vertex, so that $V\big(T\Gamma(\mathbb{Z}_8)\big) =$

$\{0,1,2,3,4,5,6,7\}$. Then the algorithm checks adjacency between vertices. For $i, j \in V(T\Gamma(\mathbb{Z}_8))$, if $i \neq j$, then continue. Next, we will check whether $i + j \in Z(\mathbb{Z}_n)$, then, i and j are adjacent. It's found when $i = 0$, $j = 2$, so that $i + j = 2 \in Z(\mathbb{Z}_8)$. Because the conditions are met, vertices 0 and 2 are adjacent. Upon resuming the looping process, it is discovered that $i = 0, j = 4$, so that $i + j = 4 \in Z(\mathbb{Z}_8)$. Because the conditions are met, vertices 0 and 4 are adjacent. As there remains an unexplored vertex $V(T\Gamma(\mathbb{Z}_8))$, the program will loop back to the process. Through the looping process, adjacencies between different vertices are identified. As a result, the looping process in this step concludes and the algorithm moves on to step 3 to draw the total graph of $\mathbb{Z}_8$.

(a) $T\Gamma(\mathbb{Z}_4)$ (b) $T\Gamma(\mathbb{Z}_8)$

Figure 8. The Total Graph of $\mathbb{Z}_{2^n}$, for (a) $n = 2$, (b) $n = 3$

Figure 8 is the result of the construction of the total graph of $\mathbb{Z}_{2^n}$, for $n = 2{,}3$. The total graph of $\mathbb{Z}_4$ consists of 4 vertices and 2 edges, while the total graph of $\mathbb{Z}_8$ consists of 8 vertices and 12 edges. Because the graph is devided into two components with each vertex in each component always connected by an edge, then for $n = 2{,}3$, $T\Gamma(\mathbb{Z}_{2^n})$ is a disconnected graph consisting of two components, where each component is a complete graph.

f. Example 3.6.

Finally, an example for the high-order case n is shown, and can be observed in an easy way to construct a total graph using an algorithm while taking the processing time of the program into account.

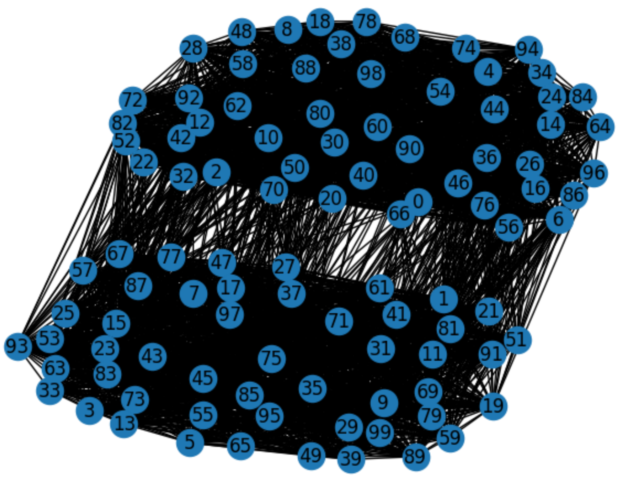


Figure 9. Total Graph of $\mathbb{Z}_n$ for greater integer values of n

Figure 9 is the result of the construction of the total graph of $\mathbb{Z}_n$ for $n = 100$. The total graph of $\mathbb{Z}_{100}$ has 100 vertices and 2950 edges. The total graph of $\mathbb{Z}_{100}$ construction process is less than 4 seconds. Undoubtedly, using an algorithm constructions program produces faster results compared to manual construction.

2. Observation of the Characteristics of $T\Gamma(\mathbb{Z}_n)$

Following are several observations that show the characteristics of $T\Gamma(\mathbb{Z}_n)$ for several n cases.

a. If $n \leq 2$, then $T\Gamma(\mathbb{Z}_n)$ is an empty graph.
b. If prime and $n \geq 3$, then $T\Gamma(\mathbb{Z}_n)$ is disconnected graph.
c. If n prime and $n \geq 3$, then $T\Gamma(\mathbb{Z}_{2n})$ and $T\Gamma(\mathbb{Z}_{4n})$ is connected graph.
d. If n is a prime and $n \geq 3$, then $T\Gamma(\mathbb{Z}_{2n})$ is a regular graph with $d(v_i) = n$ for all $v_i \in V(T\Gamma(\mathbb{Z}_n))$.
e. If n is a prime and $n \geq 3$, then $T\Gamma(\mathbb{Z}_{4n})$ is a regular graph with $d(v_i) = 2n + 1$ for all $v_i \in V(T\Gamma(\mathbb{Z}_n))$.
f. If n prime and $n \geq 3$, then $T\Gamma(\mathbb{Z}_{2n})$ and $T\Gamma(\mathbb{Z}_{4n})$ is Hamiltonian graph.
g. If $T\Gamma(\mathbb{Z}_n)$ is connected graph., then girth $gr(T\Gamma(\mathbb{Z}_n)) = 3$.
h. If $n = 2^n$ where $n \geq 2$, then $T\Gamma(\mathbb{Z}_n)$ is a disconnected graph consisting of two components, where each component is a complete graph.

D. CONCLUSION AND SUGGESTIONS

In this paper, we have defined a total graph of ring R and described the algorithm for constructing a total graph using Python. Some examples of how the GraphTotal algorithm for working in ring $\mathbb{Z}_n$ has been given. Observational results of the properties from total graph of ring $\mathbb{Z}_n$ have been shown. The results of these observations are useful as assumptions about the properties that apply to the total graph of ring $\mathbb{Z}_n$. Finally, GraphTotal algorithm to constructing $T(\Gamma(\mathbb{Z}_n))$ using the Python program is very useful because it takes only a short time to construct the graph. In future research, it is hoped that future researchers can study algorithms for constructing other graphs or graphs on other rings.

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