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Dynamical Analysis of a Predator-Prey Model Involving Intraspecific Competition in Predator and Prey Protection

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ABSTRACT

This article explains the interaction of the prey-predator model in the presence of wild harvesting and competition intra-specific predator populations and prey protection zones. Model construction is based on literature studies related to the basic theory of the model and the biological properties between predator and prey populations. This study aims to look at the dynamic conditions of the predator-prey model in the form of the existence of prey and predator populations and the impact that occurs in the long term for both populations due to changes in parameter values. The model analysis begins with the formulation of the solution conditions and boundaries model, and next with the determination of the equilibrium point. Every equilibrium point is analyzed by the characteristic of its stability is neither local or global. The model owns three equilibrium points, namely the equilibrium point of population extinction (E_0), the equilibrium point of predator extinction (E_1), and the equilibrium point of persistence of the two populations (E_2). These equilibrium points are stable locally or globally if certain conditions are met. Next, it is shown that bifurcation proceeds Which describes the changing of characteristic stability point equilibrium Which depends on the threshold parameter values h_1, Ω^*, and ρ^*. In the end, numerical simulations are presented in the form of phase, time-series, and bifurcation diagrams to support the analytical results of the model, as well as to visually show the dynamic behaviour of the interaction between the two populations based on changes in predation levels, illegal harvesting, prey refuge zones, and intra-specific competition.

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A. INTRODUCTION

The predator-prey model was first introduced by Lotka (1910) and Volterra (1926), and is widely known as the Lotka-Volterra model. The Lotka-Volterra model introduced the concept of oscillation in predator-prey dynamics. Since then, model development has continued in various considerations to adapt to more realistic biological phenomena in cases related to predator-prey interactions. Predator and prey interactions describe the dynamic relationship between individuals who act as predators and individuals who act as prey (Cresswell, 2019; Schmidt, 2019). This is an important part of the food chain and ecosystem. One of the most interesting factors to consider in developing predator-prey models is the presence of intraspecific competition between individuals. Intraspecific competition is a form of competition between individuals of the same species for important resources such as food, water, space, sunlight, or mates. Intraspecific competition occurs due to limited resources. Some papers on intraspecific competition can be found in (Gilad, 2008; Los Huertos, 2020;

Vanni et al., 2009). Predator-prey models that consider intraspecific competition are interesting to discuss because they represent the reality of many individuals, such as dee (Stone et al., 2019), birds (Tarjuelo et al., 2017), fish (Pelage et al., 2022), various types of Insects (Pekas et al., 2023), primate animals such as baboons (Patterson et al., 2021), and others. Some models have been worked out by considering intraspecific competition (Anggriani et al., 2023; Panigoro et al., 2023; Pratama et al., 2023).

Pratama et al. (2023) built a predator-prey model considering intraspecific competition using the Holling type IV response function. The model also considers the Fear and Group Defense Effect on the population dynamics of Predator and Prey. Furthermore, (Panigoro et al., 2023) built a Predator-Prey model with intraspecific competition but considering the age structure of the predator. The model discussed divides the population into 3, namely the prey population, the immature predator population, and the adult predator population. Meanwhile, Anggriani et al. (2023) built a predator-prey model with intraspecific competition and also considered the alle effect:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - axy \qquad (1)$$
$$\frac{dy}{dt} = bxy - dy - \frac{my}{y+n} - \beta y^2$$

with r, K, a, b, d, β are positive constants representing intrinsic growth rate, environmental carrying capacity, predation rate, predator birth rate due to predation process, predator natural mortality rate, intraspecific competition between predators, *and* m and n is the Allee effect constant. In addition to intraspecific competition, the phenomenon of protecting prey from predator attacks is also interesting to study. This phenomenon is called *Prey Protection*. The phenomenon of prey protection in predator-prey interactions can also be carried out by intervention in various cases such as the creation of conservation forests, wildlife sanctuaries, or some simple protections (Abraham et al., 2023). Predator-prey models with prey protection interventions have also been discussed in several studie (Djakaria et al., 2021; Panigoro et al., 2021; Rayungsari et al., 2022; Wang & Fan, 2023; Yang, 2023). So far, the consideration of the natural behavior of individuals in the model such as intraspecific competition and human intervention in the form of prey harvesting and protection has mostly been done on predators and prey separately.

In this study, a predator-prey interaction model is proposed by combining natural phenomena such as interspecific competition and human intervention phenomena such as prey harvesting and protection. The aim is to determine the dynamic nature of the predator-prey model such as the existence of prey and predator populations and the possibility that occurs in both populations if there is a change in parameter values. The proposed model is a development of the model Panigoro et al. (2023) by ignoring the alle effect. In addition, the model is developed by involving human intervention in the form of harvesting in both populations along with protection of prey. Furthermore, the model is analysed which consists of the formulation of the boundary solution of the model, the existence of the equilibrium point, the nature of local and global stability of the equilibrium point. In addition, numerical simulations were carried out using the 4th order runge-kutta method Islam (2015) to show some phenomena related to the existence of each equilibrium point in accordance with the theoretical results presented in the model analysis. Changes in parameter values were applied to the numerical simulation to see the occurrence of bifurcation and how to maintain the existence of both populations.

B. MODEL DEVELOPMENT

If in the model (1) the alle effect in the predator population is considered, then in this model, it is assumed that there is no alle effect in both populations, so model (1) is modified into

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - axy \\ \frac{dy}{dt} &= bxy - dy - \beta y^2\end{aligned} \tag{2}$$

Furthermore, harvesting assumptions are given for both populations. Harvesting for human needs is commonly done on wildlife or various types of fish. There are three types of harvesting functions commonly used in predator-prey models, namely constant harvesting (Chow et al., 2018), linear harvesting (M. Xiao & Cao, 2009), and non-linear harvesting (Zhang et al., 2018). Three common types of harvesting functions used in these models are constant harvesting, linear harvesting, and nonlinear harvesting. Constant harvesting involves a fixed rate of harvesting prey over time (Ruan, 2009). Linear harvesting, on the other hand, implies that the rate of harvesting prey increases linearly with prey density (D. Xiao et al., 2006). Nonlinear harvesting refers to more complex harvesting functions that do not follow a linear pattern, potentially involving time delays or other nonlinear dynamics (Li et al., 2016). In model (2), linear harvesting is used in both populations, respectively with parameters h_1 and h_2 so that the model becomes

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - axy - h_1x \\ \frac{dy}{dt} &= bxy - dy - \beta y^2 - h_2y\end{aligned} \tag{3}$$

Linear harvesting is often used to model scenarios where predator feeding rates increase proportionally with prey abundance, influencing the stability and oscillatory behavior of the system (Chakraborty et al., 2011). Furthermore, it is assumed that there is a protection zone on the prey with parameters ρ. Thus, model (3) changes to

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - a(1-\rho)xy - h_1x \\ \frac{dy}{dt} &= b(1-\rho)xy - dy - \beta y^2 - h_2y\end{aligned} \tag{4}$$

Predator-prey interactions with the phenomenon described in the model (4) can be found in the interaction between Orca Whales as predators and Penguins as prey (Jordaan et al., 2021). Penguins get human protection to ensure the survival of this species in its natural habitat. On the other hand, penguins are also sometimes the target of hunting carried out by humans traditionally and illegally in certain areas (BioExpedition, 2017). Meanwhile, intraspecific competition occurs in Orca Whales in several situations such as the struggle for food, social hierarchy, and the search for mating partners (Taylor, 2021). Furthermore, although Orca Whales are natural predators in the ocean, some Orca species have also been targeted by humans for hunting, either for entertainment purposes (such as the capture of live killer whales for shows in marine parks) or in some cases as a threat to fish hunted by humans (Simmonds & Fisher, 2010).

C. RESULT AND DISCUSSION

1. Positivity

Theorem 1. The solution of the system of equations (4) is positive as long as $x(0), y(0) \in \mathbb{R}_+^2$.

Proof.
It will be proved that if $x(0) \geq 0$ and $y(0) \geq 0$, then $x(t) \geq 0$ and $y(t) \geq 0$ for every $t > 0$. The condition $x(0) = 0$ will result in $\frac{dx}{dt} = 0$ which indicates that there is no change in population size x. Next review the condition $x(0) > 0$. Suppose there exists t_T with $0 < t < t_T$ such that $x(t) \geq 0$, $x(t_T) = 0$ and $x(t) < 0$ for $t > t_T$. Based on the system of equation (4) the condition $x(t_T) = 0$ result in $\frac{dx}{dt} = 0$. This contradicts the statement $x(t) < 0$ for $t > t_T$. So the supposition is false, so $x(t) \geq 0$ for every t. In the same way, it can be shown that the condition $y(0) \geq 0$ will result $y(t) \geq 0$ for every $t > 0$.

2. Existence of Equilibrium Point

The equilibrium point of system (3) is obtained by setting $\frac{dx}{dt} = \frac{dy}{dt} = 0$, i.e.

$$\begin{aligned} rx\left(1 - \frac{x}{K}\right) - a(1-\rho)xy - h_1 x = 0 \\ b(1-\rho)xy - dy - \beta y^2 - h_2 y = 0 \end{aligned} \quad (6)$$

By solving equation (6), 3 biological equilibrium points are obtained, namely:

(i) Extinction points of both populations, $E_0 = (0,0)$, always exist in R^2.

(ii) Predator extinction point, $E_1 = \left(\frac{K(r-h_1)}{r}, 0\right)$. E_1 exists if the condition $r > h_1$. This condition shows that prey will always exist if $r > h_1$, meaning that the intrinsic growth rate of prey must be greater than the harvesting rate.

(iii) Existence point of both populations $E_2 = (x^*, y^*)$, where:

$$x^* = \frac{K[\beta(r-h_1) + a(d+h_2)(1-\rho)]}{Kab(1-\rho)(1-\rho) + \beta r}$$

$$y^* = \frac{Kb(r-h_1)(1-\rho) - r(d+h_2)}{Kab(1-\rho)(1-\rho) + \beta r}.$$

E_2 exists if it satisfies the condition:

$$Kb(r-h_1)(1-\rho)-r(d+h_2)>0 \Leftrightarrow b>\frac{r(d+h_2)}{K(r-h_1)(1-\rho)}$$

3. Equilibrium Point Stability

a. Local Stability

Linearization around the equilibrium point of system (4) results in the following Jacobian matrix:

$$J(E)=\begin{bmatrix} r\left(1-\frac{2x}{K}\right)-a(1-\rho)y-h_1 & -a(1-\rho)x \\ b(1-\rho)y & b(1-\rho)x-d-2\beta y-h_2 \end{bmatrix} \tag{7}$$

The stability of the equilibrium point of system (4) refers to the eigenvalues of the Jacobian matrix (7) summarized in Theorem 2.

Theorem 2. Given $\Omega^*=\frac{r(d+h_2)}{K(1-\rho)(r-h_1)}$, the local stability of the equilibrium point of system (4) is described below:

(i) The population extinction equilibrium point, $E_0=(0,0)$ is locally asymptotically stable if $r<h_1$ and unstable otherwise.

(ii) The point of extinction equilibrium of the predator population, $E_1=\left(\frac{K(r-h_1)}{r},0\right)$ locally asymptotically stable if $r>h_1$ and $b<\Omega^*$.

(iii) The equilibrium point of the existence of the two populations, $E_2=(x^*,y^*)$ is locally asymptotically stable if $r>h_1$ and $b>\Omega^*$.

Proof

1) By substituting $E_0=(0,0)$ to equation (7), we get

$$J(E_1)=\begin{bmatrix} r-h_1 & 0 \\ 0 & -d-h_2 \end{bmatrix}$$

which results in eigenvalues, $\lambda_1=r-h_1$ and $\lambda_2=-(d+h_2)$. Since $\lambda_2<0$, then E_0 is stable $\lambda_1<0 \Leftrightarrow r<h_1$ and unstable if $\lambda_1>0 \Leftrightarrow r>h_1$.

2) By substituting $E_1=\left(\frac{K(r-h_1)}{r},0\right)$ to equation (7), the Jacobian matrix is obtained

$$J(E_1)=\begin{bmatrix} h_1-r & -\frac{Ka(1-\rho)(r-h_1)}{r} \\ 0 & \frac{Kb(1-\rho)(r-h_1)-r(d+h_2)}{r} \end{bmatrix}$$

which results in eigenvalues,

$$\lambda_1 = h_1 - r$$
$$\lambda_2 = \frac{Kb(1-\rho)(r-h_1) - r(d+h_2)}{r}$$

Based on the existence condition E_1, $r > h_1$, then $\lambda_1 < 0$. Thus E_1 will be locally asymptotically stable if and only if:

$$\frac{Kb(1-\rho)(r-h_1) - r(d+h_2)}{r} < 0 \Leftrightarrow Kb(1-\rho)(r-h_1) < r(d+h_2)$$
$$\Leftrightarrow b < \frac{r(d+h_2)}{K(1-\rho)(r-h_1)}$$

3) By subtitusing $E_2 = (x^*, y^*)$ to equation (7), the Jacobian matrix is obtained

$$J(E_2) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

where

$$a_{11} = -\frac{(r-h_1)r\beta + ar(d+h_2)(1-\rho)}{r\beta + abK(1-\rho)^2}$$
$$a_{12} = -\frac{(r-h_1)(1-\rho)\beta aK + a^2k(d+h_2)(1-\rho)^2}{r\beta + abK(1-\rho)^2}$$
$$a_{21} = \frac{Kb^2(r-h_1)(1-\rho)^2 - br(d+h_2)(1-\rho)}{r\beta + abK(1-\rho)^2}$$
$$a_{22} = \frac{\beta r(d+h_2) - \beta bK(r-h_1)(1-\rho)}{r\beta + abK(1-\rho)^2}$$

Furthermore, it is obtained

$$\det\big(J(E_2)\big) = a_{11}a_{22} - a_{12}a_{21}$$
$$= \frac{[bK(r-h_1)(1-\rho) - r(d+h_2)][(r-h_1)\beta + a(d+h_2)(1-\rho)]}{r\beta + abK(1-\rho)^2}$$

and

$$\mathrm{trace}\big(J(E_2)\big) = a_{11} + a_{22}$$
$$= -\beta\left(\frac{bK(r-h_1)(1-\rho) - r(d+h_2)}{r\beta + abK(1-\rho)^2}\right) - \frac{(r-h_1)r\beta + ar(d+h_2)(1-\rho)}{r\beta + abK(1-\rho)^2}$$

Because $r > h_1$ and $b > \frac{r(d+h_2)}{K(1-\rho)(r-h_1)}$, it is obtained $\det\big(J(E_2)\big) > 0$ and $trace\big(J(E_2)\big) < 0$, which means that the point E_2 is locally asymptotically stable.

b. Global Stability

Theorem 3. If $0 < h_1 < r < \frac{bh_1K(1-\rho)}{bK(1-\rho)-(d+h_2+y\beta)}$, then the extinction equilibrium point of the predator population $E_1 = \left(\frac{K(r-h_1)}{r}, 0\right)$ is globally asymptotically stable.

Proof. Define the Lyapunov function

$$V_1(x,y) = \frac{b}{a}\left(x - x_1 - x_1 \ln\frac{x}{x_1}\right) + y$$

where $x_1 = \frac{K(r-h_1)}{r}$.

The first derivative of V_1 is

$$\begin{aligned}
\frac{dV_1}{dt} &= \frac{\partial V_1}{\partial x}\frac{dx}{dt} + \frac{\partial V_1}{\partial y}\frac{dy}{dt} \\
&= \frac{b}{a}\left(\frac{x - x_1}{x}\right)\left[rx\left(1 - \frac{x}{K}\right) - a(1-\rho)xy - h_1x\right] + b(1-\rho)xy - dy - \beta y^2 - h_2y \\
&= \frac{b}{a}(x - x_1)\left[r\left(1 - \frac{x}{K}\right) - a(1-\rho)y - h_1\right] + b(1-\rho)xy - dy - \beta y^2 - h_2y \\
&= -\frac{b}{aK}\left[\left(h_1K + r(x-K)\right)(x - x_1)\right] + y(bx_1(1-\rho) - d - h_2 - y\beta) \\
&= -\frac{b}{aK}\frac{\left(h_1K + r(x-K)\right)^2}{r} + y\left[\frac{bK(r-h_1)(1-\rho)}{r} - (d + h_2 + y\beta)\right] \\
&< y\left[\frac{bK(r-h_1)(1-\rho)}{r} - (d + h_2 + y\beta)\right] \\
&\leq y\left[\frac{bK(r-h_1)(1-\rho)}{r} - d - h_2\right]
\end{aligned}$$

If $0 < h_1 < r < \frac{bh_1K(1-\rho)}{bK(1-\rho)-(d+h_2)}$ obtained $\frac{dV_1}{dt} < 0$. In addition $\frac{dV_1}{dt} = 0$ if $x = \frac{K(r-h_1)}{r}$ and $y = 0$. Based on LaSalle's invariance principle, E_1 is globally asymptotically stable.

Theorem 4. The equilibrium point of the existence of the two populations, $E_2 = (x^*, y^*)$ is locally asymptotically stable if $\frac{x}{x^*} > \frac{y}{y^*} > 1$.

Proof. Define Lyapunov function

$$V_2(x,y) = x - x^* - x^* \ln\frac{x}{x^*} + \frac{a}{b}\left(y - y^* - y^* \ln\frac{y}{y^*}\right)$$

where $x^*, y^* \in E_2$. The first derivative of V_2 is

$$\frac{dV_2}{dt} = \frac{\partial V_2}{\partial x}\frac{dx}{dt} + \frac{\partial V_2}{\partial y}\frac{dy}{dt}$$
$$= \left(1-\frac{x^*}{x}\right)\left[rx\left(1-\frac{x}{K}\right) - a(1-\rho)xy - h_1x\right] + \frac{a(y-y^*)}{b}[b(1-\rho)x - d - \beta y - h_2]$$
$$= (x-x^*)\left[r\left(1-\frac{x}{K}\right) - a(1-\rho)y - h_1\right] + \frac{a}{b}(y-y^*)[b(1-\rho)x - d - \beta y - h_2]$$
$$= (x-x^*)\left[r\left(1-\frac{x}{K}\right) - a(1-\rho)y - h_1 - \left(r\left(1-\frac{x^*}{K}\right) - a(1-\rho)y^* - h_1\right)\right]$$
$$+\frac{a}{b}(y-y^*)[b(1-\rho)x - d - \beta y - h_2 - (b(1-\rho)x^* - d - \beta y^* - h_2)]$$
$$= (x-x^*)\left[-\frac{r(x-x^*)}{K} - a(y-y^*)(1-\rho)\right]$$
$$+\frac{a}{b}(y-y^*)[-(y-y^*)\beta + b(x-x^*)(1-\rho)]$$
$$= -\frac{r}{K}(x-x^*)^2 - a(x-x^*)(y-y^*)(1-\rho)$$
$$+\frac{a}{b}[b(x-x^*)(y-y^*)(1-\rho) - (y-y^*)^2\beta]$$
$$= -\frac{r}{K}(x-x^*)^2 - a(x-x^*)(x^*y - xy^*)(1-\rho) - \frac{a}{b}(y-y^*)^2$$

Conditions $\frac{y}{y^*} > \frac{x}{x^*} > 1$ is equivalent to $\frac{dV_2}{dt} \leq 0$, consequently E_2 is globally asymptotically stable.

Teorema 5. Suppose $r > h_1 + d + h_2$, the model in the system of equations (4) experiences forward bifurcation at the equilibrium point E_1 when ρ moves through ρ^*.

Proof. Suppose $\rho^* = \frac{bh_1K + d\,r + h_2r - brK}{bK(h_1 - r)}$ is the bifurcation parameter and $z_1 = x$, and $z_2 = y$. The system of equations (4) becomes:

$$\begin{aligned} f_1(z_1,z_2) &= rz_1\left(1-\frac{z_1}{K}\right) - a(1-\rho)z_1z_2 - h_1z_1 \\ f_2(z_1,z_2) &= b(1-\rho)z_1z_2 - dz_2 - \beta z_2^2 - h_2z_2 \end{aligned} \tag{8}$$

Parameter $b^* = \frac{r(d+h_2)}{K(1-\rho)(r-h_1)}$ results in the jacobian matrix at the equilibrium point E_1 having one eigenvalue of zero. Based on the zero eigenvalues, the right eigenvector is obtained (u_1, u_2) and left eigenvector (v_1, v_2) as follows:

$$u_1 = -\frac{a(d+h_2)}{b(r-h_1)}$$
$$u_2 = 1$$

and

$$v_1 = \frac{b(r-h_1)}{a(d+h_2)}$$
$$v_2 = 1$$

By applying the theorem (Castillo-chavez & Song, 2004) defined

$$\psi = \sum_{k,i,j=1}^{2} v_k u_i u_j \frac{\partial^2 f_k}{\partial z_i \partial z_j}(E_1, \rho^*) \tag{9}$$
$$\chi = \sum_{k,i,j=1}^{2} v_k u_i \frac{\partial^2 f_k}{\partial z_i \partial b}(E_1, \rho^*)$$

Based on the system (8) obtained

$$\frac{\partial^2 f_1}{\partial z_1 \partial z_1}(E_1, \rho^*) = -\frac{2r}{K}$$
$$\frac{\partial^2 f_1}{\partial z_1 \partial z_2}(E_1, \rho^*) = \frac{\partial^2 f_1}{\partial z_2 \partial z_1}(E_1, \rho^*) = \frac{a(d+h_2)r}{bK(h_1-r)}$$
$$\frac{\partial^2 f_2}{\partial z_1 \partial z_2}(E_1, \rho^*) = \frac{\partial^2 f_2}{\partial z_2 \partial z_1}(E_1, \rho^*) = -\frac{(d+h_2)r}{k(h_1-r)}$$
$$\frac{\partial^2 f_2}{\partial z_2 \partial z_2}(E_1, \rho^*) = -2\beta$$
$$\frac{\partial^2 f_1}{\partial z_2 \partial \rho}(E_1, \rho^*) = a\left(K - \frac{h_1 K}{r}\right)$$
$$\frac{\partial^2 f_2}{\partial z_2 \partial \rho}(E_1, \rho^*) = \frac{bK(h_1-r)}{r}$$

By using system (9), it is obtained

$$\psi = v_1 u_1 u_1 \frac{\partial^2 f_1}{\partial z_1 \partial z_1}(E_1, \rho^*) + 2 v_1 u_1 u_2 \frac{\partial^2 f_1}{\partial z_1 \partial z_2}(E_1, \rho^*) + 2 v_2 u_1 u_2 \frac{\partial^2 f_2}{\partial z_1 \partial z_2}(E_1, \rho^*)$$
$$+ v_2 u_2 u_2 \frac{\partial^2 f_2}{\partial z_2 \partial z_2}(E_1, \rho^*)$$
$$= -\frac{2a(d+h_2)^2 r}{bK(r-h_1)^2} - 2\beta$$
$$\chi = v_1 u_2 \frac{\partial^2 f_1}{\partial z_2 \partial \rho}(E_1, \rho^*) + v_2 u_2 \frac{\partial^2 f_2}{\partial z_2 \partial \rho}(E_1, \rho^*) = \frac{bK(r-h_1)\big(r-(h_1+d+h_2)\big)}{r(d+h_2)}$$

Because $r > h_1 + d + h_2$ obtained $\psi < 0$ and $\chi > 0$, according to (Castillo-chavez & Song, 2004) the system of equations (4) experiences forward bifurcation at $\rho = \rho^*$.

3. Numerical Simulation

Numerical simulations are carried out to strengthen the results of the analysis in obtaining the dynamic behavior of the interaction between the prey and predator populations contained in the system. In addition, the 4th Order Runge-Kutta Method is used to obtain simulation results or numerical calculations of equation (4). Parameter values are given based on the stability conditions of each equilibrium point which can be seen in Table 1.

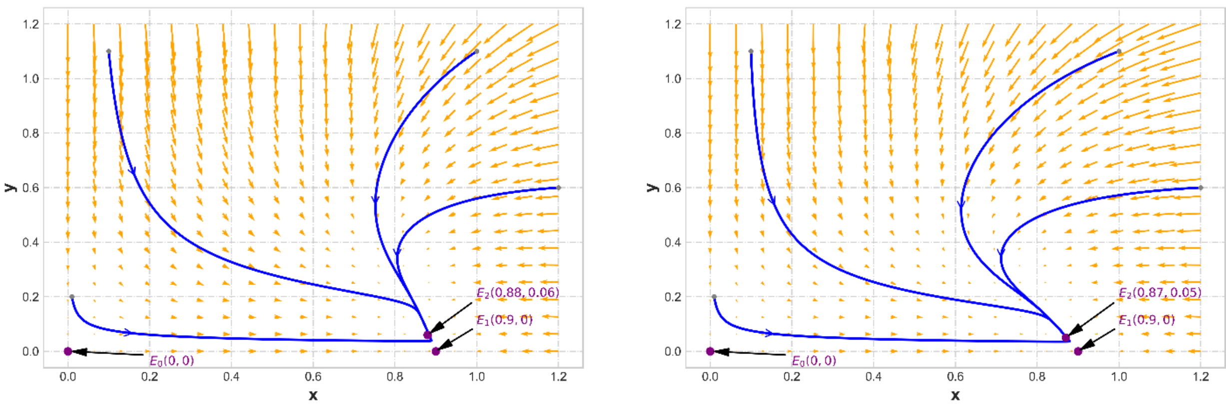
Table 1. Parameter Model

No	Parameter	Nilai	No	Parameter	Nilai
1	r	1	6	β	0,4
2	a	0,3	7	h_2	0,2
3	h_1	0,1	8	k	1
4	b	0,4	9	ρ	0,08
5	d	0,1			

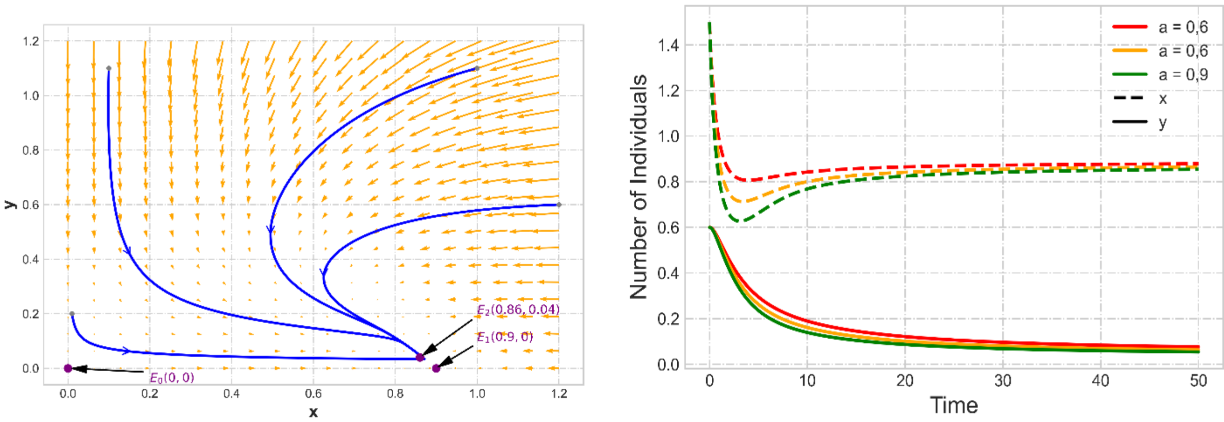
The values given in Table 1 satisfy the condition $r > h_1$ which indicates that the system will be stable at the equilibrium point E_1 or E_2.

a. Effect of Predation Rate

Calculations by increasing the level of predation while keeping other parameters constant can be seen in Figure 1. To see the dynamics of the system at this stage, three different conditions on the predation level were set. The calculation results on the three conditions obtained the value $\Omega^* = 0{,}36$ which results in the system experiencing stability at the equilibrium point E_2. However, it can be seen that, as the predation rate increases, there is a decrease in the number of individuals in the prey and predator populations. As time goes by, the number of individuals in the prey population with $a = 0{,}3$ stabilizes at the value $0{,}88$. As for the two conditions $a = 0{,}6$ and $a = 0{,}9$, the number of individuals in the prey population stabilizes at 0,87 and 0,86, respectively. The number of individuals in the predator population for the three conditions of predation level is stable at 0,06, 0,05, and 0,04, respectively.



(i) (ii)



(iii) (iv)

Figure 1. Population dynamics for (i) $a = 0{,}3$, (ii) $a = 0{,}6$, (iii) $a = 0{,}9$, and (iv) time series

b. Effects of Illegal Harvesting

The existence of illegal harvesting has a significant impact on the dynamics of both populations. This can be seen based on the calculation results in Figure 2, where an increase in illegal harvesting for the prey population can result in a decrease in the number of individuals in the prey population and the predator population. In the calculation using $h_1 = 0{,}15$ the value $\Omega^* = 0{,}38$ which results in the system experiencing stability at the equilibrium point E_2 with the number of individuals in the predator population and prey population of 0,84 and 0,03, respectively. When the wild harvesting rate is increased from the previous value, namely $h_1 = 0{,}3$, the value of $\Omega^* = 0{,}46$ causes the system to stabilize at the equilibrium point E_1 with the number of individuals in the prey population is 0.7. If the harvesting rate is increased again, the system will always be stable at the equilibrium point of the prey population E_1, as shown in Figure 5 using $h_1 = 0{,}6$.

In addition, the calculation results show that increasing the wild harvest for the predator population can increase the number of individuals in the predator population as shown in Figure 3. On condition $h_2 = 0{,}15$ the value of $\Omega^* = 0{,}3$ so that the system is stable at the equilibrium point E_2. While for the condition $h_2 = 0{,}3$ obtained value $\Omega^* = 0{,}48$ which results in the system experiencing stability at the equilibrium point E_1 with the number of prey 0,9 (increased from the previous condition) and the predator population is extinct h_2 and the predator population is extinct. The same thing also happens in the condition $h_2 = 0{,}6$ where the system is stable at the equilibrium point E_1.

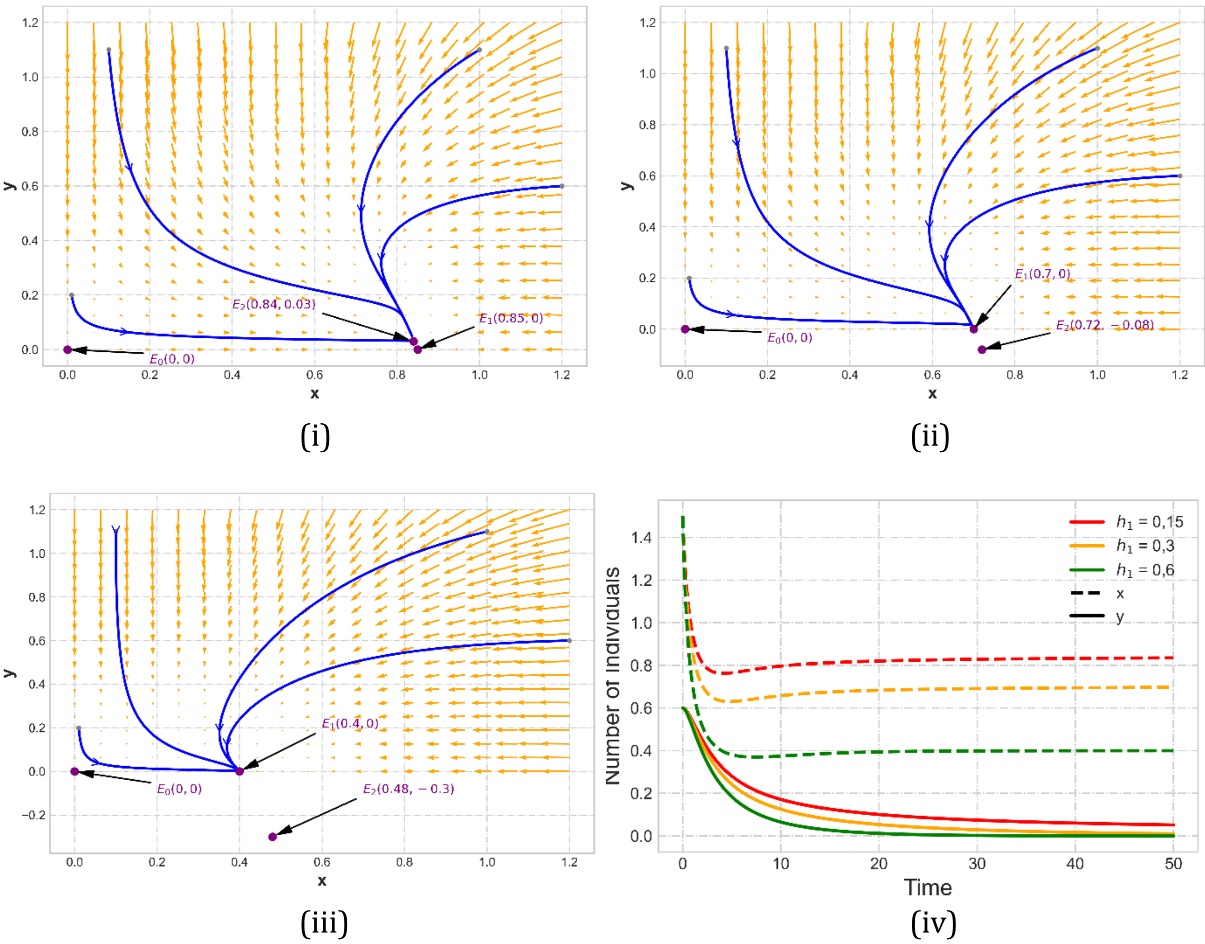
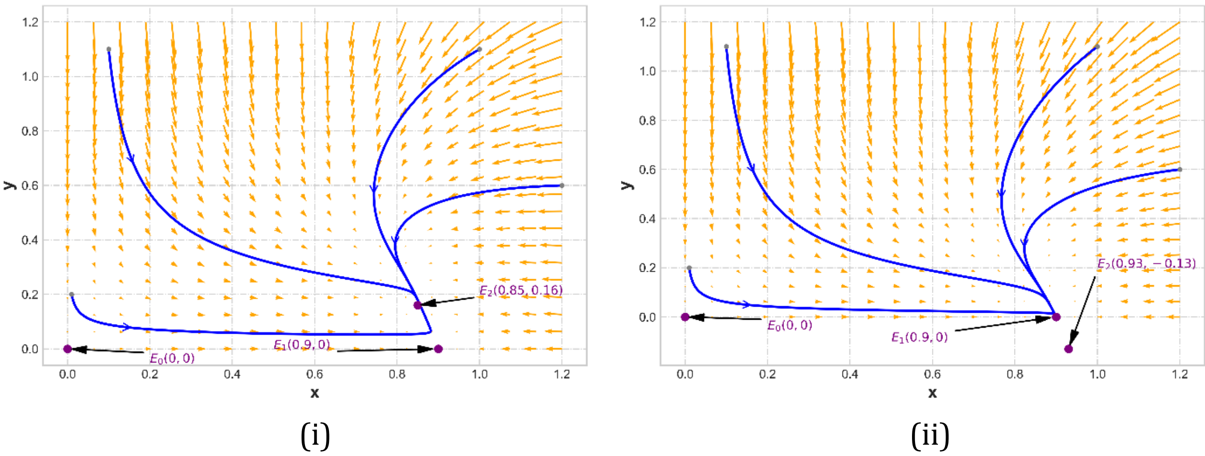
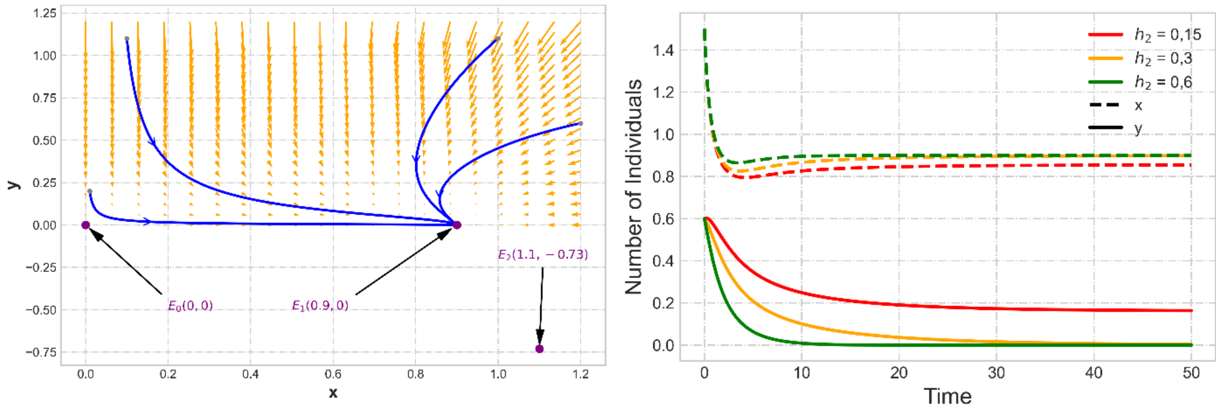


Figure 2. Population dynamics for (i) $h_1 = 0{,}15$, (ii) $h_1 = 0{,}3$, (iii) $h_1 = 0{,}6$, and (iv) time series



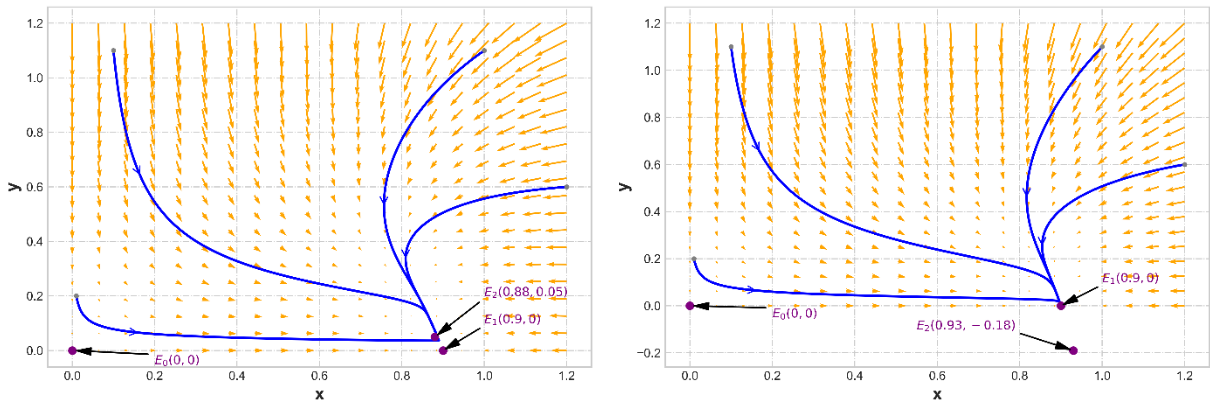


(iv) (iv)

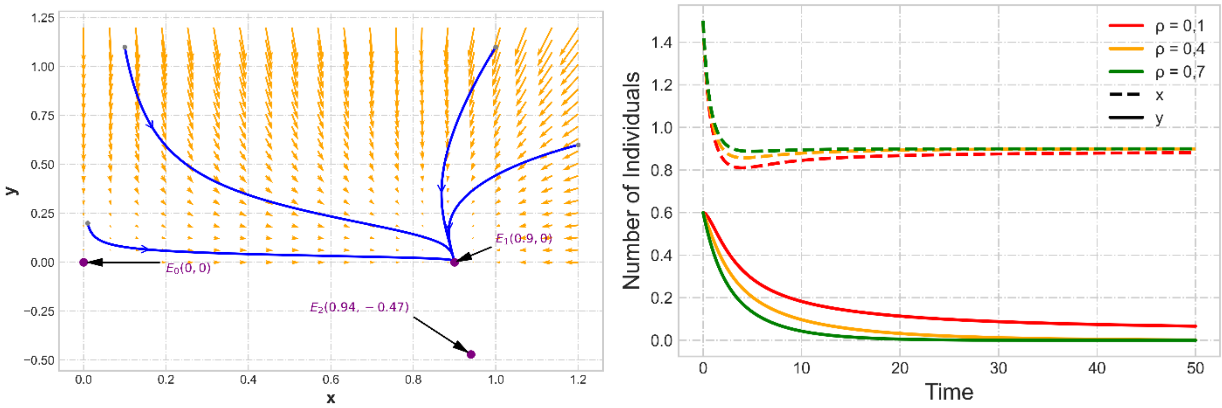
Figure 3. Population dynamics for (i) $h_2 = 0{,}15$, (ii) $h_2 = 0{,}3$, (iii) $b = 0{,}95$, and (iv) time series

c. Effect of Protection Zone on Prey

As previously described, the model assumes the existence of a protection zone on the prey to limit the interaction between prey and predators. Simulation on the change in the value of the protection zone can be seen in Figure 4. Calculation results used $\rho = 0{,}1$ obtained the value $\Omega^* = 0{,}37$ which results in a stable system at the equilibrium point E_2 with the number of individuals in the prey and predator populations of 0,88 and 0,05, respectively. However, when the protection zone is increased, where this calculation is used $\rho = 0{,}4$ a value of $\Omega^* = 0{,}55$ causes the system to stabilize at the equilibrium point E_1 which is the extinction level of the predator. The same thing also happens in the condition of $\rho = 0{,}7$ obtained value $\Omega^* = 1{,}11$ so that the number of individuals in the prey population stabilizes at 0,9 while the predator population experiences extinction, as shown in Figure 4.



(i) (ii)

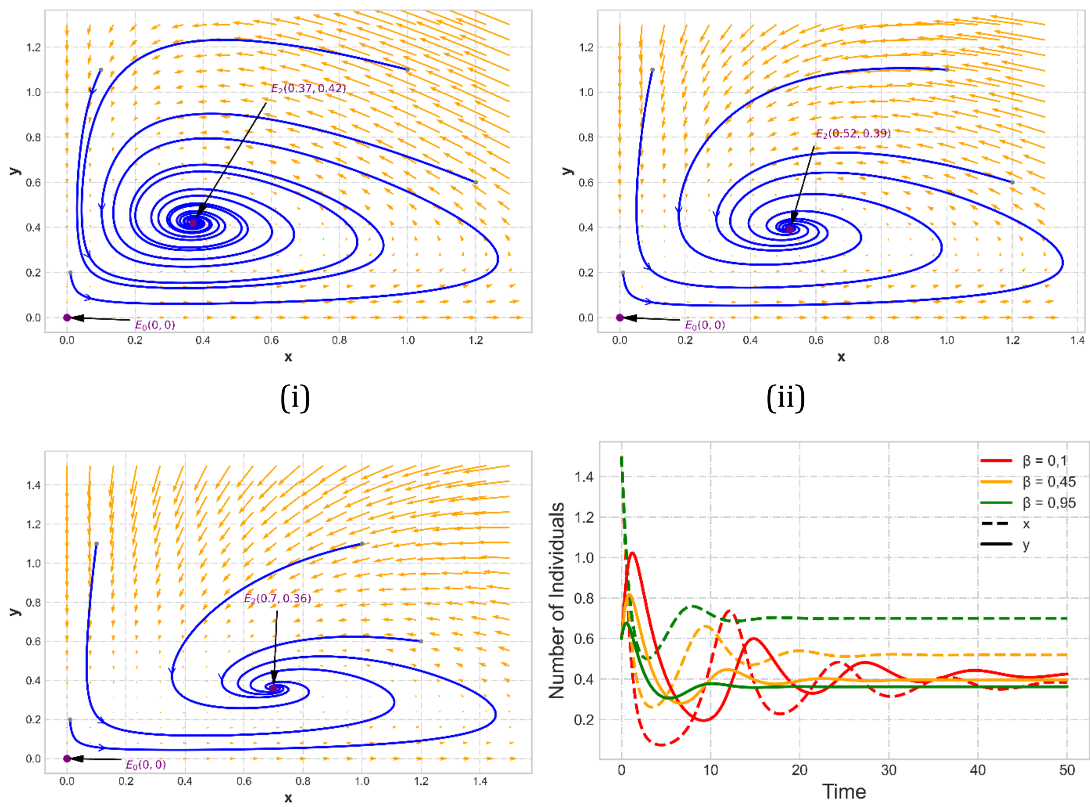


(iii) (iv)

Figure 4. Population dynamics for (i) $\rho = 0{,}1$, (ii) $\rho = 0{,}4$, (iii) $\rho = 0{,}7$, and (iv) time series

d. Effect of Predator Intra-specific Competition

In this calculation, there are three parameters whose values are changed, namely $a = 2$, $b = 1$, and $k = 3$. The results in Figure 5 show that an increase in the level of predator intra-specific competition results in an increase in the number of individuals in the prey population and a simultaneous decrease in the number of individuals in the predator population. In addition, three conditions y are given, namely $\beta = 0{,}1$, $\beta = 0{,}45$ and $\beta = 0{,}95$ values are obtained $b > \Omega^*$ for all three conditions. As a result, the system fluctuates until finally, each stabilizes at the equilibrium point of E_2, as shown in Figure 5.



(i) (ii)

(iii) (iv)

Figure 5. Population dynamics for (i) $\beta = 0{,}1$, (ii) $\beta = 0{,}45$, (iii) $\beta = 0{,}95$, and (iv) time series

e. Influence of Stability Thershold

In this section, calculations are carried out to measure how fast the bifurcation or change in system stability is caused by changes in parameter values. Based on Theorem 2 and Theorem 5, there is a stability threshold that causes the equilibrium point to be stable or unstable, namely h_1, Ω^*, and ρ^*. Thus, the calculation of system bifurcation is carried out by considering the value of the three thresholds. The calculation results can be seen in Figure 6, Figure 7, and Figure 8.

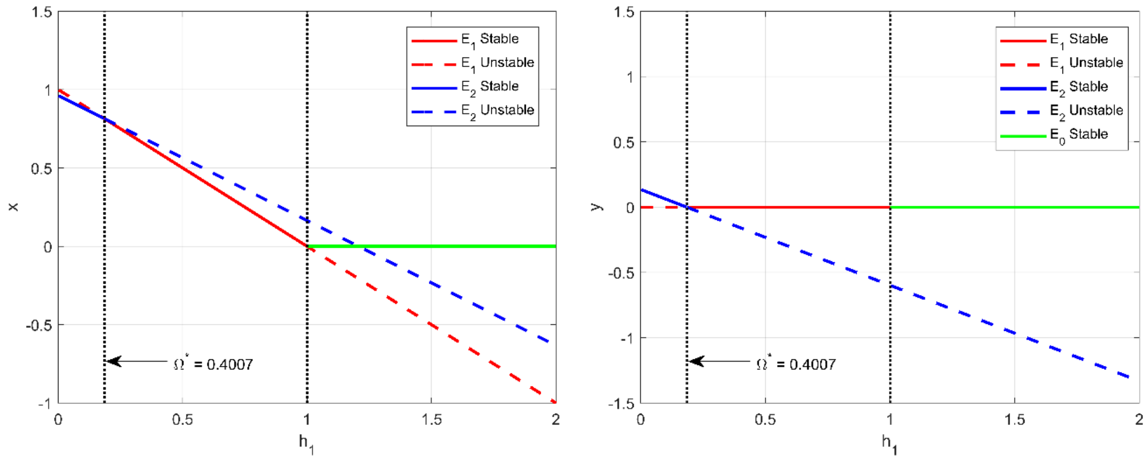


Figure 6. System Bifurcation Based on Change h_1

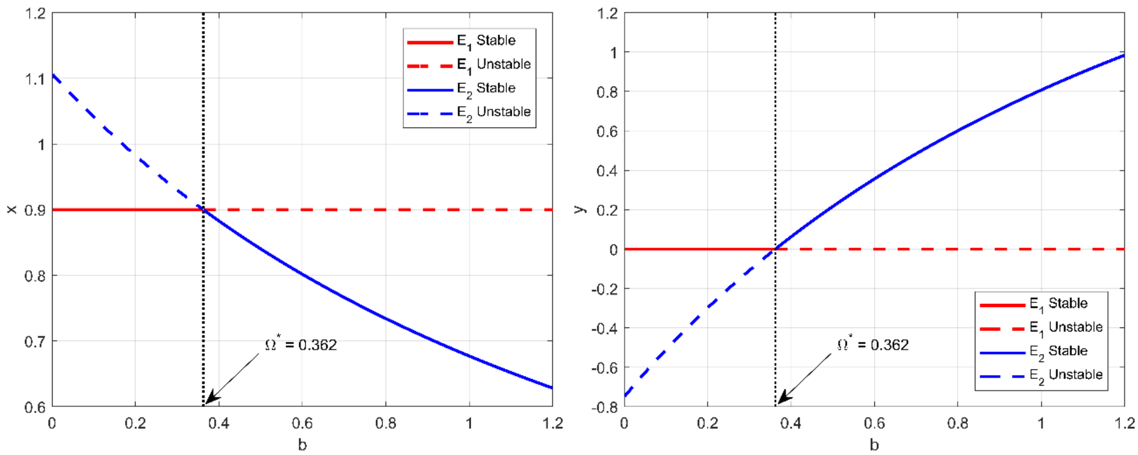


Figure 7. System Bifurcation Based on Change b

Figure 6 shows that increasing the value of h_1 causes the system to bifurcate twice. Initially, the system stabilizes at the equilibrium point E_2 until finally there is a shift in stability to the equilibrium point E_1 when $h_1 = 0{,}186$ which is equivalent to $\Omega^* = 0{,}4007$. If the value of h_1 is increased again, the stability of the system will shift to the equilibrium point E_0 when $h_1 > 1$ or $h_1 > r$. This shows that a continuous increase in wild harvesting of the prey population can lead to the extinction of both the prey population and the predator population.

The increase in the predator birth rate due to the predation process causes the system to change stability from the equilibrium point E_1 to the equilibrium point E_2 as can be seen in Figure 7. The system is stable at the equilibrium point E_2 when the value $b >$

$\Omega^* = 0{,}362$. In addition, it can be seen that when the predator birth rate due to the predation process increases, the number of individuals in the predator population also increases but simultaneously reduces the number of individuals in the prey population, as shown in Figure 8.

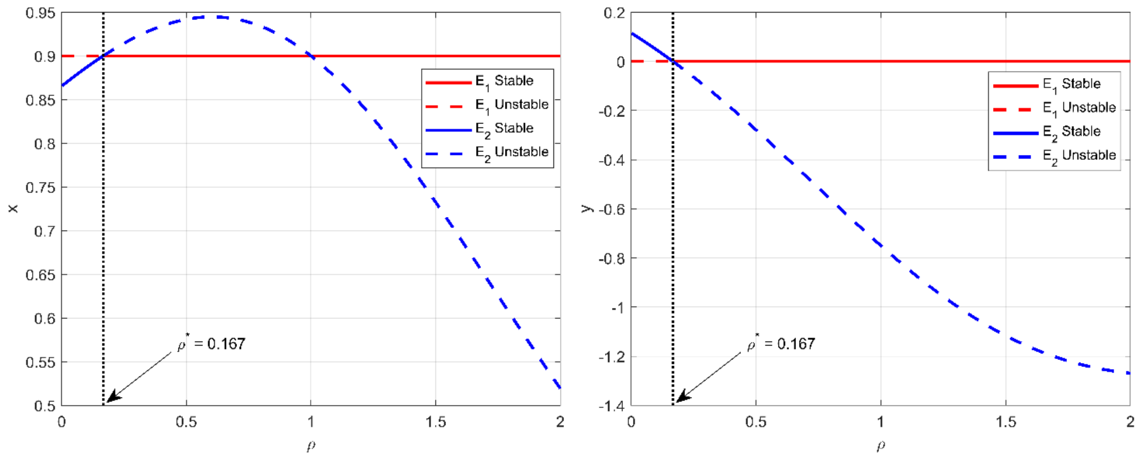


Figure 8. System Bifurcation Based on Change ρ

Changes in the stability properties of the system also occur when there is a change in the value of the prey protection zone as found in Figure 8. When the prey protection zone is minimized, the system will experience stability at the equilibrium point E_2. Conversely, if the prey protection zone is enlarged until it passes through $\rho^* = 0{,}167$, the system will experience stability at the equilibrium point E_1. This shows that if the prey protection zone is enlarged, then individuals in the predator population will experience extinction.

D. CONCLUSION AND SUGGESTIONS

The model discussed in this article is a predator-prey model involving scenarios of illegal harvesting in prey and predator populations, intra-specific competition in predator populations, and prey protection zones. Analysis of the model resulted in three equilibrium points, namely the extinction point of both populations (E_0), the extinction point of the predator population (E_1), and the point of existence of both populations (E_2). The results of the analysis show that the prey and predator populations will experience extinction if $r < h_1$. On the other hand, both populations will exist if $r > h_1$ provided that $b > \Omega^*$. Furthermore, it is shown that there is a forward bifurcation at the extinction point of the predator population (E_1) which depends on the change in the value of the protection rate of the prey (ρ). Numerical simulations show that an increase in illegal harvesting could lead to the extinction of both populations. In addition, the prey protection zone needs to be minimised to maintain the existence of both populations.

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