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Analysis Stability of the Model $\mathrm{SEI_1I_2I_3R}$ on the Spread of TikTok User

Susila Bahri[1*], Sri Afdianti[1], Zulakmal[1]
[1]Department of Mathematics and Data Science, Andalas University, Indonesia
susilabahri@sci.unand.ac.id

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ABSTRACT

TikTok addicts, namely content creators (I_1), people who shop (I_2), and people who watch TikTok content (I_3) in Indonesia, continue to increase from year to year. The dynamics of TikTok addicts are analogous to cases of disease transmission. This can be done because TikTok users (S) can gradually increase to become account owners (E), then because there is interaction between account owners and groups of TikTok addicts, TikTok addicts increase, increase or spread. However, TikTok addicts can recover (R) and become vulnerable again over time. Each TikTok addict can have different negative impacts, such as wasting time, getting inaccurate information that can spread people, being deceived, and can disrupt mental health such as depression. Therefore, it is necessary to know which group of TikTok addicts has the greatest influence on the increase in TikTok users in Indonesia so that the government can take action to reduce the increase in TikTok users. In this research, the mathematical dynamics model ($SEI_1I_2I_3R$) was first constructed. Analysis of the stability of the model's equilibrium point is carried out by determining the eigenvalues and the Jacobian matrix to obtain an equilibrium point that is free from the influence of Tik Tok addicts, asymptotically stable if $\mathrm{R}_0 = 0.941019 < 1$. This means that the influence of TikTok addicts on increasing TikTok users is slowly decreasing and will disappear from the population as time goes by. The endemic equilibrium point is asymptotically stable if $\mathrm{R}_0 = 1.011756 > 1$. This means that the influence of TikTok addicts on the increase in TikTok users will remain in the population and will increase over time. Numerical simulations were carried out using MAPLE software.

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A. INTRODUCTION

TikTok is currently the most widely used social media platform (Al-Khasawneh et al., 2022). TikTok is used to carry out various activities such as creating and distributing content (videos), shopping online, and watching content (Abi et al., 2023). The activities of TikTok addicts in these three activities have a positive impact, including getting entertainment, looking for information, filling free time and getting profit or income (Tang, 2021). The negative impacts caused by each group are specific and different, including groups of TikTok content creators who often feel pressure to create interesting content so that it can go viral, which can cause stress and fatigue. This group also likes to do controversial and extreme things to gain lots of followers. The negative impact caused by online shopping groups is that fraud often occurs because the products purchased do not match the product descriptions offered on TikTok. Meanwhile, the negative impact of individuals who watch content is that the individual experiences desensitization or is less sensitive to violence and considers violence as something normal (Li & Kang, 2020). The differences in negative impacts caused by each group of sufferers require special attention and treatment by the government for the comfort of the community.

We Are Social Indonesia Digital Report states that TikTok addicts (content creator groups, shopping groups and content watching groups) continue to increase every year (Riyanto, 2023). However, it is not yet known how much influence each group of TikTok addicts has on the increase in TikTok users. The magnitude of this influence needs to be known so that the government can anticipate the negative impacts caused by each group of addiction sufferers through policies and preventive measures that will be implemented. Apart from that, by knowing the largest group of TikTok addicts, the government can prioritize dealing with the negative impacts of this group of TikTok addicts.

Various mathematical models of disease epidemics such as SIR, SICR, SIS, and SEIR have been used to analyze the spread of TikTok in the real world, as proposed in (Wang, 2023). The dynamics of TikTok addicts are analogous to cases of disease transmission. This can be done because TikTok users (Susceptible) can gradually increase to become account owners (Exposed), then because there is interaction between account owners and groups of TikTok addicts (namely content creators (Infected content), people who shop (Infected eCommerce), and people who watch TikTok content (Infected Viewers), then TikTok addicts will increase, increase or spread. TikTok addicts who have negative impacts can be cured or no longer use TikTok (Recovery) over time. The real conditions of the dynamics of TikTok addicts can be modeled mathematically in a model. This mathematical model has several parameters that can influence changes in the number of TikTok users in Indonesia, namely influence the total compartment subpopulation, the population rate of TikTok users *(*μ*)*, the rate of movement of people exposed to TikTok to become content creators (β), the rate of movement of people exposed to TikTok to become people who shop on TikTok (γ), the rate of movement of people exposure turns into people who watch TikTok (ε), the rate at which content creators move into people who recover or stop using TikTok (τ), the rate at which people who shop on TikTok move into people who stop using TikTok (ρ), the rate at which people move those who watch content become people who stop using TikTok (η), the rate of movement of people who stop using TikTok into susceptible people (λ), the rate of population that stops using TikTok (α), the rate of infection of the population of content creators (π_1), the rate of infection of the population who shop on TikTok (π_2), and the rate of infection of the population who watch content (π_3). The research on TikTok that the proposer will carry out is analogous to several of the proposer's previous studies which also discuss real world problems and dynamic system models, namely regarding the transmission of Malaria, Covid-19 and Omicron diseases which were put forward in (Bahri et al., 2024)(Bahri et al., 2023)(Khairunnisa et al., 2021)(Lasif et al., 2021)(Putri et al., 2023)(Putri et al., 2023), about the problem of young gamblers which he expressed in (Bahri et al., 2023) as well as research on the problem of predator prey in (Hazisyah et al., 2021).

B. METHODS

In this research, several stages were carried out, first collecting TikTok user data for six subpopulations or groups of users (susceptible, exposed, content creators, those shopping, those watching content and those recovering), then constructing a compartment diagram of the six subpopulations, and constructing a mathematical model, then determine the model solution, namely two equilibrium points (free of TikTok addiction and endemic to TikTok addiction). The TikTok addict-free equilibrium point is determined by making the subpopulations of content creators, those who shop, and those who watch content equal zero ($I_1 = I_2 = I_3 = 0$). The endemic equilibrium point is obtained by assuming that the subpopulations of content creators, shoppers, and content watchers are not equal to zero ($I_1 = I_2 = I_3 > 0$). Then to determine the basic reproduction number the Next Generation Matrix method is used (Van Den Driessche & Watmough, 2008). Next, the type of stability of the two equilibrium points is determined using eigenvalues and the Jacobian matrix (Escalante & Odehnal, 2020). In the final stage, a numerical simulation is carried out which describes the dynamics of each population using MAPLe software, and draws conclusions. The results of this research can be a basis for the government in creating regulations that can inhibit the spread of negative content on TikTok and at the same time monitor the available content, so that it does not cause unrest in the community.

C. RESULT AND DISCUSSION

1. Mathematical Model

The population of the model is divided into six categories: TikTok users (Susceptible), denoted by S; people who have a TikTok account (Exposed), denoted by E; people addicted to using or involved in infection of TikTok to create content (Infected Content) is denoted by $\boldsymbol{I_1}$; people are addicted to using or infection TikTok to shopping(Infected eCommerce) is denoted by $\boldsymbol{I_2}$; people are addicted to use or be infected TikTok to watch content (Infected Viewers) is denoted by $\boldsymbol{I_3}$; and people stopped using TikTok (Recovered) is denoted by R. Individuals susceptible class can move to the exposed class if interact with the infected class. Subsequently, the exposed class can move to the infection class if they interact with the infected class (or a group of TikTok addicts). Infection classes can move to the recovered classes if they have high self-control. However, recovered classes can move to susceptible classes if they use TikTok. The total population is denoted by N. Based on the case, the diagram of the mathematical model of the influence of TikTok addiction increasing TikTok users, as shown in Figure 1.

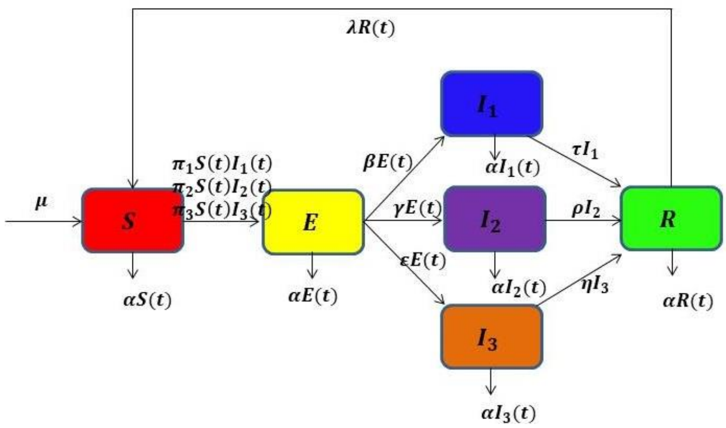


Figure 1. Compartmental Diagram of the Model $\boldsymbol{SEI_1I_2I_3R}$

The diagram in Figure 1 can be constructed into the following system of nonlinear differential equations (Ross, 1984):

$$
\begin{aligned}
\frac{dS(t)}{dt} &= \mu + \lambda R(t) - S(t)(\alpha + \pi_1 I_1(t) + \pi_2 I_2(t) + \pi_3 I_3(t) \\
\frac{dE(t)}{dt} &= S(t)(\pi_1 I_1(t) + \pi_2 I_2(t) + \pi_3 I_3(t) - E(t)(\alpha + \beta + \gamma + \varepsilon) \\
\frac{dI_1(t)}{dt} &= \beta E(t) - I_1(t)(\alpha + \tau) \\
\frac{dI_2(t)}{dt} &= \gamma E(t) - I_2(t)(\alpha + \rho) \\
\frac{dI_3(t)}{dt} &= \varepsilon E(t) - I_3(t)(\alpha + \eta) \\
\frac{dR(t)}{dt} &= \tau I_1(t) + \rho I_2(t) + \eta I_3(t) - R(t)(\alpha + \lambda)
\end{aligned}
\tag{1}
$$

with initial conditions at $t = 0$ are :

$$S(0) \geq 0, E(0) \geq 0, I_1(0) \geq 0, I_2(0) \geq 0, I_3(0) \geq 0, R(0) \geq 0$$

In this case, the parameters $\mu, \lambda, \alpha, \pi_1, \pi_2, \pi_3, \beta, \gamma, \varepsilon, \tau, \rho, \eta$ are positive constants. The description of each parameter can be found in the following Table 1.

Table 1. The Description of parameter's model

Parameter	Description	Value	Reference
μ	The rate TikTok user population	0.5	(Alemneh & Alemu, 2021)
α	The population stop using TikTok	0.25	(Alemneh & Alemu, 2021)
λ	The rate of transition for recovered individuals to the susceptible group	0.35	(Alemneh & Alemu, 2021)
π_1	The infection rate of the population content-creating TikTok	0.6	(Alemneh & Alemu, 2021)
π_2	The infection rate of the population shopping in TikTok	0.2	Asumsi
π_3	The infection rate of the population watching-creating TikTok	0.2	Asumsi
β	The rate of transition for exposed individuals to the infected content group	0.7	(Alemneh & Alemu, 2021)
γ	The rate of transition for exposed individuals to the infected ecommerce group	0.2	Asumsi
ε	The rate of transition for exposed individuals to the infected viewer's group	0.1	Asumsi
τ	The rate of transition for infected content individuals to the recovered group	0.7	(Alemneh & Alemu, 2021)
ρ	The rate of transition for infected ecommerce individuals to the recovered group	0.2	Asumsi
η	The rate of transition for infected viewer's individuals to the recovered group	0.1	Asumsi

2. Analysis of the Model

a. Equilibrium Point

The equilibrium points of the system (1) can be obtained by setting each equation equal to zero (Kuttler, 2017):

$$
\begin{aligned}
&\mu + \lambda R(t) - S(t)(\alpha + \pi_1 I_1(t) + \pi_2 I_2(t) + \pi_3 I_3(t) = 0 \\
&S(t)(\pi_1 I_1(t) + \pi_2 I_2(t) + \pi_3 I_3(t) - E(t)(\alpha + \beta + \gamma + \varepsilon) = 0 \\
&\beta E(t) - I_1(t)(\alpha + \tau) = 0 \\
&\gamma E(t) - I_2(t)(\alpha + \rho) = 0 \\
&\varepsilon E(t) - I_3(t)(\alpha + \eta) = 0 \\
&\tau I_1(t) + \rho I_2(t) + \eta I_3(t) - R(t)(\alpha + \lambda) = 0
\end{aligned} \tag{2}
$$

Once completed, two equilibrium points obtained that is TikTok addiction-free equilibrium points and equilibrium points endemic.

1) TikTok addiction-free equilibrium point

The TikTok addiction-free equilibrium point denoted $T_1^0 = (S^0, E^0, I_1^0, I_2^0, I_3^0 R^0)$ states a condition where no individual is exposed or addicted to TikTok. Therefore, $I_1 = 0, I_2 = 0, I_3 = 0$. TikTok's addiction-free equilibrium point for system (1) is:

$$T_1^0 = (\frac{\mu}{\alpha}, 0, 0, 0, 0, 0)$$

2) Endemic Equilibrium Point

The endemic equilibrium point $T_1^* = (S^*, E^*, I_1^*, I_2^*, I_3^* R^*)$ states the condition where the population is addicted to TikTok. Therefore, $I_1 > 0, I_2 > 0, I_3 > 0$, the endemic equilibrium point for system (1) is:

$$S^* = \frac{\mu + \lambda R(t)}{\left(\alpha + \pi_1\left(\frac{\beta}{\alpha+\tau}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right) + \pi_2\left(\frac{\gamma}{\alpha+\rho}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right) + \pi_3\left(\frac{\varepsilon}{\alpha+\eta}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right)\right)}$$

$$E^* = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$I_1^* = (\frac{\beta}{\alpha+\tau})(\frac{-B \pm \sqrt{B^2-4AC}}{2A})$$

$$I_2^* = (\frac{\gamma}{\alpha+\rho})(\frac{-B \pm \sqrt{B^2-4AC}}{2A})$$

$$I_3^* = (\frac{\varepsilon}{\alpha+\eta})(\frac{-B \pm \sqrt{B^2-4AC}}{2A})$$

$$R^* = \frac{\tau\left(\frac{\beta}{\alpha+\tau}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right) + \rho\left(\frac{\gamma}{\alpha+\rho}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right) + \eta\left(\frac{\varepsilon}{\alpha+\eta}\right)\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right)}{\alpha + \lambda}$$

where

$$A = \alpha(\alpha+\beta+\gamma+\varepsilon)\left(\frac{\pi_1\beta}{\alpha+\tau}+\frac{\pi_2\gamma}{\alpha+\rho}+\frac{\pi_3\varepsilon}{\alpha+\eta}\right)+\lambda\,(\alpha+\beta+\gamma+\varepsilon)\left(\frac{\pi_1\beta}{\alpha+\tau}+\frac{\pi_2\gamma}{\alpha+\rho}+\frac{\pi_3\varepsilon}{\alpha+\eta}\right)$$
$$-\left(\frac{\tau\lambda}{\alpha+\tau}\right)\left(\frac{\pi_1\beta^2}{\alpha+\tau}+\frac{\pi_2\beta\gamma}{\alpha+\rho}+\frac{\pi_3\beta\varepsilon}{\alpha+\eta}\right)-\left(\frac{\rho\lambda}{\alpha+\rho}\right)\left(\frac{\pi_1\gamma\beta}{\alpha+\tau}+\frac{\pi_2\gamma^2}{\alpha+\rho}+\frac{\pi_3\gamma\varepsilon}{\alpha+\eta}\right)$$
$$-\left(\frac{\eta\lambda}{\alpha+\eta}\right)\left(\frac{\pi_1\varepsilon\beta}{\alpha+\tau}+\frac{\pi_2\varepsilon\gamma}{\alpha+\rho}+\frac{\pi_3\varepsilon^2}{\alpha+\eta}\right)$$
$$B = \alpha^2(\alpha+\beta+\gamma+\varepsilon)+\alpha\lambda(\alpha+\beta+\gamma+\varepsilon)-\mu\alpha\left(\frac{\pi_1\beta}{\alpha+\tau}+\frac{\pi_2\gamma}{\alpha+\rho}+\frac{\pi_3\varepsilon}{\alpha+\eta}\right)$$
$$-\mu\lambda\left(\frac{\pi_1\beta}{\alpha+\tau}+\frac{\pi_2\gamma}{\alpha+\rho}+\frac{\pi_3\varepsilon}{\alpha+\eta}\right)$$
$$C = 0$$

b. Basic Reproduction Number

The basic reproduction number for system (1) can be determined from the infection subpopulation using the Next Generation Matrix Method (Diekmann et al., 1990), that is:

$$\begin{aligned}
\frac{dE(t)}{dt} &= S(t)(\pi_1 I_1(t)+\pi_2 I_2(t)+\pi_3 I_3(t)-E(t)(\alpha+\beta+\gamma+\varepsilon)\\
\frac{dI_1(t)}{dt} &= \beta E(t)-I_1(t)(\alpha+\tau)\\
\frac{dI_2(t)}{dt} &= \gamma E(t)-I_2(t)(\alpha+\rho)\\
\frac{dI_3(t)}{dt} &= \varepsilon E(t)-I_3(t)(\alpha+\eta)
\end{aligned}\tag{3}$$

The first step is to determine transmission matrix F and transition matrix V, then look for the spectral radius of $K = FV^{-1}$ so that obtained (Bahri et al., 2023):

$$F = \begin{pmatrix} 0 & \frac{\mu\pi_1}{\alpha} & \frac{\mu\pi_2}{\alpha} & \frac{\mu\pi_3}{\alpha} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{and } V = \begin{pmatrix} (\alpha+\beta+\gamma+\varepsilon) & 0 & 0 & 0 \\ -\beta & (\alpha+\tau) & 0 & 0 \\ -\gamma & 0 & (\alpha+\rho) & 0 \\ -\varepsilon & 0 & 0 & (\alpha+\eta) \end{pmatrix}$$

Then, the next generation matrix $K = FV^{-1}$ is

$$K = FV^{-1}$$
$$= \begin{pmatrix} 0 & \frac{\mu\pi_1}{\alpha} & \frac{\mu\pi_2}{\alpha} & \frac{\mu\pi_3}{\alpha} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(\alpha+\beta+\gamma+\varepsilon)} & 0 & 0 & 0 \\ \frac{\beta}{(\alpha+\beta+\gamma+\varepsilon)(\alpha+\tau)} & \frac{1}{(\alpha+\tau)} & 0 & 0 \\ \frac{\gamma}{(\alpha+\beta+\gamma+\varepsilon)(\alpha+\rho)} & 0 & \frac{1}{(\alpha+\rho)} & 0 \\ \frac{\varepsilon}{(\alpha+\beta+\gamma+\varepsilon)(\alpha+\eta)} & 0 & 0 & \frac{1}{(\alpha+\eta)} \end{pmatrix} = \begin{pmatrix} P_1 & P_2 & P_3 & P_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where:

$$P_1 = \left(\frac{\mu\pi_1\beta}{\alpha\big((\alpha+\beta+\gamma+\varepsilon)(\alpha+\tau)\big)} + \frac{\mu\pi_2\gamma}{\alpha\big((\alpha+\beta+\gamma+\varepsilon)(\alpha+\rho)\big)} + \frac{\mu\pi_3\varepsilon}{\alpha\big((\alpha+\beta+\gamma+\varepsilon)(\alpha+\eta)\big)}\right),$$

$$P_2 = \left(\frac{\mu\pi_1}{\alpha(\alpha+\tau)}\right), P_3 = \left(\frac{\mu\pi_2}{\alpha(\alpha+\rho)}\right), P_4 = \left(\frac{\mu\pi_3}{\alpha(\alpha+\eta)}\right).$$

The basic reproduction number ($\Re_0$) is the spectral radius or the maximum absolute value of the eigenvalues of the next generation matrix K, such that:

$$\det(K - \lambda I) = 0$$

$$det\begin{pmatrix} P_1 - \lambda & P_2 & P_3 & P_4 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix} = 0$$

$$\lambda_1 = \frac{\mu\pi_1\beta}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\tau))} + \frac{\mu\pi_2\gamma}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\rho))} + \frac{\mu\pi_3\varepsilon}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\eta))}, \qquad \lambda_2 = 0$$

Thus, the basic reproduction number is :

$$\Re_0 = \frac{\mu\pi_1\beta}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\tau))} + \frac{\mu\pi_2\gamma}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\rho))} + \frac{\mu\pi_3\varepsilon}{\alpha((\alpha+\beta+\gamma+\varepsilon)(\alpha+\eta))}$$

c. Stability Analysis at Equilibrium point

1) Stability Analysis at TikTok Addiction-free Equilibrium Point

The TikTok addiction- free equilibrium point of the model (1) is asymptotically stable if the eigen values of the Jacobian matrix are negative. The jacobian matrix of model (1) at TikTok addiction-free equilibrium point (T_1^0) is given by

$$J_{T_1^0} = \begin{pmatrix} h & 0 & -a & -b & -c & \lambda \\ 0 & i & a & b & c & 0 \\ 0 & \beta & j & 0 & 0 & 0 \\ 0 & \gamma & 0 & k & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & l & 0 \\ 0 & 0 & \tau & \rho & \eta & m \end{pmatrix}$$

With:

$$h = -\alpha, i = -(\alpha+\beta+\gamma+\varepsilon), j = -(\alpha+\tau), k = -(\alpha+\rho), l = -(\alpha+\eta),$$
$$m = -(\alpha+\lambda), a = \frac{\mu\pi_1}{\alpha}, b = \frac{\mu\pi_2}{\alpha}, c = \frac{\mu\pi_3}{\alpha}$$

The characteristic equation can be obtained by $\det\left(J_{T_1^0} - \lambda I\right) = 0$

$$\begin{vmatrix} h-\lambda & 0 & -a & -b & -c & \lambda \\ 0 & i-\lambda & a & b & c & 0 \\ 0 & \beta & j-\lambda & 0 & 0 & 0 \\ 0 & \gamma & 0 & k-\lambda & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & l-\lambda & 0 \\ 0 & 0 & \tau & \rho & \eta & m-\lambda \end{vmatrix} = 0$$

$$(h-\lambda)(m-\lambda)\begin{vmatrix} i-\lambda & a & b & c \\ \beta & j-\lambda & 0 & 0 \\ \gamma & 0 & k-\lambda & 0 \\ \varepsilon & 0 & 0 & l-\lambda \end{vmatrix} = 0$$

$$(h-\lambda)(m-\lambda)(\lambda^4 - p\lambda^3 + q\lambda^2 + r\lambda + s) = 0 \qquad (4)$$

where

$p = -(i + j + k + l)$
$q = (ij + ik + il + jk + jl + kl) - \mu(\beta\pi_1 + \epsilon\pi_3 + \gamma\pi_2)$
$r = -(ijk + ijl + ikl + jkl) + \mu(\beta k\pi_1 + \beta l\pi_1 + \epsilon j\pi_3 + \epsilon k\pi_3 + \gamma j\pi_2 + \gamma l\pi_2)$
$s = (\alpha ijkl + \beta kl\mu\pi_1 - \epsilon jk\mu\pi_3 - \gamma jl\mu\pi_2)/\alpha$

from equation (4) two eigenvalues are obtained, namely

$$\lambda_1 = h = -\alpha < 0$$
$$\lambda_2 = m = -(\alpha + \lambda) < 0$$

Next, using the Routh-Hurwitz the root of the equation will be negative if:
$i.\ p > 0$
$ii.\ s > 0$
$iii.\ pq - r > 0$
$iv.\ \dfrac{r^2 + p^2 s - pqr}{r - pq} > 0$

Thus, the TikTok addict-free equilibrium point is asymptotically stable if:
$i.\ -\alpha < 0$
$ii.\ -(\alpha + \lambda) < 0$
$iii.\ p > 0$
$iv.\ s > 0$
$v.\ pq - r > 0$
$vi.\ \dfrac{r^2 + p^2 s - pqr}{r - pq} > 0$

2) Stability Analysis at Endemic Equilibrium Point
The TikTok addiction endemic equilibrium point of the model (1) is asymptotically stable if the eigen values of the Jacobian matrix are negative. The jacobian matrix of model (1) at TikTok addiction-endemik equilibrium point (T_1^*) is given by

$$J_{T_1^*} = \begin{pmatrix} H & 0 & -a & -b & -c & \lambda \\ U & I & a & b & c & 0 \\ 0 & \beta & J & 0 & 0 & 0 \\ 0 & \gamma & 0 & K & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & L & 0 \\ 0 & 0 & \tau & \rho & \eta & M \end{pmatrix}$$

with:

$H = -\left(\alpha + \pi_1 I_1^*(t) + \pi_2 I_2^*(t) + \pi_3 I_3^*(t)\right), I = -(\alpha + \beta + \gamma + \varepsilon), J = -(\alpha + \tau),$
$K = -(\alpha + \rho), L = -(\alpha + \eta), M = -(\alpha + \lambda), U = \pi_1 I_1^*(t) + \pi_2 I_2^*(t) + \pi_3 I_3^*(t)$
$a = S^*(t)\pi_1, b = S^*(t)\pi_2, c = S^*(t)\pi_3$

The characteristic equation can be obtained by $\det\left(J_{T_1^*} - \lambda I\right) = 0$

$$\begin{vmatrix} H-\lambda & 0 & -a & -b & -c & \lambda \\ U & I-\lambda & a & b & c & 0 \\ 0 & \beta & J-\lambda & 0 & 0 & 0 \\ 0 & \gamma & 0 & K-\lambda & 0 & 0 \\ 0 & \varepsilon & 0 & 0 & L-\lambda & 0 \\ 0 & 0 & \tau & \rho & \eta & M-\lambda \end{vmatrix} = 0$$

$$\lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6) = 0 \tag{5}$$

where

$a_1 = -(H + J + K + L + M + I),$
$a_2 = (-\gamma\pi_2 S^* - \beta S^*\pi_1 - \epsilon S^*\pi_3 + H(J + K + L + M) + IM + J(K + L + M)$
$\quad + K(I + L + M) + IL + LM + IJ + IH),$
$a_3 = \beta\pi_1 S^*(H + K + L + M + U) + \gamma\pi_2 S^*(H + J + L + M + U)$
$\quad + \epsilon\pi_3 S^*(H + J + K + M + U)$
$\quad - I(KM + LM + HJ + HK + HL + HM + JK + JL + JM + KL)$
$\quad - H(LM + JK + JL + JM + KL + KM) - J(KL + KM + LM) - KLM,$
$a_4 = HJKL + HJKM + HJLM + HKLM + JKLM + HJKI + HJLI + HJMI + HKLI + HKMI$
$\quad + HLMI + JKLI + JKM\,l + JLMI + KLMI - U\delta\gamma\rho - U\beta\delta\tau - U\delta\epsilon\eta$
$\quad - HJ\epsilon\pi_3 S^* - LM\gamma\pi_2 S^* - LM\beta\pi_1 S^* - KM\epsilon\pi_3 S^* - KM\beta\pi_1 S^* - KL\beta\pi_1 S^*$
$\quad - JM\gamma\pi_2 S^* - JM\epsilon\pi_3 S^* - JL\gamma\pi_2 S^* - JK\epsilon\pi_3 S^* - HM\gamma\pi_2 S^* - HM\beta\pi_1 S^*$
$\quad - HM\epsilon\pi_3 S^* - HL\gamma\pi_2 S^* - HK\epsilon\pi_3 S^* - HL\beta\pi_1 S^* - HJ\gamma\pi_2 S^* - HK\beta\pi_1 S^*$
$\quad - MU\beta\pi_1 S^* - MU\epsilon\pi_3 S^* - KU\beta\pi_1 S^* - KU\epsilon\pi_3 S^* - LU\beta\pi_1 S^* - JU\epsilon\pi_3 S^*$
$\quad - J\gamma U\pi_2 S^* - L\gamma U\pi_2 S^* - M\gamma U\pi_2 S^*,$
$a_5 = KLM\beta\pi_1 S^* + JLM\gamma\pi_2 S^* + HKM\beta\pi_1 S^* + HKM\epsilon\pi_3 S^* + HJM\gamma\pi_2 S^* + HKL\beta\pi_1 S^*$
$\quad + HJL\gamma\pi_2 S^* + HJM\epsilon\pi_3 S^* + HJK\epsilon\pi_3 S^* + HJL\gamma\pi_2 S^* + HLM\beta\pi_1 S^*$
$\quad + HLM\gamma\pi_2 S^* + JKM\epsilon\pi_3 S^* + JM\gamma U\pi_2 S^* + LM\gamma U\pi_2 S^* + JM\epsilon U\pi_3 S^* - HJKLM$
$\quad + JU\delta\epsilon\eta + JU\delta\gamma\rho + KU\beta\delta\tau + KU\delta\epsilon\eta + LU\beta\delta\tau + LU\delta\gamma\rho + KLU\beta\pi_1 S^*$
$\quad + JKU\epsilon\pi_3 S^* - HJKLI - HJKMI - HJLMI - HKLMI - JKLmI + JLG\gamma U\pi_2 S^*$
$\quad + KMU\beta\pi_1 S^* + KMU\epsilon\pi_3 S^* + LMU\beta\pi_1 S^*$
$a_6 = -HJKM\epsilon\pi_3 S^* - HJLM\gamma\pi_2 S^* - HKLM\beta\pi_1 S^* - JKMU\epsilon\pi_3 S^* - JLM\gamma U\pi_2 S^* - KLMU\beta\pi_1 S^*$
$\quad + HJKLMI - JKU\delta\epsilon\eta - JL\gamma U\delta\rho - KLU\beta\delta\tau.$

Next, using the Routh-Hurwitz the root of the equation will be negative (stable asymptotically) if:

$i.\ \alpha_1 > 0$
$ii.\ a_6 > 0$
$iii.\ a_1 a_2 - a_3 > 0$
$iv.\ Pa_3 - a_1 Q > 0$
$v.\ RQ - PS > 0$
$vi.\ TS - Ra_6 > 0$

Where

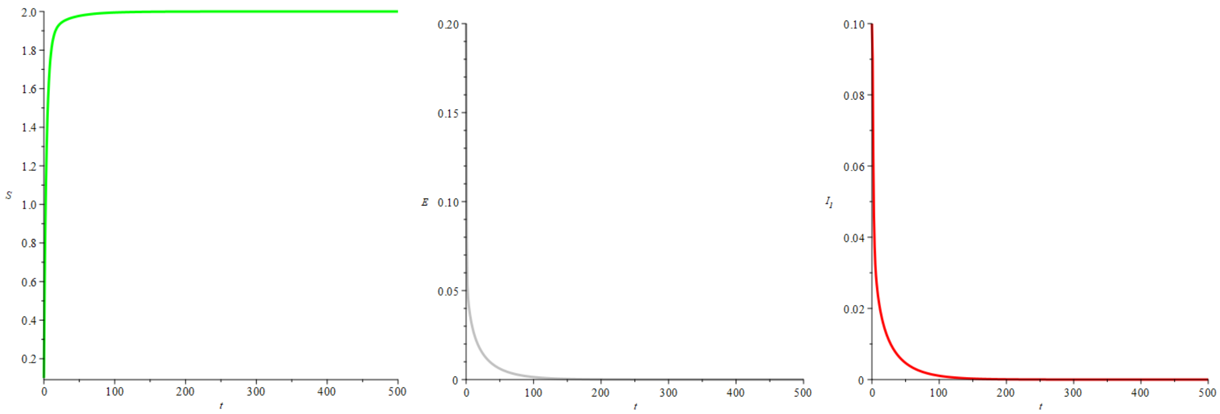
$$P = \frac{a_1 a_2 - a_3}{a_1}$$
$$Q = \frac{a_3 a_4 - a_2 a_5}{a_3}$$
$$R = \frac{P a_3 - a_1 Q}{P}$$
$$S = \frac{Q a_5 - a_3 a_6}{Q}$$
$$T = \frac{RQ - PS}{R}$$

3. Numerical Simulation

According to the parameter values in Table 1, we first calculate the value of $\Re_0, \Re_0 = 0.9410$, and we obtain $\Re_0 < 1$. Then, we calculate the value of the TikTok addiction-free equilibrium point, $T_1^0 = (2;\ 0;\ 0;\ 0; 0; 0)$. The simulation uses initial values to simulate the TikTok addiction-free equilibrium point $(0) = 0{,}1; E(0) = 0{,}2;\ I_1(0) = 0{,}1;\ I_2(0) = 0{,}1; I_3 = 0.1; R(0) = 0.2$. To analyze the stability of the equilibrium point from model (1), substitute the value of each parameter in Table (1) into

$i.\ -\alpha = -0.25 < 0$
$ii. -(\alpha + \lambda) = -0.6 < 0$
$iii.\ p = 0.75 > 0$
$iv.\ s = 0.0027578125 > 0$
$v.\ pq - r = 0.352150000 > 0$
$vi. \dfrac{r^2 + p^2 s - pqr}{r - pq} == 0.1137747278 > 0$

Because $\lambda_1, \lambda_2 < 0$ and $p, s, pr - r, \frac{r^2 + p^2 s - pqr}{r - pq} > 0$ then the equilibrium point of model (1) T_1^0 asymptotically stable when $R_0 < 1$. Based on the parameter values and initial values above, graphs for each group against time t are obtained, as shown in Figure 2.



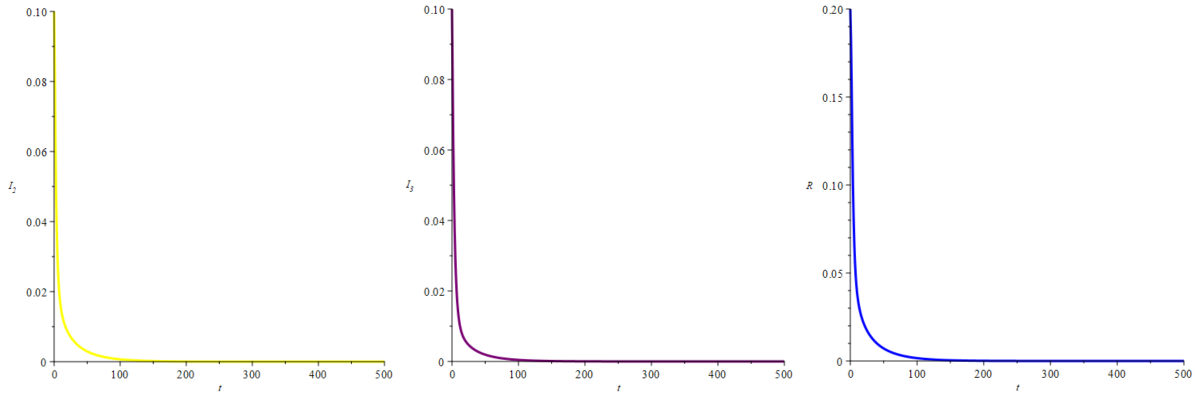


Figure 2. Trajectories off all the model compartments at the TikTok addiction-free equilibrium point

From the figure above, it can be seen that the subpopulations susceptible increases when the exposed subpopulations, infected content, infected ecommerce, infected viewers, and recovery declining. Then over time the susceptible subpopulation will increase towards equilibrium point $s^0 = 2$ while the subpopulations of exposed, infected content, infected ecommers, infected viewers, recovery will decrease leading to equilibrium point $E^0 = \mathrm{I}_1^0 = \mathrm{I}_2^0 = \mathrm{I}_3^0 = \mathrm{R}_0 = 0$. Next, the simulation is done for parameter variations π_1, π_2, π_3 with other parameters fixed. By taking $\pi_1 = 0.66$, $\Re_0 = 1.011756057 > 1$ is obtained. Because $\Re_0 > 1$, content creators will increase TikTok users. Then, we have the endemic equilibrium point value for TikTok addiction is $T_1^* = (1.976761; 0.006597; 0.00486126; 0.0029322; 0.0018; 0.006963)$. To analyze the stability of the endemic equilibrium point from model (1), substitute the value of each parameter in Table (1) into

$i.\ \alpha_1 = 3.85417 > 0$
$ii.\ a_6 = 0.00169798016 > 0$
$iii.\ a_1a_2 - a_3 = 0.352150000 > 0$
$iv.\ Pa_3 - a_1Q = 2.100482350 > 0$
$v.\ RQ - PS = 0.5630560638 > 0$
$vi.\ TS - Ra_6 = 0.07346449988 > 0$

Because $\alpha_1, a_6, a_1a_2 - a_3, Pa_3 - a_1Q, RQ - PS, TS - Ra_6 > 0$ then the endemic equilibrium point of model (1) T_1^* asymptotically stable when $R_0 > 1$. Based on the parameter values and initial values above, graphs for each group against time t are obtained, as shown in Figure 3.

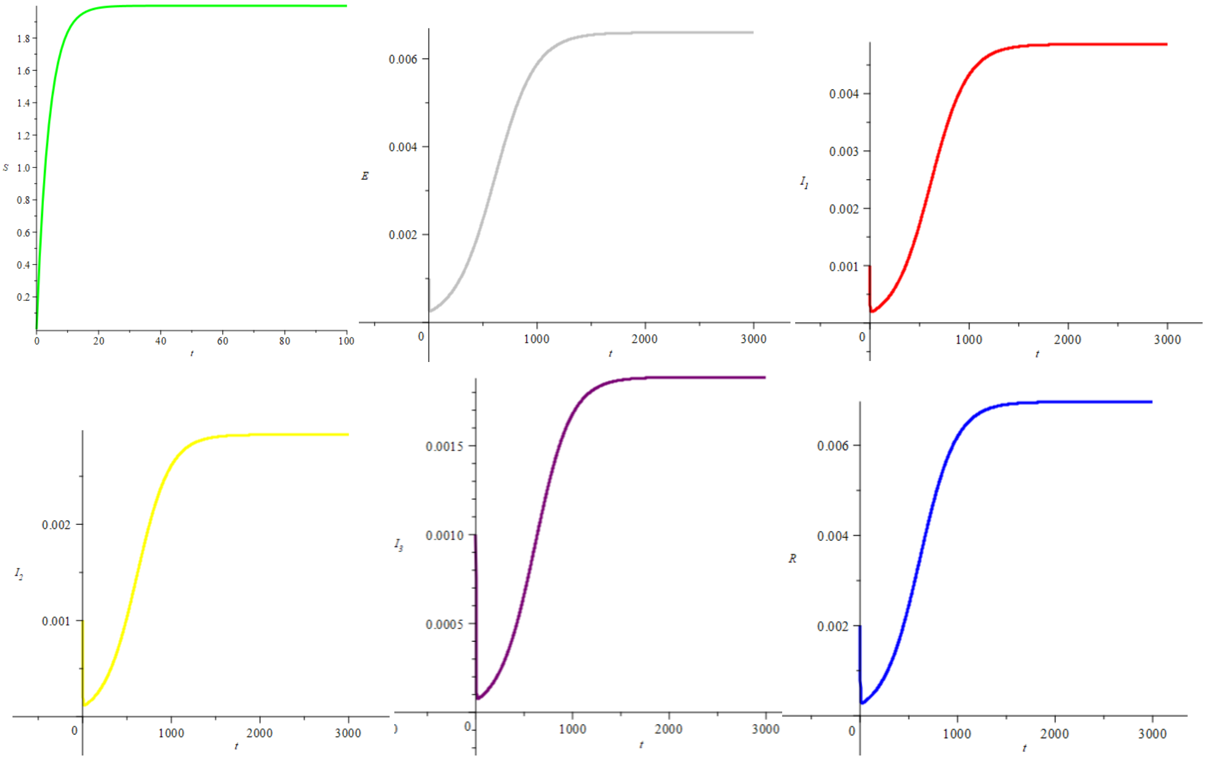


Figure 3. Trajectories with $\pi_1 = 0.66$ for all the compartments

In the same way, simulations for parameters π_2 and other parameters fixed, are also carried out. The simulation produces $\Re_0 > 1$, which means that the transmission rate of groups of individuals shopping will increase TikTok users. Based on the parameter values and initial values above, graphs for each group against time t are obtained, as shown in Figure 4.

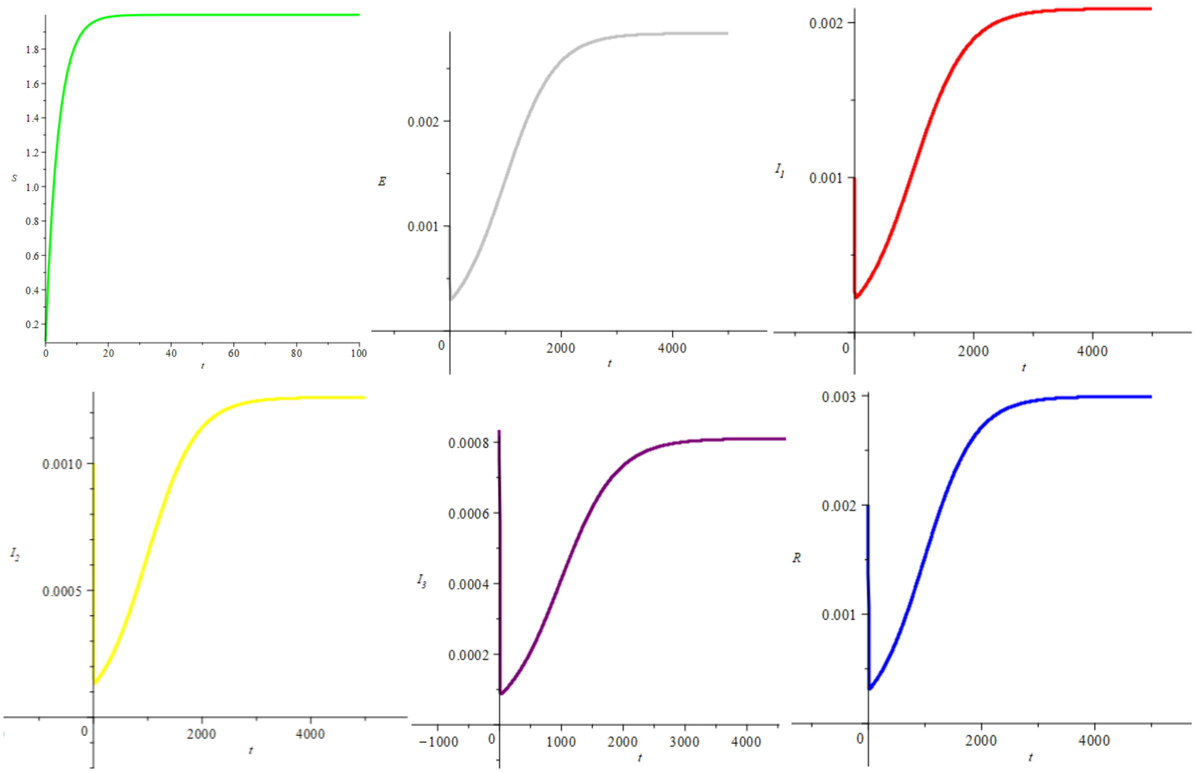


Figure 4. Trajectories with $\pi_2 = 0.29$ for all the compartments

Next, for simulation $\pi_3 = 0.33$ with other parameters fixed also result $\Re_0 > 1$, namely $\Re_0 = 1.000447786$. Therefore, the transmission rate of individual content viewers will result in an increase in TikTok users. Based on the parameter values and initial values above, graphs for each group against time t are obtained, as shown in Figure 5.

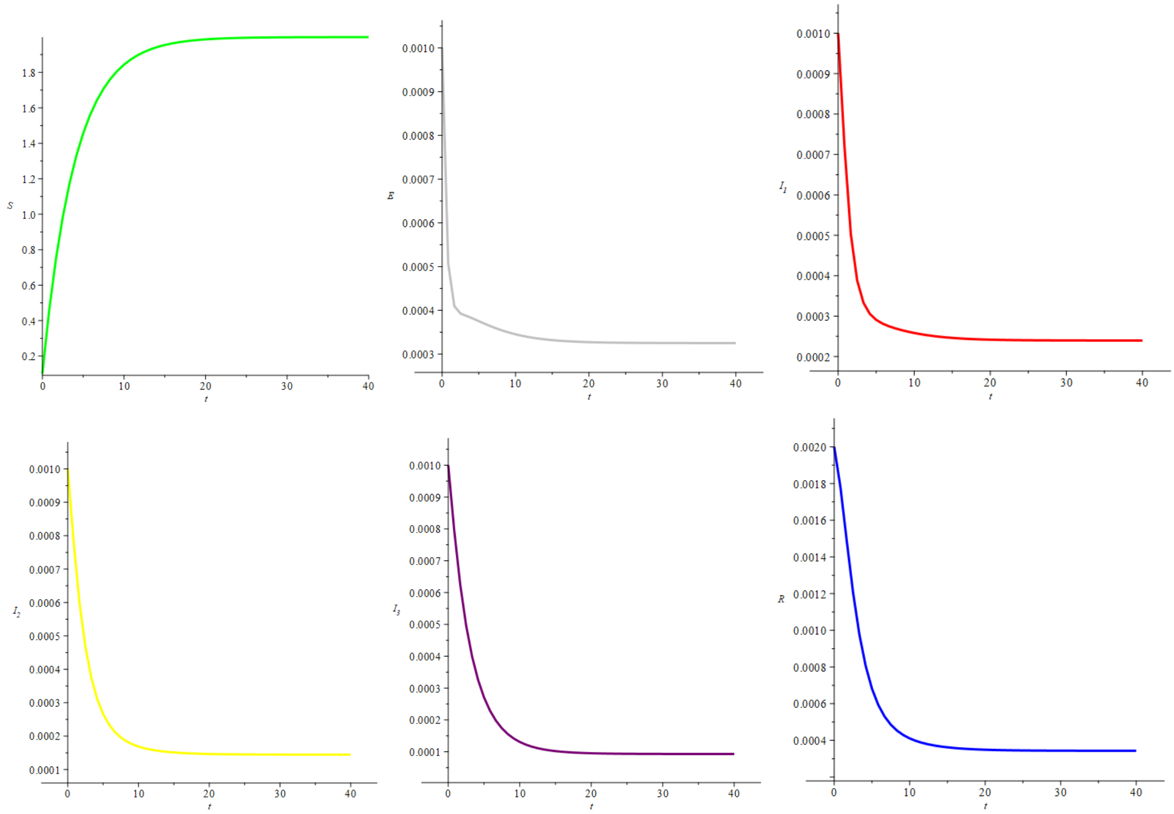


Figure 5. Trajectories with $\pi_3 = 0.33$ for all the compartments

It can be seen from the results of the data simulation that the three groups of TikTok addicts (π_1, π_2, π_3), influence the spread of TikTok users. The order of magnitude of influence of parameters or groups of TikTok addicts on the distribution of TikTok users based on the results of the basic reproduction number is the group of creators content, groups who shop and groups watching content on TikTok.

D. CONCLUSION AND SUGGESTIONS

Based on the system stability analysis, it can be concluded that if $\boldsymbol{R_0 < 1}$, system (1) is asymptotically stable at the free equilibrium point disease $\boldsymbol{T_1^0}$. As a result, the influence of TikTok addicts on increasing TikTok users is slowly decreasing and will disappear from the population as time goes by. However, if $\boldsymbol{R_0 > 1}$, system (1) is stable asymptotic at the endemic equilibrium point $\boldsymbol{T_1^*}$. As a result, the influence of addicted TikTok on the increase in TikTok users will remain in the population and will increase over time. The results of the data simulation showed that the order of magnitude of the influence of TikTok addiction parameters or groups on the increase in TikTok users based on the basic reproduction quantity was the creator group $(\boldsymbol{\pi_1})$, groups who shop $(\boldsymbol{\pi_2})$, and groups viewers content $(\boldsymbol{\pi_3})$ on TikTok, can be seen from real conditions in everyday life that content creators have a greater influence than others because it is from this group of content creators that TikTok users get content that has a negative impact, such as content about eating large amounts of spicy food such as larval meetballs which can damage the stomach. Therefore, the government should also prioritize handling negative effects based on the order of magnitude of these parameters' influence.

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