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Development of Mathematical Maturity through the Amalgamation of Computational Thinking and Technology-Enhanced Learning

Harsa Wara Prabawa[1], Rizky Rorjanuardi[1], Elah Nurlaelah[1]
[1]Department of Mathematics Education, University of Pendidikan Indonesia, Indonesia
harsawara@upi.edu

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ABSTRACT

Advancements in technology and global information infrastructure have transformed education, leveraging a constructivist approach that enhances knowledge construction through student interaction with computer applications. The research aimed to explore the facilitation of enhancing mathematical maturity through the integration of technology in mathematics education using computational thinking. This research employed a qualitative method with a case study approach. The participants in this study were 15 Grade XI students from various classes of the Vocational High School, selected from a total of 159 students. The data collection techniques used in this research included observation and open-ended test instruments. The data analysis techniques involved data reduction, data presentation, and conclusion drawing. The research results show: (1) technology enhances mathematics education; (2) computers deepen mathematical understanding; (3) technology creates contextual learning environments for problem-solving; (4) combining computational thinking and technology aids in mastering mathematical concepts. The conclusion is that integrating computational thinking with technology-enhanced learning significantly fosters mathematical maturity among students.

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A. INTRODUCTION

The rapid advancement of technology and the widespread growth of the global information infrastructure have fundamentally transformed the behaviors and habits of human endeavors. In relation to the realm of education, technology can be perceived as the predominant tool for constructing a knowledge-based society, and as a means for developing educational systems and procedures that lead to superior quality education (Sangrà & González-Sanmamed, 2010). The challenge in educational system design is to integrate technology to significantly enhance administrative efficiency and student learning experiences (Seehorn, et al., 2011). This includes designing technology-responsive curricula, developing interactive digital learning platforms, and continuously evaluating their impact on academic achievement and skill development (Gustafsson, 2016). A constructivist approach in technology application is also crucial as it allows students to actively build knowledge through exploration and collaboration rather than merely consuming information (Lodi & Martini, 2021).

The research conducted in the past two decades has revealed evidence of the positive impact of technology on student learning (Albirini, 2007). The integration of technology in learning generally refers to the constructivist teaching paradigm, where the use of technology plays a crucial role in the process of knowledge construction through students' interactions with computer applications (Chomal & Saini, 2013). The widespread adoption of information technology in the 21st century further highlights the fact that literacy is determined by change. The computational literacy is an identity to characterize the need for using specific technologies or computational tools to explore ideas, solve problems, and express solutions, similar to how language and mathematical literacy are typically used (diSessa, 2018).

This situation poses an interesting research question, particularly regarding the extent to which technology can be utilized widely in mathematics education. However, integrating technology into the classroom has always been a challenging task for many educator (So & Kim, 2009). Teachers feel unprepared to learn and teach with technology, mainly due to a lack of theoretical framework mastery concerning the use of technology for learning (Inan & Lowther, 2010). The study on computer literacy, teachers in Indonesia generally possess relatively high levels of basic computer literacy. However, they only use these skills for a few applications, with word processing and presentation applications being the most prominent ones (Son et al., 2011).

The development of computing as a field of knowledge has a strong connection to the study of mathematics. This relationship is evident in the fact that the first theoretical models of computing, such as the Turing machine or the calculus, were created to mathematically demonstrate the feasibility of automating numerical calculations (Barcelos et al., 2018). He Computer Science Teachers Association (CSTA) offers a reasonable justification for this connection, stating that education is a highly complex environment with competing priorities, thoughts, pedagogies, and ontologies vying for attention (Seehorn, et al., 2011). Therefore, it is imperative to position technology engagement as one of the practical approaches to problem-solving that can be widely applied across subject areas (Kramarski & Michalsky, 2010). By engaging with technology in learning as a tool or thinking skill, students are better equipped to master concepts, analyze and develop problem-solving solutions in every dimension of their lives (Seehorn, et al., 2011). This aligns with Cockcroft's model of the learning problem, where technology can be instrumental in the identification and diagnosis of learning issues, providing tools for remediation and continuous evaluation, and enabling effective monitoring to ensure sustained academic progress and mastery across various subject areas (Kramarski & Michalsky, 2010).

Mathematics is often perceived by students as a complex and challenging subject, with many finding it uninteresting. Cockroft's three-dimensional model (Turmudi, 2008), provides a useful framework for examining the state of mathematics education. The model comprises three main components: mathematics as the subject matter, method as the instructional approach, and students as the learners (Harrell & Bynum, 2018). Cockroft places mathematics on a continuum ranging from concrete to abstract. In terms of instructional methods, the inquiry-based approach is located on the left end of the continuum, while the textbook-oriented approach is located on the right. With regard to students, Cockroft places those who are viewed as passive recipients of information and subjected to rote learning and drill exercises at the upper end of the continuum. Conversely, at the lower end of the continuum, students are viewed as individuals with unique interests, needs, and developmental conditions who deserve respect. Using Cockroft's model, it is possible to analyze how mathematics is taught and the type of mathematics learning that takes place in Indonesia. The Cockcroft's model's upper left corner, which is octant 4, considers mathematics as an abstract science. The scientific aspect is deemed 'rigid' and not practical, while the approaches used are textbook-oriented, teacher-centric, and students are viewed as objects to be graded or ranked instead of being valued as learners with their unique learning interests and tendencies. Consequently, mathematics becomes a stagnant subject, and its learning techniques become outdated, teaching the subject as it is.

Mathematical knowledge can be considered a distinct form of knowledge, as compared to other sciences, owing to its tendency to involve abstract concepts (Acharya et al., 2022). This is due to the fact that mathematical objects are not directly accessible, necessitating the use of semiotic representations to gain access to them (Gustafsson, 2016). These representations often take the form of agreed-upon symbols (Duval, 2006). These symbols can be transformed in two ways: treatments, which involve transformations within a semiotic system, such as repeating a sentence or solving an equation, and conversions, which involve transformations that change the semiotic system while retaining the same conceptual reference. Examples of conversions include changing the representation of an algebraic function from a graphical form to an algebraic one. It is these conversions that prove particularly challenging for students, as they often struggle when a mathematical situation is presented in a different form. For instance, a graphical representation of an algebraic equation may be changed to an algebraic representation based on the provided graph, leading to confusion among students (Kramarski & Michalsky, 2010).

Mathematical modeling is often necessary to expand conversions under specific circumstances. To solve a mathematical problem, it is essential for students to simplify the problem and begin with the most basic problem (Lester & Kehle, 2003). Technology, specifically computers, is considered a powerful tool for overcoming the obstacles to learning that Piaget referred to as "barriers." With technology, knowledge that was previously inaccessible can now be approached in a concrete manner (Papert, 2020). The subsequent step involves manipulating the mathematical representation to arrive at a mathematical solution. Finally, the solution must be translated back into the original problem situation (Lodi & Martini, 2021).

It is acknowledged that certain aspects of computational thinking (CT) such as decomposition, generalization, abstraction, and algorithmic thinking have long been a part of the field of mathematics. Decomposition is recognized as a technique that can demonstrate a student's comprehension of a given problem and not only enhances computational proficiency but also improves mathematical performance (Sztajn et al., 2020). This is the case with abstraction ability. Abstraction skills are developed through the process of identifying the necessary concepts or properties required to solve a problem (Palatnik & Koichu, 2018). The abstraction ability is the capacity to establish connections among diverse ideas, facts, or procedures and propose two methods to foster this ability: exploring similarities and differences and establishing comprehensive relationships (Laski et al., 2014). An analytical framework for interpreting levels of abstraction: Recognition of mathematical structure through perceptual abstraction, Application of mathematical structure through internalization, and Construction of new mathematical structure through internalization. It is important to note that the concept of an algorithm in computer science and mathematics is distinct (Gustafsson, 2016). Algorithms in computer science are formalised records of actions, whereas algorithms in mathematics are formalized actions themselves (Sadykova & Il'bahtin, 2020).

"Computational literacy" and "computational thinking" have a close relationship in the context of education and modern digital life (Ye et al., 2023). Computational literacy refers to an individual's ability to effectively use computational tools and technologies in various contexts (Hong & Kim, 2016). It encompasses a basic understanding of how computational technology works, the ability to use software and applications productively, and the ability to critically evaluate digital information (Miller, 2019). Meanwhile, computational thinking is the mental ability to formulate problems and solve theme using computational concepts and methods (Acharya et al., 2022). It involves the ability to design algorithms, identify patterns in data, and use various computational approaches to analyse and solve complex problems (Pei et al., 2018). In education, developing computational literacy helps students acquire practical skills in using technology, while developing computational thinking enhances critical and analytical thinking skills needed to solve problems in both digital and non-digital contexts (Acharya et al., 2022). Together, they equip individuals with relevant skills to face challenges in the information and technology era (Ye et al., 2023).

The integration of Computational Thinking (CT) into mathematics education, when combined with specific programming tools highlights the similarities between the practices of CT, science and mathematics (Papert, 2020). Such similarities include data handling, modeling and simulation, computational problem solving, and thinking processes (Ye et al., 2023). Moreover, CT has the potential to augment the conceptual foundation and practice of computationally enhanced mathematics education (Ng & Cui, 2021). The growing trend of incorporating computer programming into mathematics instruction can be attributed to the technology's relative ease and accessibility (DeJarnette, 2019). So, this article aims to explore the role of computational thinking in enhancing mathematical maturity through the integration of technology in mathematics education.

B. METHODS

The research aims to explore the role of computational thinking in enhancing mathematical maturity through the integration of technology in mathematics education. In order to address the research aim, a case study research design was utilized, which is considered the most suitable for gathering descriptions of teaching and learning experiences within a bounded system (i.e., the thoughts and actions of participants in a specific educational setting, such as students working on mathematical problem-solving exercises using computational aids, such as laptops or tablets, in a specific subject) (Schwandt & Gates, 2017). We have opted for case study methodology as our analytical approach. This research inquiry involves the use of multiple sources of evidence to derive pertinent conclusions (Campbell & Yin, 2018), which further supplement deep analyses with extended, open-ended and meticulous consideration of data. In light of the responses provided by the students, we have gathered video data that documents their problem-solving processes. Additionally, we have scrutinized the commands used in the relevant software to ascertain whether or not the students were able to arrive at solutions for the given problems. This has enabled us to identify any potential challenges encountered by the students in their problem-solving process. We have also analyzed all the examples in which the students have expressed their difficulties or have faced obstacles while attempting to solve a problem. This analysis has been facilitated by employing a constant comparative strategy in accordance with the chosen conceptual framework (Sadykova & Il'bahtin, 2020).

The participants in this study were 15 Grade XI students from various classes of the Vocational High School, selected from a total of 159 students. Participants were selected using purposive sampling technique. Purposive sampling is a sampling technique where researchers choose samples based on specific criteria deemed relevant to the research objectives (Schwandt & Gates, 2017). The criteria for participants include: (1) Grade XI students; (2) willingness to participate in the research; (3) sufficient foundational knowledge of mathematics; (4) access and ability to use technology; and (5) diverse backgrounds and representation from various classes for comprehensive data.

Data collection employed participant observation and test techniques. Participant observation allows researchers to engage in the research process to obtain deeper insights. Furthermore, open-ended test questions were designed to assess understanding of mathematical concepts, problem-solving abilities, and application of computational thinking. Instrument validation includes content validation involving expert judgment to ensure that the instruments cover all relevant aspects of the research objectives.

Data analysis in this study involves three main stages: data reduction, data presentation, and drawing conclusions. First, data reduction simplifies and organizes raw data into a format that is easier to understand and interpret. This process involves sorting relevant data through coding and categorizing to identify patterns and main themes. Second, data presentation includes visualizing data in tables, graphs, or diagrams and using descriptive narratives to provide context and deeper explanations of findings. Effective presentation aids interpretation and enhances understanding of relationships among various data components. Third, drawing conclusions involves interpreting presented data to answer research questions. This process includes critical analysis of findings, testing of proposed hypotheses, and identifying implications for theory and practice in mathematics education (Campbell & Yin, 2018).

The research procedure can be described as follows. The study involved a Technology-Enhanced Learning Activity in Learning Mathematics in Computer Science, which was conducted over a period of two weeks, with two meetings per week. This activity was designed as a supplementary exercise to the main learning process, which was conducted on a scheduled basis. The aim of this activity was to examine the impact of using software support in solving mathematical problems and to assess the extent to which software support could influence the students' comprehension of mathematics. The study focused on students who were not familiar with using programming to solve mathematical problems, particularly in integral materials. These students relied on traditional methods such as paper and pencil. Since the students were graduates of a Vocational High School with a background in Software Engineering, it was unnecessary to teach them the basics of programming. Therefore, our instruction in the Python programming language was limited to conceptual modifications of a fundamental nature.

The technology being used is the Python programming language within the Jupyter Notebook Environment. We are comparing the MapleSoft application to the results obtained through Python coding. The choice to use Python was based on its widespread use in programming courses. Therefore, we assume that students are proficient in identifying features and applying them in their daily lives. The teacher will encourage cooperative work during each session and lead a class discussion at the beginning and end of each session to explain the learning activity and introduce relevant computational thinking and mathematical concepts.

The lesson will commence with an introductory segment that will present a basic scenario involving the computation of the area of a field. Consider a scenario where a tenant farmer is required to measure the area of a field before planting crops. The plot of land in question has the shape of a right-angled trapezium with a width of 5 units, and the lengths of its two parallel sides are 5 and 8 units, respectively. At this juncture, the pupils are requested to recollect the notion of determining the magnitude of a surface, specifically the magnitude of a right trapezoid. To facilitate the comprehension of the students, the instructor shall sketch a right trapezoid and subsequently furnish it with the requisite data to calculate the area of a right trapezoid.

The problem at hand pertains to determining the area of a plane with an unknown shape. This is demonstrated in Figure 1 (i). The teacher facilitates a discussion with the students to brainstorm potential solutions to this problem. Ultimately, the trapezium approach is suggested as the most effective solution. The students are also encouraged to divide the area in order to minimize wasted space, as shown in Figure 1 (ii). The problem is then expanded upon by introducing several other functional forms.

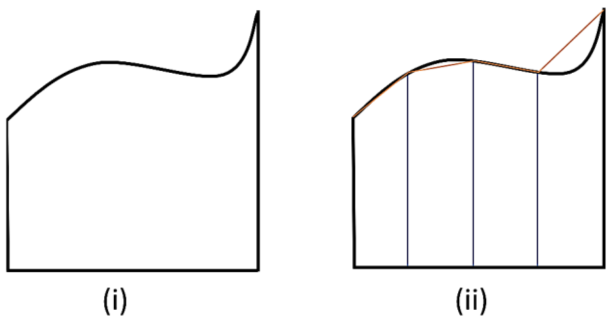

Figure 1. (i) Modified plane shape of a right-angled trapezium; (ii) Calculation of the area using the approach for calculating the area of a trapezium

C. RESULT AND DISCUSSION

1. Co-Development of CT and Technology-Enhanced Learning

The initial stage of learning involves guiding the students to recognize mathematical attributes present in a given problem through discussion and a question-and-answer session. This can be achieved by drawing an illustration on the board and highlighting the area that needs to be calculated. For instance, consider the problem of calculating the integral $\int_0^\pi x \sin(x)$. The teacher can engage the students by posing questions such as: (i) Is it possible to use the trapezoidal approach to solve this problem? If yes, how? (ii) What mathematical properties are necessary to solve this problem using the trapezoidal approach? (iii) What is the required number of trapeziums to calculate the area, and does this affect the outcome?

Regarding question (i), the majority of students exhibited confusion regarding the appropriate response. Several students inquired as to why a trapezium shape was necessary and if other shapes could be utilized. Such a query presents a challenge that can enhance students' comprehension of mathematical concepts. To address this, the educator directed pupils to select a specific two-dimensional shape that could facilitate the calculation of the area beneath the curve. One student opted for a triangular shape. However, this resulted in a random triangle, thereby complicating the automation of the calculation process and leading to the exclusion of numerous areas from the area calculation. Similarly, those who chose to employ circles experienced difficulty in determining the radius length. Certain students utilized rectangles; nonetheless, in this particular scenario, utilizing this shape did not encompass more area than that which the trapezium contained.

The presented issue pertains to the determination of integral limits and functions. Some classmates have put forth the notion that the resolution of this problem can be achieved by connecting it to the 'help' shape that was previously discussed in the first question, wherein the area of the two-dimensional figure can be computed. As for the third query, a number of pupils have expressed uncertainty regarding the answer, and have suggested the necessity of employing arithmetic tools to establish a proof.

The teacher and students collaborated to conduct a test using the Python programming language in order to verify the conjectures posed in each question. The incorporation of technology-supported learning in problem-solving within this domain facilitated a crucial skill the ability for students to simulate a substantial number of solution ideas, which would be nearly impossible to accomplish manually. Combining manual calculations with appropriate technological support is an essential skill that enables students to develop their mathematical maturity alongside their computational concepts and practices. For instance, when students are evaluated on corresponding flat shapes that aid in determining area, it demonstrates their comprehension of utilizing the trapezoidal rule. By processing a significant number of technology-supported experiments, students were able to test their hypotheses and bridge the gap between manual-theoretical calculations and technology-supported experiments, which is one of the primary challenges in learning Integral. Those students who effectively computed the area using the Python program code concluded that subdividing intervals (a) and (b) into more parts (smaller x) would result in a higher level of precision. The observed trends indicate that as the input size increases, the runtime of the algorithms will also increase proportionally. This pattern is observed consistently across some of the tested functions.

In this activity, students are presented with the opportunity to expand their knowledge by substituting the provided function with a more intricate or diverse one. The task involves computing the area enclosed by by $x = 0$; $x = 7$; the x-axis, and the function $f(x) = x^2e^{-x}$, which can be expressed as the integral $\int_0^7 x^2e^{-x}dx$. The process involved in the task is similar, making it easier for students to adapt to the given function and integral boundaries. However, some students encountered difficulties in translating mathematical functions into Python program code, which is a critical aspect of the process. This issue is largely related to the student’s level of proficiency in Python, which can be developed through extensive interaction with the language. Some students attempted to replace the integral limits with other values, while others experimented with more complex functions. From this experiment, two students struggled with functions involving fractions with power ($\int_0^2 \frac{x}{(x-3)^2} dx$) while one student had difficulty computing trigonometric functions ($\int_0^{\frac{\pi}{6}} \cos\left(2x + \frac{x}{3}\right) dx$). Additionally, one student inquired about the possibility of employing Python program code to solve improper integral problems.

The fundamental idea behind the integral that is studied in mathematics is that of continuous addition. This means that, when used to determine the area of a surface, the integral can be calculated by performing specific sums. To simplify the calculation process, technology can be incorporated into the learning process. In the Maple application experiment, it was observed that processing information based on prior work steps was comparably more convenient for the students. However, it is essential to develop a new habituation to comprehend the commands. To perform Riemann sum calculations in Maple, the command is occasionally executed by inputting the following command:

```
> with(Student[Calculus1]):
> ApproximateInt(x.sin(x),x=0..Pi,method=trapezoid,output=plot)
```

The function utilized in the calculation is x.sin(x), which also serves as the boundary of the area on the y-axis. The boundary of the area on the x-axis is x=0..Pi. The method employed to calculate the area of the divided area is trapezoid, although there are alternative methods such as left, lower, midpoint, right, upper, and Simpson's. The command output=plot is utilized to generate the image. Figure 2 depicts the trapezoidal method's calculation of the area under a curve with a divided area of 10. The instructor subsequently instructs the pupils to carry out computations involving a divisible range of 100 and juxtapose the outcomes with those of a divisible range of 10 utilizing directives:

```
> ApproximateInt(x.sin(x),x=0..Pi,method=trapezoid,output=plot, partition= 100)
```

The outcome of this instruction is depicted in Figure 3. During an activity using the Maple application, several students endeavored to calculate the area of a given function using different rules. One student opted to use the "left" command to divide the function into numerous regions of equal width. Upon comparing their results, the teacher posed the question: "Why is it that with the same number of areas divided, the results of the area calculations are different? What causes the difference in the results?" One student posited that the disparity in results could be attributed to the fact that the "left" rule generates a greater amount of wasted area than the trapezoidal rule. The teacher then proceeded to ask further questions to enhance the student's understanding and knowledge, such as "Why does a smaller number of divided regions always produce a different result than a larger number of divided regions?" and "What do you believe explains this phenomenon?" as shown in Figure 2.

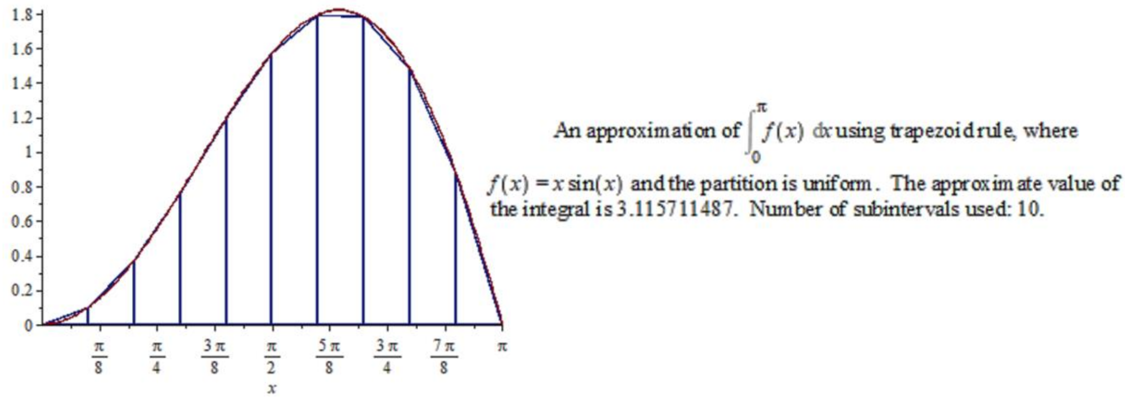

Figure 2. Maple result with trapezoidal rule with a divided area of 10

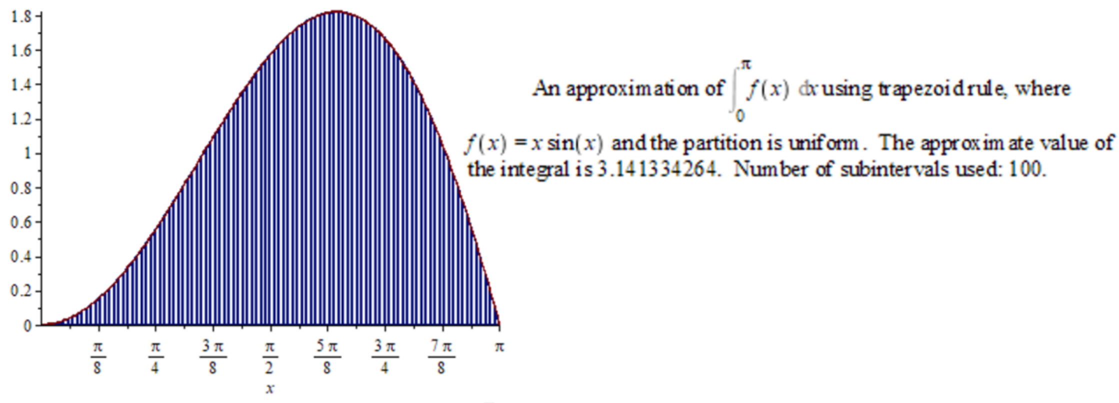

Figure 3. Maple result with trapezoidal rule with a divided area of 100

Numerous studies have demonstrated that the strategic utilization of technological tools can effectively support students in exploring, identifying, and developing advanced mathematical concepts, procedures, and skills, including problem-solving, reasoning, and justification (NCTM, 2015). The positive outcomes are likely to occur when information technology is utilized for teaching and learning purposes (Pierce et al., 2010), provided that clearly defined implementation methods and a sustained and planned focus are present (Kissane et al., 2015). In specific contexts, technology may serve as a tool for developing students' thinking through problem-solving activities (Kitchen & Berk, 2016). Additionally, technology can provide a contextualized learning environment that facilitates mathematical problem-solving (Noss & Hoyles, 1996), as well as the opportunity to conduct studies related to changes in mathematical objects resulting from changes in their elements, represented by media object (Condie et al., 2007).

2. The Potential of Computational Thinking in Facilitating the Development of Mathematical Maturity through the Utilization of Technology-Enhanced Mathematics Learning

This lesson aims to acquaint students with the Riemann Sum as a means of calculating the definite integral. The Riemann Sum is introduced in the initial lecture to aid students in comprehending the fundamental aspect of determining the surface area. The objective is to provide students with a relatively concrete understanding of integral concepts before progressing to other integral topics. The employment of numerical integration for solving integral problems can serve as a basis for formulating a procedure that can be further refined based on the specific requirements of the calculation, as shown in Figure 4.

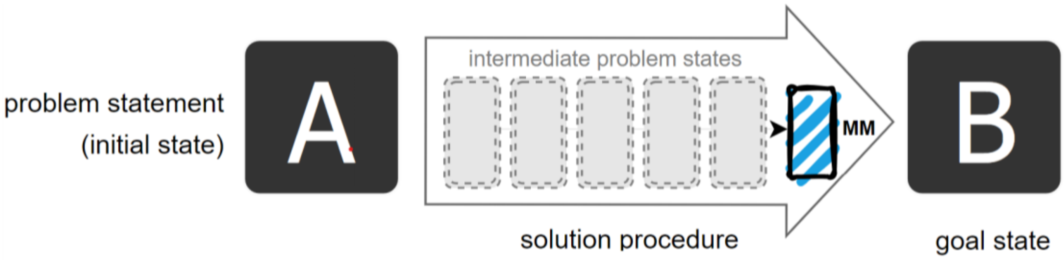

Figure 4. Computational Thinking and Technology-enhanced Learning Amalgamation in Developing Mathematical Maturity

The development of such a procedure is undoubtedly of significant value as it teaches students to break down a complex problem into multiple independent components and subsequently combine them in a more adaptable manner to facilitate the resolution of other problems that involve relatively similar computational processes. This process of breaking down a problem is referred to as decomposition. The technology employed is implemented in accordance with the problem-solving requirements. Decomposition can also be viewed as a hypothetical model that depicts the mental structures and mechanisms that students may need to develop to comprehend specific concepts (Arnon, et al., 2014). The decomposition process is typically influenced by the conceptual, theoretical, and practical learning experiences that students have acquired. Therefore, it is natural that decomposition can also be utilized to explain the extent to which students' knowledge has grown and developed, as well as to illustrate individual differences in knowledge construction development (Arnon, et al., 2014). Regarding the level of decomposition ability, it is a fact that the use of technology can also assist pupils in increasing their decomposition level from domain level to technical level (Gadanidis & Geiger, 2009). Decomposition has an effect not only on computational strategies but also on the enhancement of mathematical performance (Laski et al., 2014).

The analytical framework of mathematical abstraction, the process of recognizing the mathematical attributes necessary for problem-solving falls within the first level of abstraction, namely the recognition of mathematical structure through perceptual abstraction (Gadanidis & Geiger, 2009). To calculate the area of a divided surface using the trapezoid rule, one must identify the boundaries of the surface (lower-limit and upper-limit), the number of surfaces to be divided (n), the width of each divided surface (delta), which is also the height of the trapezium, and the y-values for each boundary of the divided surface (x), which are also the two parallel sides of the trapezium. The construction and explication process involves using these known attributes to calculate the area of a trapezoid. This requires students to apply the integral concept, which involves continuous addition. The calculation_value is included for comparison purposes to highlight the approximate nature of the calculation_approach. This demonstrates the existence of the second level of abstraction in the analytical framework of mathematical abstraction, namely the application of mathematical structure through internalization. At this educational tier, pupils actively implement previously acquired mathematical frameworks to tackle problem-solving tasks (Gadanidis & Geiger, 2009).

This activity also facilitates the analytical framework of mathematical in the third stage of abstraction, which is the construction of novel mathematical structures through interiorization (Gadanidis & Geiger, 2009). During this stage, students actively generalize previously learned mathematical concepts to solve mathematical problems, both the same and distinct, and attain a new mathematical structure when attempting to solve the problem. Students are expected to construct and explain the process of calculating the area using the trapezoidal rule based on previously learned attributes and function values in the context of the presented case. Students must distinguish the existence of specific features/concepts required to solve a problem as part of the abstraction process (Hong & Kim, 2016). The ability to abstract is the capacity to link distinct concepts, facts, or procedures. In general, technology-based learning processes can facilitate the development of abstraction skills, particularly in terms of identifying similarities and differences and establishing comprehensive relationships between available concepts (White & Mitchelmore, 2010).

The acquisition of theoretical knowledge and problem-solving experience in the classroom is crucial for students' knowledge structure. The cognitive mental constructions of students are influenced by the instructional strategies employed for a particular concept. To facilitate a comprehensive understanding, teachers must encourage students to approach a problem from a distinctive perspective. It is imperative to provide opportunities for students to construct their knowledge while solving a problem, followed by a group check. Students tend to focus on a specific solution pattern while solving a problem by referring to examples taught in class. Although this approach is not entirely incorrect, it is necessary to teach students how to construct arguments concerning the problem-solving process that may remain unchanged or altered in relation to the overall stages taught. This approach assists students in recognizing the possible differences and similarities in the problem-solving process, thereby enhancing their understanding of a particular concept.

The ability to recognize and locate patterns that fit together is a valuable skill that can bring order, cohesion, and predictability to complex situations. It also allows students to make generalizations beyond the knowledge they already possess. While pattern recognition is often associated with content, mental habits and processes play a larger role. When students engage in pattern generalization, they use their perceptual, inferential, and symbolic abilities to create algebraic structures that are both plausible and useful. Generalization is closely linked to the abstraction process, which involves selecting essential characteristics from recurring situations to create a general overview. This enables students to transform generalized characteristics into specialized objects for further action. As a result, identifying the integral learning components and enhancing their processing capacities through targeted instruction is a crucial aspect of effective teaching, particularly in the context of Algorithmic Thinking. The utilization of algorithms is considered a methodical depiction of problem-solving methodologies. Students can foster a more methodical problem-solving approach by employing algorithms, thereby simplifying their ability to solve mathematical problems. These algorithms can be universally applied to any area calculation problem, thereby simplifying the process for students to tackle problems, such as calculating the area under a different curve.

D. CONCLUSION AND SUGGESTIONS

Based on the research results and discussion, it can be concluded that: (1) the use of technology in mathematics learning can provide substantial contributions; (2) computers offer a deeper mathematical experience when comprehensive understanding is required before solutions can be constructed; (3) technology enables the creation of contextual learning environments that help students solve mathematical problems and analyse variations in mathematical objects; and (4) the integration of computational thinking as a cognitive tool and technology as a facilitator increases the likelihood of students achieving mastery of mathematical concepts. In the context of mathematical learning, technology can serve as both a learning aid and a resource for exploration. Incorporating computational thinking skills into teaching and learning can provide students with the opportunity to develop a mastery of concepts, analyze problems, and construct solutions that apply to various aspects of their lives. Ultimately, this approach can enhance students' ability to represent and solve mathematical problems across various disciplines.

Based on the research conclusions, several things can be recommended for related parties. Governments and policy-makers should collaborate with educators to develop math curricula integrating computational thinking (CT) and technology-based learning (TEL). Emphasis should be on fostering problem-solving and critical thinking skills through digital tools. Allocate resources for comprehensive curricula across grades, focusing on creative problem-solving with computational methods. Provide continuous professional development for teachers to effectively integrate CT and TEL. Implement monitoring and evaluation systems to assess impact on student outcomes, ensuring curricula adjustments are evidence-based. This approach equips students with essential skills for the digital era, preparing them to thrive in an increasingly technology-driven world. Governments should provide ongoing training and professional development programs for mathematics teachers to ensure they have the skills and knowledge necessary to integrate CT and TEL into their teaching. For teachers, it is recommended to implement learning programs that combine CT and TEL concepts in the mathematics teaching process through collaborative projects, the use of computer simulations, and technology-based problem-solving. For future researchers, it is recommended to develop new learning methods and tools that integrate CT and TEL, including the development of educational software, digital learning aids, and innovative curricula that can be applied at various levels of education.

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