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Price Model with Generalized Wiener Process for Life Insurance Company Portfolio Optimization using Mean Absolute Deviation

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ABSTRACT

The Financial Services Authority (OJK) has issued Regulation of the Financial Services Authority of the Republic of Indonesia Number 5 Year 2023. Article 11 paragraph 1d explains the limitations of assets allowed in the form of investment, investment in the form of shares listed on the stock exchange for each issuer is a maximum of 10% of the total investment and a maximum of 40% of the total investment. The investment manager of a life insurance company needs to adjust its investment portfolio. In 1991, Konno and Yamazaki proposed an approach to the portfolio selection problem with Mean Absolute Deviation (MAD) model. This model can be solved using linear programming, effectively solving high-dimensional portfolio optimization problems. Another problem in stock portfolio formation is that the ever-changing financial markets demand the development of models to understand and forecast stock price behavior. One method that has been widely used to model stock price movements is the generalized Wiener Process. The generalized Wiener process provides a framework that can accommodate the stochastic nature of stock price changes, thus allowing portfolio managers to be more sensitive to unanticipated market fluctuations. The stock price change model using the Generalized Wiener Process is very good at predicting stock price changes. The results of this stock price prediction can then be used to find the optimal portfolio using the MAD model. The portfolio optimization problem with the MAD model can be solved using linear programming to obtain the optimal stock portfolio for life insurance companies.

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A. INTRODUCTION

In general, the income of life insurance companies can be divided into several categories, including premium income as the primary source of income, investment returns obtained from investment activities, reinsurance claims, and other income. The government has issued Minister of Finance Regulation No. 53/2012 on the Financial Health of Insurance Companies and Reinsurance Companies, particularly in terms of income from investment returns. The purpose of this regulation is to prevent life insurance companies from making aggressive investments that may be contrary to the characteristics of life insurance companies.

The Financial Services Authority (OJK), as the institution that organizes the regulatory system and supervises all activities in the financial services sector, issued Regulation of the Financial Services Authority of the Republic of Indonesia Number 5 of 2023. This POJK determines the amount of funds that life insurance companies can invest, the types of investment instruments allowed, and their limits. Article 11, paragraph 1d explains the limitations of assets allowed in the form of investment, investment in the form of shares listed on the stock exchange for each issuer is a maximum of 10% (ten per cent) of the total investment and a maximum of 40% (forty per cent) of the total investment. Thus, investment managers of life insurance companies need to adjust their investment portfolios, especially stock investment portfolios.

This portfolio problem is an important issue for investment managers Banihashemi & Navidi (2017) in risk management in finance who aim to find the optimal allocation among multiple assets (Aksaraylı & Pala, 2018; Deng et al., 2012). In general, investment managers have a wide range of possibilities for portfolio composition, and the problem is to choose the composition that maximizes investment returns and minimizes risk (Li et al., 2019; Liu, 2011).

Markowitz (1952) proposed an approach to the portfolio selection problem with the mean-variance (MV) model based on two assumptions, namely, historical prices reflect future prices, and there is a correlation between stocks (Kalayci et al., 2020). This model is based on the mean or average approach to calculate returns and uses variance to measure the risk of a portfolio (Huang & Yang, 2020; Li & Zhang, 2021). The Markowitz-type portfolio selection problem is to minimize variance limited by portfolio budget constraints and desired returns (Grechuk & Zabarankin, 2014; Lv et al., 2016; Ramos et al., 2023). The Markowitz portfolio optimization model has yet to be used extensively to construct large-scale portfolios. One reason is the computational difficulties associated with solving large-scale quadratic programming problems with dense covariance matrices (Qin et al., 2016), making it computationally ineffective for solving complex portfolio models (Erwin & Engelbrecht, 2023).

Konno & Yamazaki (1991) introduced the Mean Absolute Deviation (MAD) model as an alternative to the mean-variance model (Hosseini-Nodeh et al., 2023; Ma et al., 2023; Zhang & Zhang, 2014). This model has a linear approximation, which eliminates most of the difficulties associated with the mean-variance model (Qin, 2017), so it can be solved using linear programs (Vanti & Supandi, 2020), and is effective for solving high-dimensional portfolio optimization problems.

Another problem in stock portfolio construction is that the ever-changing financial markets demand the development of models to understand and forecast the behavior of stock prices. The value of stock prices constantly changes over time and in an uncertain direction. One method that has been used extensively to model stock price movements is the Generalized Wiener Process. Wiener processes provide a framework that can accommodate the stochastic nature of stock price changes, thus making portfolio managers more sensitive to unanticipated market fluctuations.

Researchers are interested in knowing the steps of the Generalized Wiener Process methodology as a basis for stock price models and the formulation of optimal portfolio formation using the Mean Absolute Deviation (MAD) model. By integrating this model into the investment decision-making process, life insurance companies can find an optimal portfolio solution that maximizes returns while effectively controlling risk. The Mean Absolute Deviation (MAD) model with additional buy-in threshold and cardinality constraints in forming optimal portfolios. The portfolio obtained can be a recommendation for investment managers when compiling their stock investment portfolio. The buy-in threshold constraint aims to avoid the proportion of shares that are too small or too large, and this aims to fulfill the share proportion limit set in POJK No. 5 of 2023. Meanwhile, the cardinality constraint aims to limit the number of assets in the optimal portfolio (Le Thi & Moeini, 2014).

B. METHODS

This stock portfolio formation research can be used for stock portfolio recommendations used by life insurance companies. Portfolios will be formed using the Mean Absolute Deviation (MAD) model with additional buy-in threshold and cardinality constraints. The initial step of portfolio formation is to evaluate the performance of the stocks used based on the return and risk of each stock. Furthermore, stock price predictions are made using the generalized winner process. Stock prices are assumed to follow a generalized Wiener process. The Wiener model can be found by first finding the price change model. Assuming a constant rate of return and constant volatility the stock price change model is

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t} \tag{1}$$

where, S is stock price; ΔS is change in stock price; Δt is change in time t; μ is expected return; σ is volatility of the stock price; and ε is Wiener process $\sim \emptyset(0,1)$. The predicted stock price is used as an estimate of the expected return of each stock in the portfolio. The MAD model then optimizes the weights of stocks in the portfolio with the objective of minimizing the average absolute deviation of actual returns to predicted returns.

1. Type and Source of Data

The type of data used in this study is quantitative secondary data. Data obtained from the Indonesia Stock Exchange (IDX) website www.idx.co.id in the form of a list of company shares listed in the LQ45 Index from January 2018 to Januari 2024 on the IDX and yahoo.finance.com in the form of historical monthly closing stock price data. The supporting data relevant to the research is obtained from literature, research reports, and electronic media.

The data time period from January 2018 to January 2024 was selected for stock portfolio optimization as it encompasses a comprehensive range of market conditions, including the pre- and post-pandemic phases, as well as the post-pandemic recovery period. This time span allows for a comprehensive examination of stock performance under diverse economic cycles and market conditions, thereby ensuring that the optimization model is capable of capturing long-term dynamics and significant market fluctuations. Data from the IDX (Indonesia Stock Exchange) was selected to provide specific information on the Indonesian stock market, which is relevant to local investors. Meanwhile, Yahoo Finance provides historical and real-time data covering global markets, enabling comparative analysis and benchmarking. The use of data from these two sources will support stock performance evaluation, risk and return measurement, and optimization model, thus helping to achieve the research objectives by combining local and global views in effective portfolio design.

2. Data Collection Technique

The sample was drawn by purposive sampling with the following criteria: (a) Consistent stocks that become research samples must always be listed on the LQ45 index from January 2018 to January 2024; and (b) Ten stocks with the highest return.

3. Number of Sample

The population in this study consists of stocks that are members of the LQ45 Index consisting of 45 stocks. Of the 45 stocks, only a few meet the first and second criteria. The data used to calculate return and risk is monthly stock price data for five years, from January 2018 to Desember 2023. Based on the resulting return value, ten issuers with the highest return were selected to form a portfolio.

4. Data Processing and Analysis

The optimal portfolio of stocks based on the Mean Absolute Deviation model and the Mean Semi absolute Deviation model with additional buy-in threshold and cardinality constraints is formed with the following steps:

a. Selecting stable stocks in the LQ45 Index for the last six periods.
b. Calculating the actual returns of the selected individual stocks.
c. Calculating the average return of individual stocks.
d. Selecting the stocks that have the highest returns.
e. Develop the mean absolute deviation with additional buy-in threshold and cardinality constraints.
f. Solving the model to obtain the optimum portfolio model using linear programming.
g. Calculate portfolio returns and stock portfolio risk from both models.

The portfolio optimization problem using the MAD model is a problem with the Linear Programming (LP). Linear programming is a mathematical method used to optimize an objective function by meeting a number of specified constraints, where the objective function and constraints are expressed in linear form. In the context of portfolio optimization using the MAD model, linear programming can be applied to minimize the average deviation of expected returns in an investment portfolio. The MAD model focuses on reducing the average deviation of actual returns to expected returns. The solution of the MAD model portfolio optimization problem can be solved using the JuMP package in Julia programming. JuMP is a collection of support packages and modeling languages for solving mathematical optimization problems in Julia.

The Mean-Absolute Deviation model is used to improve the Markowitz mean-variance model both computationally and theoretically. Konno and Yamazaki proposed a linear programming portfolio selection model using mean absolute deviation as an alternative measure of risk. Simplicity and ease of computation are considered the most essential advantages of the Mean-Absolute Deviation model. Hence, this model can quickly optimize a portfolio even when considering many assets. In particular, the mean absolute deviation model has been applied to problems with asymmetric distributions of the rate of return (Gupta et al., 2014). The mean absolute deviation (MAD) is a statistical measure representing the average distance between each data value and the mean of a data set. Portfolio risk measured as absolute deviation denoted by m(x) is expressed as follows:

$$m(x_1, x_2, \dots, x_n) = E\left[\left|\sum_{i=1}^{n} R_i x_i - E\left[\sum_{i=1}^{n} R_i x_i\right]\right|\right] \tag{2}$$

where R_i is Random variable representing the rate of return of asset-i; and x_i is proportion of aset-i. Thus, the portfolio optimization problem with risk measures using mean absolute deviation is as follows:

$$\min E\left[\left|\sum_{i=1}^{n} R_i x_i - E\left[\sum_{i=1}^{n} R_i x_i\right]\right|\right] \tag{3}$$

subject to

$$\begin{gathered} \sum_{i=1}^{n} r_i x_i = r_0 \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, \qquad i = 1, 2, \dots, n \end{gathered} \tag{4}$$

The expected value of a random variable can be approximated by the average derived from historical data or future projrctions. Suppose r_{it} is the realization of the random variable R_i over the period t ($t = 1, \dots, T$), which is assumed to be available through historical data or future projections. In this case, r_{it} comes from the stock price prediction data that has been obtained. In addition, the expected value of the random variable can be approximated by an average derived from these data, expressed as follows:

$$r_i = E[R_i] = \sum_{t=1}^{T} \frac{r_{it}}{T} \tag{5}$$

The portfolio risk $m(x_1, x_2, \dots, x_n)$ can be approximated as follows:

$$m(x_1, x_2, \dots, x_n) = E\left[\left|\sum_{i=1}^{n} R_i x_i - E\left[\sum_{i=1}^{n} R_i x_i\right]\right|\right] = \frac{1}{T}\sum_{i=1}^{T}\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| \tag{6}$$

The portfolio optimization problem leads to the following minimization problem.

$$\min \frac{1}{T}\sum_{i=1}^{T}\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right| \tag{7}$$

subject to

$$\sum_{i=1}^{n} r_i x_i = r_0$$
$$\sum_{i=1}^{n} x_i = 1 \tag{8}$$
$$x_i \geq 0, \qquad i = 1, 2, \dots, n$$

The optimization problem is nonlinear and non-smooth due to the existence of absolute-valued functions, so these functions must be reconstructed to eliminate their influence and simplify the optimization problem. The functions can be expressed in the following form.

$$\min \frac{1}{T} \sum_{t=1}^{T} p_t \tag{9}$$

subject to

$$p_t + \sum_{t=1}^{T} (r_{it} - r_i) x_i \geq 0, \qquad t = 1, 2, \dots, T$$
$$p_t - \sum_{t=1}^{T} (r_{it} - r_i) x_i \geq 0, \qquad t = 1, 2, \dots, T$$
$$\sum_{i=1}^{n} r_i x_i = r_0 \tag{10}$$
$$\sum_{i=1}^{n} x_i = 1$$
$$p_t \geq 0, t = 1, 2, \dots, T$$
$$x_i \geq 0, i = 1, 2, \dots, n$$

where,

$$p_t = \left| \sum_{i=1}^{n} (r_{it} - r_i) x_i \right| = \max \left(\sum_{i=1}^{n} (r_{it} - r_i)\, x_i, - \sum_{i=1}^{n} (r_{it} - r_i) x_i \right) \tag{11}$$

The optimization problem will be added buy-in threshold constraints and cardinality constraints. The buy-in threshold constraint aims to avoid the proportion of shares that are too small or too large. This aims to fulfill the share proportion limit set in POJK No. 5 of 2023. While the cardinality constraint aims to limit the number of assets in the optimal portfolio. Suppose ε_i is the lower limit of the i-th share proportion, η_i is the upper limit of the i-th share proportion, and K is the desired number of shares in the portfolio. The buy-in threshold constraint and cardinality constraint can be written as follows,

$$\begin{gathered}\varepsilon_i z_i \leq x_i \leq \eta_i z_i, i = 1, 2, \dots, n \\ \sum_{i=1}^{n} z_i = K\end{gathered} \tag{12}$$

where,

$$z_i = \begin{cases} 1, & \text{if the } i - \text{th stock is included in the portfolio} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

The complete portfolio optimisation model using the Mean Absolute Deviation model with additional buy-in threshold constraints and cardinality constraints is written as follows,

$$\min \frac{1}{T} \sum_{t=1}^{T} p_t \tag{14}$$

subject to

$$\begin{gathered}p_t + \sum_{t=1}^{T} (r_{it} - r_i) x_i \geq 0, \qquad t = 1, 2, \dots, T \\ p_t - \sum_{t=1}^{T} (r_{it} - r_i) x_i \geq 0, \qquad t = 1, 2, \dots, T \\ \sum_{i=1}^{n} r_i x_i = r_0 \\ \sum_{i=1}^{n} x_i = 1 \\ \sum_{i=1}^{n} z_i = K \\ \varepsilon_i z_i \leq x_i \leq \eta_i z_i, i = 1, 2, \dots, n \\ p_t \geq 0, t = 1, 2, \dots, T \\ x_i \geq 0, i = 1, 2, \dots, n \\ p_t = \left| \sum_{i=1}^{n} (r_{it} - r_i) x_i \right| = \max \left(\sum_{i=1}^{n} (r_{it} - r_i)\, x_i, - \sum_{i=1}^{n} (r_{it} - r_i) x_i \right)\end{gathered} \tag{15}$$

C. RESULT AND DISCUSSION

1. Stock selection for portfolio optimization in the LQ45 Index

The stock portfolio optimization model for life insurance companies is formed based on historical data of monthly closing prices of company shares that are consistently listed on the LQ45 index during the period February 2019 to January 2024 and will be selected 10 stocks that have the highest mean return. The list of companies can be seen in Table 1.

Table 1. List of portfolio candidate stocks

Stock code	Company name	Mean return	Volatility
ANTM	Aneka Tambang Tbk.	0.02547	0.16513
ADRO	Adaro Energy Tbk.	0.01938	0.12913
INCO	Vale Indonesia Tbk.	0.01237	0.12313
BBRI	Bank Rakyat Indonesia (Persero) Tbk.	0.01213	0.07717
BMRI	Bank Mandiri (Persero) Tbk.	0.01173	0.07995
BBCA	Bank Central Asia Tbk.	0.01125	0.05197
BBNI	Bank Negara Indonesia (Persero) Tbk.	0.00962	0.10726
EXCL	XL Axiata Tbk.	0.00546	0.10398
TLKM	Telekomunikasi Indonesia (Persero) Tbk.	0.00282	0.06371
KLBF	Kalbe Farma Tbk.	0.00273	0.06062

Mean return shows the average monthly return during the observation period. Volatility is a statistical measure that shows changes in stock prices within a certain period. Prior to stock portfolio optimization, the stocks listed in Table 1 will first form a stock price model using Wiener process generalization. This stock price model will then be used to predict monthly stock prices from January 2024 to December 2024. The results of this stock price prediction will be used for portfolio optimization using the MAD and Semi-MAD models.

2. Stock Price Model using Generalized Wiener Processes

Equation (1) is used to model the stock prices on the list of candidate portfolio stocks. Based on the mean return and volatility data in table 1. After obtaining the stock price change model, a Monte Carlo simulation will be carried out to obtain a stock price prediction. The step of Wiener process generalization and Monte Carlo simulation in stock price prediction involves two key methods in stochastic analysis and forecasting. Wiener processes, or Brownian processes, are used to model stock price movements as a continuous series of random fluctuations, allowing to capture market volatility and price dynamics. In this generalization, the model is extended to include additional factors such as drift and variable volatility, which can be matched to historical stock data. Monte Carlo simulation is then applied by generating a large number of simulated future stock price paths based on the generalized Wiener model. By calculating various possible future price scenarios, Monte Carlo simulation provides an in-depth probability distribution of stock prices, aiding in risk estimation and more informed investment decision-making. The stock price model can be seen in Table 2.

Table 2. Stock price change model

Stock code	Stock price change model	MAPE
ANTM	$\Delta S = 0.002122S + 0.047669S\varepsilon$	4.8192
ADRO	$\Delta S = 0.001615S + 0.037278S\varepsilon$	18.4489
INCO	$\Delta S = 0.001031S + 0.035544S\varepsilon$	7.8658
BBRI	$\Delta S = 0.001011S + 0.022278S\varepsilon$	6.26689
BMRI	$\Delta S = 0.000978S + 0.023079S\varepsilon$	5.4020
BBCA	$\Delta S = 0.000938S + 0.015003S\varepsilon$	3.9945
BBNI	$\Delta S = 0.000802S + 0.030963S\varepsilon$	4.1188
EXCL	$\Delta S = 0.000455S + 0.0300160\varepsilon$	7.5818
TLKM	$\Delta S = 0.000938S + 0.015003S\varepsilon$	4.3207
KLBF	$\Delta S = 0.000938S + 0.015003S\varepsilon$	8.9577

Mean absolute percentage error (MAPE) is a statistical measure that assesses the accuracy of a forecasting method. If the MAPE value is less than 10%, it can be said that the ability of the forecasting model is very good to forecast changes in stock prices. If the MAPE value is less than 10% - 20%, it can be said that the ability of the forecasting model is good to forecast changes in stock prices. However, high MAPE values, such as in the case of ADRO stock (18.4489), indicate that the predicted stock price has a significant deviation from the actual price, about 18.45% from the average actual price. The implication of the high MAPE value is that investors should be aware of the high level of uncertainty in the ADRO stock price prediction, as this may lead to inappropriate investment decisions. To overcome or minimize the high MAPE, it is recommended to improve the quality of data used in the prediction model, consider additional factors that may affect ADRO's stock price such as industry news or unexpected market conditions, and improve or adjust the prediction model used to more accurately reflect actual stock price movements. Thus, investors can improve their investment decisions by minimizing the impact of high prediction uncertainty. Based on Table 2, all stock price change models can be used to predict monthly stock prices in the period January - December 2024. The prediction result data can be seen in Table 3.

Table 3. Stock price return prediction

Stock code	Monthly Return												Mean Return
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
ANTM	0.00497	0.00510	-0.00599	0.04852	0.11426	0.01703	0.07877	0.03522	-0.00095	0.05170	0.03508	-0.04801	0.02798
ADRO	0.01891	0.03016	0.02129	0.03642	0.00430	0.02998	-0.03065	0.00557	0.07510	0.01139	0.04200	0.02937	0.02282
BBRI	-0.00194	0.00770	0.03082	0.03232	0.03264	0.00881	0.04747	-0.01991	0.03365	0.02245	0.00834	-0.00132	0.01675
INCO	0.07339	0.03704	-0.00341	-0.04251	0.03093	0.04289	0.02468	-0.00334	-0.05415	0.02069	0.01781	0.04553	0.01580
BMRI	-0.00613	-0.00940	0.01153	-0.01103	0.03749	0.06156	0.02708	0.05323	0.00039	0.03869	0.00828	-0.02419	0.01563
BBNI	0.02029	0.07301	-0.00309	-0.08837	-0.05211	-0.01837	-0.00150	0.08144	0.02240	0.00751	0.05407	0.06205	0.01311
BBCA	0.03470	0.01683	-0.00091	0.02925	0.02227	0.01534	-0.00839	0.00818	0.00862	-0.01173	0.02224	0.01648	0.01274
EXCL	-0.04697	-0.02918	0.02798	-0.00118	-0.00198	0.04422	0.00933	-0.01388	0.00338	0.03295	0.02958	0.04400	0.00819
KLBF	-0.02114	-0.00396	0.01360	-0.00808	0.00625	-0.00785	0.00575	-0.00044	0.01553	0.04676	-0.02065	0.01290	0.00322
TLKM	-0.00121	-0.02048	0.01219	0.00723	-0.03996	0.00961	0.02228	0.01502	0.01034	0.00385	0.00981	0.00582	0.00288

3. Portfolio Optimization Using the MAD Model

The optimal portfolio selection model is employed to analyze the results of monthly stock price predictions in 2024. In addition, assumptions regarding the lower and upper limits of the proportion of shares are made to fulfil the proportion limits regulated in POJK No. 5 of 2023. The assumption is that the amount of stock investment is 40% of the total investment. Consequently, the upper limit of the proportion for each issuer is 25% (μ = 0.25), while the lower limit is set at 5% (ε = 0.05). The stock portfolio optimization problem, which seeks to identify the optimal asset allocation using the MAD model, is formulated as follows:

$$\min \frac{1}{T}\sum_{t=1}^{12} p_t \tag{16}$$

subject to

$$\begin{aligned}
&p_1 - 0.02300x_1 - 0.00391x_2 - 0.01869x_3 + 0.05759x_4 - 0.02176x_5 + 0.00718x_6 \\
&\quad + 0.02196x_7 - 0.05516x_8 - 0.02436x_9 - 0.00408x_{10} \geq 0 \\
&p_2 - 0.02287x_1 + 0.00734x_2 - 0.00905x_3 + 0.02124x_4 - 0.02503x_5 + 0.05990x_6 \\
&\quad + 0.00409x_7 - 0.03737x_8 - 0.00718x_9 - 0.02335x_{10} \geq 0 \\
&p_3 - 0.03396x_1 - 0.00153x_2 + 0.01407x_3 - 0.01921x_4 - 0.00410x_5 - 0.01620x_6 \\
&\quad - 0.01365x_7 + 0.01979x_8 + 0.01038x_9 + 0.00932x_{10} \geq 0 \\
&p_4 + 0.02055x_1 + 0.01360x_2 + 0.01557x_3 - 0.05831x_4 - 0.02666x_5 - 0.10148x_6 \\
&\quad + 0.01651x_7 - 0.00937x_8 - 0.01130x_9 + 0.00435x_{10} \geq 0 \\
&p_5 + 0.08629x_1 - 0.01852x_2 + 0.01589x_3 + 0.01513x_4 + 0.02186x_5 - 0.06522x_6 \\
&\quad + 0.00953x_7 - 0.01017x_8 + 0.00303x_9 - 0.04284x_{10} \geq 0 \\
&p_6 - 0.01094x_1 + 0.00716x_2 - 0.00794x_3 + 0.02709x_4 + 0.04593x_5 - 0.03148x_6 \\
&\quad + 0.00260x_7 + 0.03603x_8 - 0.01107x_9 + 0.00674x_{10} \geq 0 \\
&p_7 + 0.05080x_1 - 0.05347x_2 + 0.03072x_3 + 0.00888x_4 + 0.01145x_5 - 0.01461x_6 \\
&\quad - 0.02113x_7 + 0.00114x_8 + 0.00253x_9 + 0.01941x_{10} \geq 0 \\
&p_8 + 0.00725x_1 - 0.01725x_2 - 0.03666x_3 - 0.01914x_4 + 0.03760x_5 + 0.06833x_6 \\
&\quad - 0.00456x_7 - 0.02207x_8 - 0.00366x_9 + 0.01214x_{10} \geq 0 \\
&p_9 - 0.02892x_1 + 0.05228x_2 + 0.01690x_3 - 0.06995x_4 - 0.01524x_5 + 0.00929x_6 \\
&\quad - 0.00412x_7 - 0.00481x_8 + 0.01231x_9 + 0.00746x_{10} \geq 0 \\
&p_{10} + 0.02373x_1 - 0.01143x_2 + 0.00570x_3 + 0.00489x_4 + 0.02306x_5 - 0.00560x_6 \\
&\quad - 0.02447x_7 + 0.02476x_8 + 0.04354x_9 + 0.00098x_{10} \geq 0 \\
&p_{11} + 0.00711x_1 + 0.01918x_2 - 0.00841x_3 + 0.00201x_4 - 0.00735x_5 + 0.04096x_6 \\
&\quad + 0.00950x_7 + 0.02139x_8 - 0.02387x_9 + 0.00694x_{10} \geq 0 \\
&p_{12} - 0.07598x_1 + 0.00655x_2 - 0.01807x_3 + 0.02973x_4 - 0.03982x_5 + 0.04894x_6 \\
&\quad + 0.00374x_7 + 0.03581x_8 + 0.00968x_9 + 0.00294x_{10} \geq 0 \\
&p_1 + 0.02300x_1 + 0.00391x_2 + 0.01869x_3 - 0.05759x_4 + 0.02176x_5 - 0.00718x_6 \\
&\quad - 0.02196x_7 + 0.05516x_8 + 0.02436x_9 + 0.00408x_{10} \geq 0 \\
&p_2 + 0.02287x_1 - 0.00734x_2 + 0.00905x_3 - 0.02124x_4 + 0.02503x_5 - 0.05990x_6 \\
&\quad - 0.00409x_7 + 0.03737x_8 + 0.00718x_9 + 0.02335x_{10} \geq 0 \\
&p_3 + 0.03396x_1 + 0.00153x_2 - 0.01407x_3 + 0.01921x_4 + 0.00410x_5 + 0.01620x_6 \\
&\quad + 0.01365x_7 - 0.01979x_8 - 0.01038x_9 - 0.00932x_{10} \geq 0
\end{aligned}$$

$$p_4 - 0.02055x_1 - 0.01360x_2 - 0.01557x_3 + 0.05831x_4 + 0.02666x_5 + 0.10148x_6 - 0.01651x_7 + 0.00937x_8 + 0.01130x_9 - 0.00435x_{10} \geq 0$$
$$p_5 - 0.08629x_1 + 0.01852x_2 - 0.01589x_3 - 0.01513x_4 - 0.02186x_5 + 0.06522x_6 - 0.00953x_7 + 0.01017x_8 - 0.00303x_9 + 0.04284x_{10} \geq 0$$
$$p_6 + 0.01094x_1 - 0.00716x_2 + 0.00794x_3 - 0.02709x_4 - 0.04593x_5 + 0.03148x_6 - 0.00260x_7 - 0.03603x_8 + 0.01107x_9 - 0.00674x_{10} \geq 0$$
$$p_7 - 0.05080x_1 + 0.05347x_2 - 0.03072x_3 - 0.00888x_4 - 0.01145x_5 + 0.01461x_6 + 0.02113x_7 - 0.00114x_8 - 0.00253x_9 - 0.01941x_{10} \geq 0$$
$$p_8 - 0.00725x_1 + 0.01725x_2 + 0.03666x_3 + 0.01914x_4 - 0.03760x_5 - 0.06833x_6 + 0.00456x_7 + 0.02207x_8 + 0.00366x_9 - 0.01214x_{10} \geq 0$$
$$p_9 + 0.02892x_1 - 0.05228x_2 - 0.01690x_3 + 0.06995x_4 + 0.01524x_5 - 0.00929x_6 + 0.00412x_7 + 0.00481x_8 - 0.01231x_9 - 0.00746x_{10} \geq 0$$
$$p_{10} - 0.02373x_1 + 0.01143x_2 - 0.00570x_3 - 0.00489x_4 - 0.02306x_5 + 0.00560x_6 + 0.02447x_7 - 0.02476x_8 - 0.04354x_9 - 0.00098x_{10} \geq 0$$
$$p_{11} - 0.00711x_1 - 0.01918x_2 + 0.00841x_3 - 0.00201x_4 + 0.00735x_5 - 0.04096x_6 - 0.00950x_7 - 0.02139x_8 + 0.02387x_9 - 0.00694x_{10} \geq 0$$
$$p_{12} + 0.07598x_1 - 0.00655x_2 + 0.01807x_3 - 0.02973x_4 + 0.03982x_5 - 0.04894x_6 - 0.00374x_7 - 0.03581x_8 - 0.00968x_9 - 0.00294x_{10} \geq 0$$

$$0.02798x_1 + 0.02282x_2 + 0.01675x_3 + 0.01580x_4 + 0.01563x_5 + 0.01311x_6 + 0.01274x_7 + 0.00819x_8 + 0.00322x_9 + 0.00288x_{10} = r_0$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1$$
$$z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9 + z_{10} = K$$
$$\varepsilon_i z_i \leq x_i \leq \eta_i z_i, i = 1, 2, \dots, n$$
$$p_t \geq 0, t = 1, 2, \dots, T$$
$$x_i \geq 0, i = 1, 2, \dots, n$$

The above portfolio optimization problem is solved using the JuMP package in Julia programming. Jump is a collection of support packages and modeling languages for solving mathematical optimization problems. The solution of the portfolio optimization problem can be seen in Table 4.

Table 4. The solution of the portfolio optimization

Parameter	Portfolio Allocation					Risk
$K = 5$	ANTM	ADRO	INCO	BBRI	BMRI	0.0084399
$r = 0.014897$	0	0.17268	0	0.25000	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.25000	0	0.12882	0.19850	0	
$K = 6$	ANTM	ADRO	INCO	BBRI	BMRI	0.0078346
$r = 0.014897$	0.12330	0	0	0.25000	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.19331	0.10339	0	0.25000	0.08000	
$K = 7$	ANTM	ADRO	INCO	BBRI	BMRI	0.0073414
$r = 0.014897$	0.09742	0.05907	0	0.25000	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.25000	0.09470	0	0.14800	0.10081	

Parameter	Portfolio Allocation					Risk
$K = 5$	ANTM	ADRO	INCO	BBRI	BMRI	0.0120440
$r = 0.018621$	0.19145	0.25000	0	0.25000	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.25000	0	0	0	0.05855	
$K = 6$	ANTM	ADRO	INCO	BBRI	BMRI	0.0103772
$r = 0.018621$	0.21198	0.23490	0	0.17968	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.21464	0.10880	0	0	0.05000	
$K = 7$	ANTM	ADRO	INCO	BBRI	BMRI	0.0103772
$r = 0.018621$	0.21198	0.23490	0	0.17968	0	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.16464	0.10880	0	0.05000	0.05000	
$K = 5$	ANTM	ADRO	INCO	BBRI	BMRI	0.0163923
$r = 0.02$	0.25000	0.25000	0	0.25000	0.18614	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0	0	0	0	0.06386	
$K = 6$	ANTM	ADRO	INCO	BBRI	BMRI	0.0150769
$r = 0.02$	0.25000	0.25000	0	0.25000	0.14048	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0	0	0	0.05952	0.05000	
$K = 7$	ANTM	ADRO	INCO	BBRI	BMRI	0.0141316
$r = 0.02$	0.25000	0.25000	0.07760	0.25000	0.05832	
	BBCA	BBNI	EXCL	TLKM	KLBF	
	0.06408	0	0	0	0.05000	

Table 4 shows the results of portfolio optimization with variations in the parameters K (number of stocks in the portfolio) and r (expected rate of return). Each combination of parameters results in different portfolio allocations to specific stocks, as well as varying levels of risk. First, note that different K parameters (5, 6, and 7) affect the number of stocks taken into the portfolio, which in turn affects the diversification and potential risk of the portfolio. At each value of K, the portfolio allocation is divided between different stocks with varying weights. For example, in the case of $K = 5$ with $r = 0.02$, the portfolio has the highest allocation to ADRO, BBRI, and BMRI stocks with a weight of 25% each of the total portfolio, indicating a preference for these stocks under conditions of higher expected returns.

Secondly, the analysis of changes in the r parameter shows how the level of expected return affects the asset allocation in the portfolio. The higher the level of r, the higher the allocation to stocks with higher expected returns. For example, at $r = 0.02$, stocks such as ADRO and ANTM receive a larger allocation than when $r = 0.014897$, reflecting the increased expected return on the investment. Third, portfolio risk is assessed through allocations to stocks that have different volatility and price fluctuations. For example, a reduced allocation to a particular stock may reduce the overall portfolio risk, depending on the correlation between the stocks in the portfolio.

D. CONCLUSION AND SUGGESTIONS

The stock price change model using the Generalized Wiener Process provides a MAPE value between 4% - 18%, so it can be said to be good at predicting stock price changes. The results of this stock price prediction can then be used to find the optimal portfolio using the MAD model.

The Mean Absolute Deviation (MAD) model takes into account all portfolio return deviations from the expected value, both below and above the expected value. So the purpose of this MAD model is to minimize the MAD value as a portfolio risk. In addition, this MAD model is a problem with linear equations, so it can be solved using linear programming. The portfolio results obtained depend on the parameter values used. In this study used 3 different values for each parameter K and r used, the results show different portfolios for each parameter value as shown in table 4. The results of this portfolio can be used by investment managers of insurance companies in compiling their stock investment portfolios.

Further research can be done by reconstructing the portfolio optimization problem into a Multi-objective linear programming (MOLP) optimization problem. MOLP is a linear programming problem that has more than one objective function at a time. The objective function that can be optimized is maximizing portfolio return and minimizing portfolio risk simultaneously. So it helps integrate various objectives and constraints in the investment decision-making process. However, the implementation of Multi-Objective Linear Programming (MOLP) in portfolio construction may face significant challenges, including high calculation complexity and difficulty in determining and quantifying preferences between various objectives.

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