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Indonesian National Mortality Rates using the Whittaker-Henderson Graduation Method

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ABSTRACT

In this paper, we aim to present a graph depicting the quantified mortality rates for the entire population of Indonesia, derived from the 2019 World Health Organization (WHO) mortality data for Indonesia. First, the mortality rates which consisted of five-year age groups were interpolated to determine the rates for each individual age. Next, these rates were extrapolated to extend the data from age 85 up to age 110. The resulting crude rates were adjusted with the Whittaker-Henderson smoothing technique by utilizing Python and MS Excel. The refined results were then compared to the insured lives from the fourth Indonesian Mortality Rates Table (TMI IV). This assessment supplied the government with insights to help shape health policies and inform economic forecasts. The results indicated that male mortality rates were higher than those of females, although no significant difference was observed among the younger generation. On the contrary, mortality rates of old people were significantly greater compared to the insured lives which was due to WHO's limited data availability and more comprehensive data collection process, compared to TMI IV's insured lives through the underwriting process.

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A. INTRODUCTION

The insurance business, namely life insurance, contributed positively and significantly to the economic growth of Indonesia, as highlighted by (Safitri, 2019; Şenol et al., 2020). However, its density and penetration rates were still considered low, compared to the total population, during the period of 2016-2020 (Nazhifa, 2022). Compared to other ASEAN countries, according to Ferezagia (2018), Indonesia was still behind Singapore, Malaysia, and Thailand in terms of insurance penetration. Nevertheless, there was a 52% change in insurance penetration from 1990 to 2016. This signified that Indonesian residents were becoming aware of the importance of having insurance as a financial protection and as a form of risk awareness. Financially secured individuals are more likely to have access to better health care services and can afford private health insurance, which leads to lower mortality rates. The lower to middle income families have limited access to health care and tend to be provided solely by the universal health coverage (UHC) of the national health insurance, although it did not work well due to Indonesia's vast geography, health accessibility, and culture (Agustina et al., 2018). Since its inception in 2014, the UHC was heavily influenced by political factors (Pisani et al., 2017). The system currently covers roughly 200 million people, while the remaining 40 million were not yet covered (Perdana et al., 2022). This poses a challenge to its initial promise of providing affordable healthcare for all Indonesian citizens. This disparity in healthcare access leads to different mortality rates for the wealthy and the impoverished. Low mortality rates led to higher labour productivity (Schultz, 1993), which eventually achieve economic stability. On developed countries, such as Australia, England & Wales, and the United States, national life tables were produced based on the mortality experience of the entire residents of a country by using records of deaths and census data. Other life tables based on some group of individuals are produced to accommodate for different purposes (Dickson et al., 2020). In Indonesia, there are numerous life tables produced: the mortality table based on aggregate data from life insurance companies, the most recent release was on the fourth edition (TMI IV) (Suwondo et al., 2019).

Several previous studies have aimed to project mortality rates as follows (Nigri et al., 2019). Incorporated deep learning techniques to the Lee-Carter model. Hainaut (2018) utilized neural network to forecast mortality which showed significant prowess compared to the traditional Lee-Carter model (Liu et al., 2017). Introduced an improved version of "nuga hedging" to handle longevity risks in the locally linear Cairns-Blake-Dowd model (Ahmadi, S.; Li, 2014). Proposed development of the generalized linear models in which estimates of mortality rates are generated using the model structure, without undergoing stochastic processes (Odhiambo et al., 2022). used credibility regression approach for mortality modelling with the assumption of the number of actual deaths following a Poisson distribution, which accounted for a large population with small probability of death. Another method is also proposed by Odhiambo (2023) by incorporating the Gaussian distributional assumption on the error term of the Lee-Carter model, and the deep-learning method is used to estimate the parameter values. Manejero et al. (2020) used the variational formulation to solve partial differential equation problem numerically by first treating the Whittaker-Henderson method as a minimization problem in a Sobolev space.

There were several limitations to the previous studies mentioned above. Unlike in developed countries, Indonesia has limited data pertaining to experience study of the entire population. The actual number of actual deaths are not readily available. This means that the parameters of the Poisson distribution cannot be estimated directly from the historical data (Odhiambo et al., 2022). Subsequently, the dataset used in this study consisted of mortality rates for 5-year age brackets, but our goal is to generate mortality rates for each individual year of age. As a result, stochastic processes cannot be used directly in this context. The deep-learning techniques presented by Hainaut, (2018); Nigri et al. (2019); Odhiambo (2023) were also not applicable due to the limited availability of data. To produce mortality tables based on the experience of the entire country, there must be data collection of death records and surveys of healthcare facilities conducted on the entire population. In comparison, the mortality rates from TMI IV consisted of insured lives which were not representative of the entire population of Indonesia. These individuals were screened from certain medical conditions and were financially secured to be able to purchase insurance contracts. Thus, this paper aimed to estimate the Indonesian mortality rates by obtaining data from the World Health Organization (WHO) (Organization, 2023). The collected data was in 5-year age brackets for people aged 0-85. Thus, to align with the mortality rates in TMI IV, which presented the mortality rates for each age, we conducted interpolation and extrapolation techniques for projecting the 5-year intervals into mortality rates of each age for ages 0-110. The initial crude rates were refined by a graduation method. The resulting graduated rates were then compared with TMI IV to assess its reasonableness. This assessment is vital for the government to make informed decisions about healthcare policies, especially regarding its UHC program, and strategies for economic projections. The paper is to be explained in the following order. The next section discusses the underlying methods conducted in this study. The third section presents the results and analysis, along with a comparison of these results to the mortality rates from TMI IV. The last section concludes the findings of this paper and suggestions for future research.

B. METHODS

To estimate the Indonesian mortality rate, the quantitative process workflow is shown in Figure 1. First, we analyse the 2019 secondary tabular data from Indonesian residents obtained from WHO database, based on surveys from household, health facilities, and civil records (Organization, 2023). Figure 2 illustrates the initial crude mortality rate. From this figure, the 2019 Indonesian mortality data is incomplete, and some values are missing. The data showed mortality rates in intervals of group of individuals within the age brackets, not for each individual age. These brackets are as follow: ages 0-1, ages 1-4, ages 5-9, ages 10-14, ..., ages 80-84, and age 85+, moreover this signifies people older than 85 were grouped into the 85+ age group. Therefore, we must clean and prepare this incomplete data before applying the graduation method. To do this, we used the cubic spline interpolation method to estimate mortality rates all individual age for each age brackets. This method was preferred for its versatility and ease in capturing fluctuations in mortality rates between ages, especially when compared to other methods such as linear interpolation, which may oversimplify the graph, or radial basis function interpolation, which can be overly complex to implement. Subsequently, the mortality rates were extended to age 110 using the Gompertz mortality law to align with mortality rates of TMI IV from the life insurance companies. This law was selected because it specifically addressed age-related mortality trends, whereas the Makeham mortality law incorporated the influence of a constant mortality rate as well (Macdonald et al., 2018). After these interpolation and extrapolation steps, the Whittaker-Henderson graduation method was applied to smooth the mortality rates.

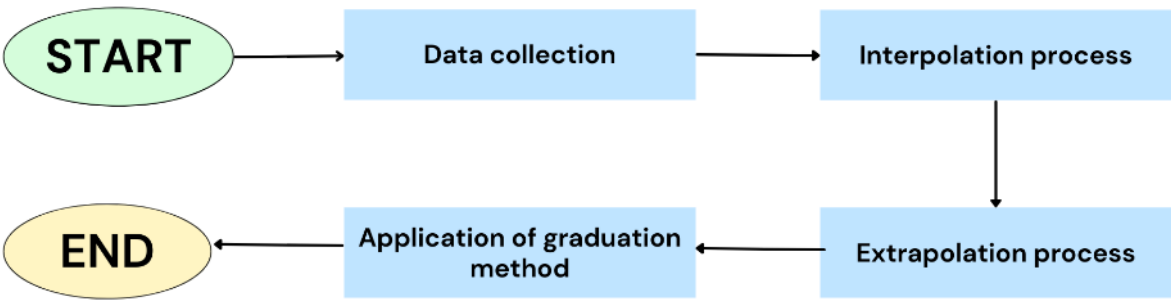

Figure 1. Flowchart of mortality rate estimation process based on 2019 WHO data.

1. Interpolation Process

Interpolation is the process of adding data samples by estimating values within the range of known data. In this study, the cubic spline method was used, which estimates values at points between two known data points using a third-degree polynomial function (Kong et al., 2020), defined the function as:

$$f_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i, i = 1, 2, \dots, n-1 \tag{1}$$

with a_i, b_i, c_i, d_i denoted as constants with unknown value. To solve equation (1), these four constants were estimated using:

$$f_i(x_i) = y_i, \qquad i = 1, 2, \dots, n-1, \tag{2}$$

$$f_i(x_{i+1}) = y_{i+1}, \qquad i = 1, 2, \dots, n-1, \tag{3}$$

$$f_i'(x_{i+1}) = f_{i+1}'(x_{i+1}), \qquad i = 1, 2, \dots, n-2, \tag{4}$$

$$f_i''(x_{i+1}) = f_{i+1}''(x_{i+1}), \qquad i = 1, 2, \dots, n-2, \tag{5}$$

$$f_1''(x_1) = 0, \tag{6}$$

$$f_{n-1}''(x_n) = 0, \tag{7}$$

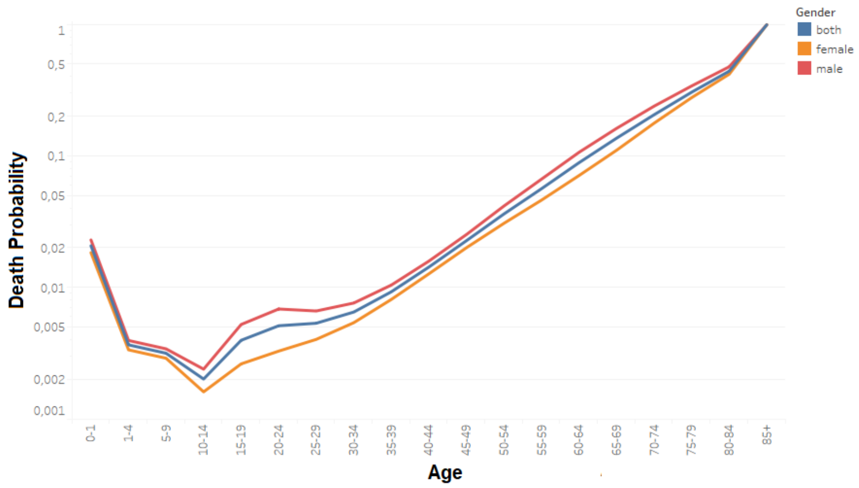

Figure 2. The 2019 Indonesian mortality rate based on WHO data.

where x_i signified independent variable and y_i denotes the dependent variable. Equations (2) and (3) ensured $f_i(x)$ remained continuous on $[x_i, y_i]$. Equations (4) and (5) assured the first and second derivatives of $f_i(x)$ also remained continuous and there was no acute angle at any data point. Lastly, equations (6) dan (7) guaranteed that the second derivative of the curve was a linear function. This method is applied to Indonesia's mortality rate data to provide an accurate estimate of annual mortality rates without overfitting. Cubic spline interpolation can also address negative changes in mortality rates. This process was implemented with the help of CubicSpline package from the Python software. The package produced the death probabilities (q_x) for each age and gender.

2. Extrapolation Process

Extrapolation is the process of adding data samples by estimating values from data points outside the known range. In this study, extrapolation is performed using the Gompertz law. The Gompertz law is a mathematical model used to model mortality rates as age increases. The use of the Gompertz law is based on the observation that the risk of death for an individual tends to increase exponentially with age. The mortality probability is defined by the Gompertz distribution (London, 1997; Macdonald et al., 2018) by:

$$q_x = 1 - \exp\left[-\frac{Bc^x}{\ln(c)}\,(c-1)\right] \tag{8}$$

where parameters B and c are evaluated numerically using the root mean squared error (Hodson, 2022):

$$error_x = \sqrt{(q_{x,estimate} - q_{x,real})^2}, x = 1,2,\ldots,84$$

The parameter B represented the baseline mortality rate in which individuals die at younger ages, whereas the parameter c denoted the rate at which mortality risks rise for older ages. The interval values were set between $10^{-5} \leq B \leq 10^{-4}$ with increments of 10^{-6} and $1{,}0001 \leq c \leq 1{,}5$ with increments of 10^{-4} (Fatimah et al., 2016). The optimal values for B and c were used for the extrapolation process to estimate the mortality rates for people aged 85 to 110.

3. Graduation Method

The graduation method is a commonly used technique for smoothing mortality rates. This method is necessary to create a smoother mortality rate where there is less fluctuation in the values. In actuarial science, smooth mortality rates are important for various purposes, such as estimating future mortality rates, calculating life expectancy, determining the value of annuities in insurance products, and calculating other financial and insurance products that depend on mortality rates. The Whittaker-Henderson graduation method is the most common method used to smooth crude rates (Chanco, 2016; Dzupire et al., 1952; Whittaker, 1922). This method balances the smoothness of the mortality rate with the accuracy of the data. The graduation method is defined as:

$$M = \sum_{x=0}^{n} w_x (q_x - \hat{q}_x)^2 + h \sum_{x=0}^{n-z} (\nabla^z \hat{q}_x)^2, \tag{9}$$

where w_x denotes the exposure or exposed to risk of death Asamoah (2019) which is essentially a weight value between 0 and 1, q_x is the death probability of someone aged x until $x+1$, h denotes the smooth rate, z symbolizes the order of differencing, and ∇ calculates the subtraction in the differencing method.

By equation (9), the first expression evaluated the accuracy of the data by quantifying the quadratic sum of the difference between the crude rates and the graduated rates, then it was multiplied by the weights which were randomly generated as there was no exposure data available from the WHO database. The second expression arranged the smooth rate of the graduation method using the differencing technique, where h values were predetermined to be 10, 50, and 100. The bigger the value of h, the more smoothing occurs. In this method, the crude rates were graduated until the value of M reaches its minimum. Using the solver function on Excel, we obtained the minimum value of M, along with the graduated rates $\hat{q}_x$.

C. RESULT AND DISCUSSION

1. Result of the Interpolation Process

For the 2019 WHO mortality rates for Indonesia, the data only included mortality probabilities for certain age ranges. This posed a problem because mortality probabilities for every age were needed to calculate graduated mortality rates. Therefore, interpolation was performed to expand the data so that each age has its own mortality rate. The interpolation method applied to the data was cubic spline interpolation as in equations (2) to (7). The mortality probability given by the data at age 85 was 1, meaning that all lives were expected to survive until a maximum age of 86 years old. The results of the interpolation for the 2019 WHO mortality rates for Indonesian residents are shown in Figure 3.

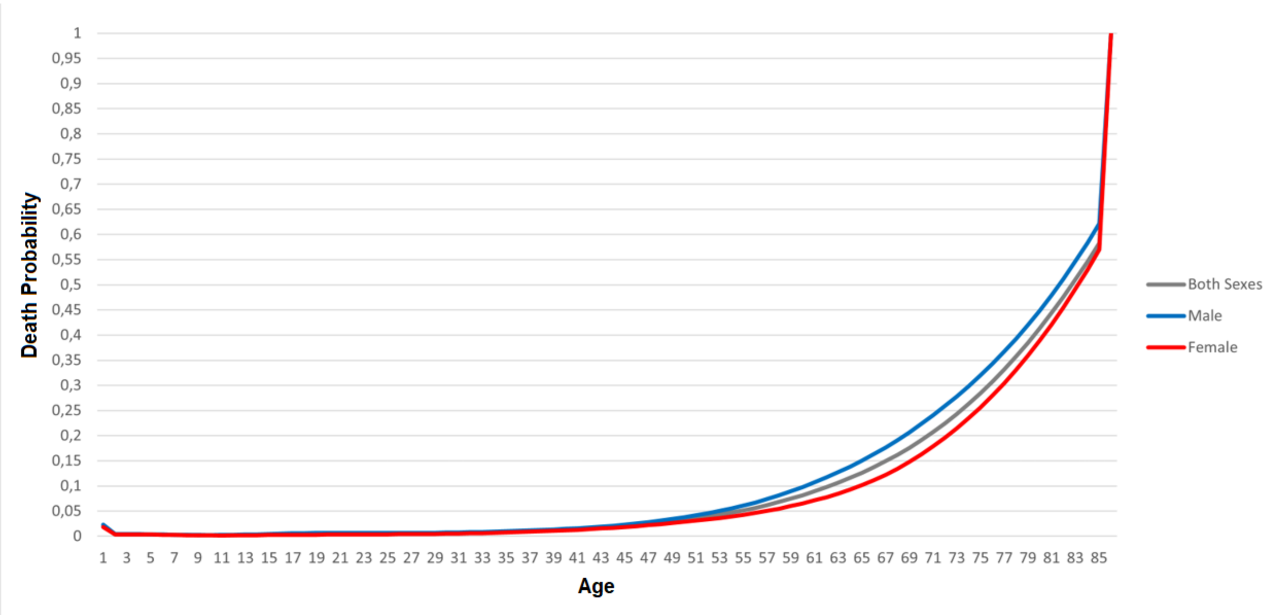

Figure 3. Graph of interpolation result of the Indonesian mortality rate based on 2019 WHO data.

2. Result of the Extrapolation Process

To assess the reasonableness of the WHO mortality data, we compared it with TMI IV which contains mortality rates for ages 0-110. The WHO data, after interpolation, covered ages 0-85 only. Therefore, we must extrapolate the WHO data numerically using the Gompertz mortality law. Obtained from Python, Table 1 presents the optimal parameter values for B and c through minimized root mean squared error calculation between predicted rate from Gompertz law and actual rate from WHO data.

Table 1. Optimal parameter values based on Gompertz mortality law.

Parameter	Mixed	Gender Male	Female
B	10^{-4}	10^{-4}	10^{-4}
c	1.1144	1.1162	1.1123

The values in Table 1 are used in the extrapolation process of mortality rates for ages 85-110. It is also assumed that no one lives beyond the age of 111, therefore the death probability of a person aged 111 is 1. The extrapolation result is shown in Figure 4. An alternative approach for achieving smoother results is through curve fitting (Chapra, 2018) by utilizing the *cftool* function in the MATLAB software.

3. Whittaker-Henderson Graduation Method

In the Whittaker-Henderson graduation method, there were two parameters to be optimally determined: h, which controlled the degree of smoothing, and z, which governed the order of the differencing method. In this calculation, the values used for h were {10, 50, 100} and the values for z were {2, 3, 4}. The optimal parameters value for h and z were calculated based on equation (9) that resulted when M reached its minimum value. The results were shown in Table 2, Table 3 and Table 4.

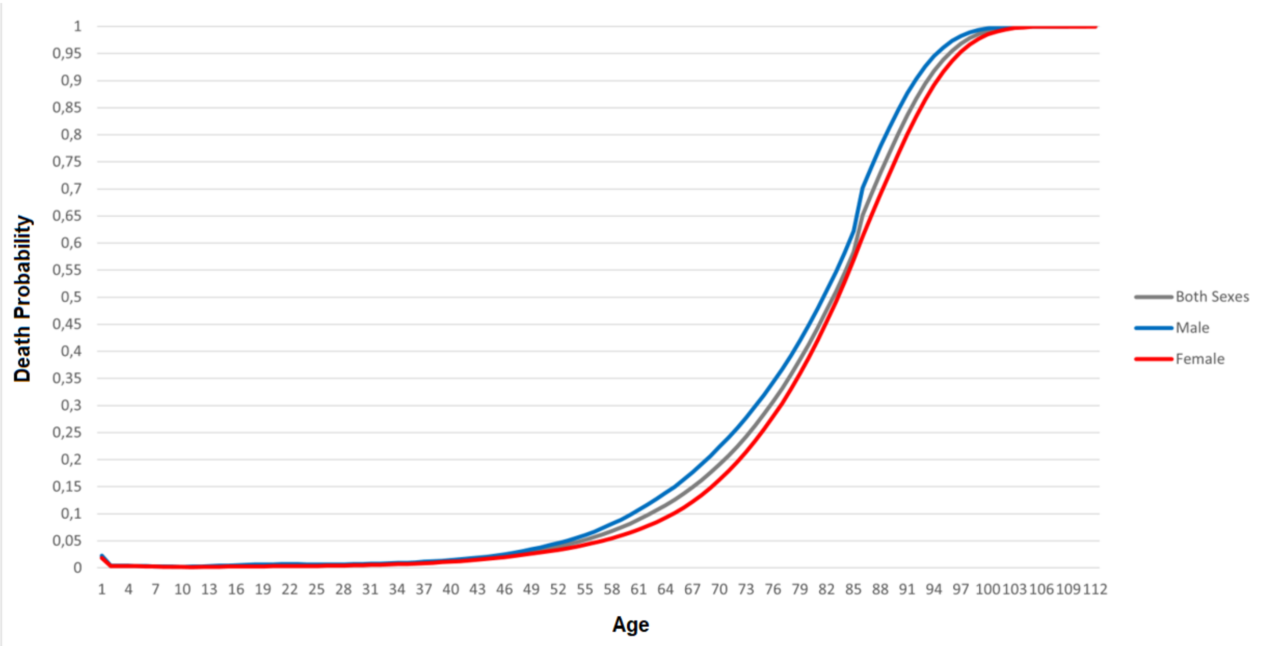

Figure 4. Graph of extrapolation result of the Indonesian mortality rate based on 2019 WHO data.

The result of graduated mortality rates is shown in Figure 5. It can be observed that the highest mortality rates are among males, with all Indonesian men expected to die between the ages of 109 and 110, whereas for all Indonesian women are expected to die between the ages of 110 and 111. When reviewing the combined data for both genders, it shows that all residents of Indonesia are expected to die between the ages of 110 and 111.

Table 2. Parameter value of M for $\boldsymbol{h} = \mathbf{10}$.

Gender	2	z 3	4
Mixed	0.002483	0.000724	0.000681
Male	0.003178	0.001125	0.001324
Female	0.001881	0.000416	0.000528

Table 3. Parameter value of M for $\boldsymbol{h} = \mathbf{50}$.

Gender	2	z 3	4
Mixed	0.009433	0.001652	0.002490
Male	0.010360	0.002585	0.001769
Female	0.008053	0.001471	0.014963

Table 4. Parameter value of M for $\boldsymbol{h} = \mathbf{100}$.

Gender	2	z 3	4
Mixed	0.017391	0.002742	0.004433
Male	0.019955	0.003854	0.006664
Female	0.014730	0.001655	0.002812

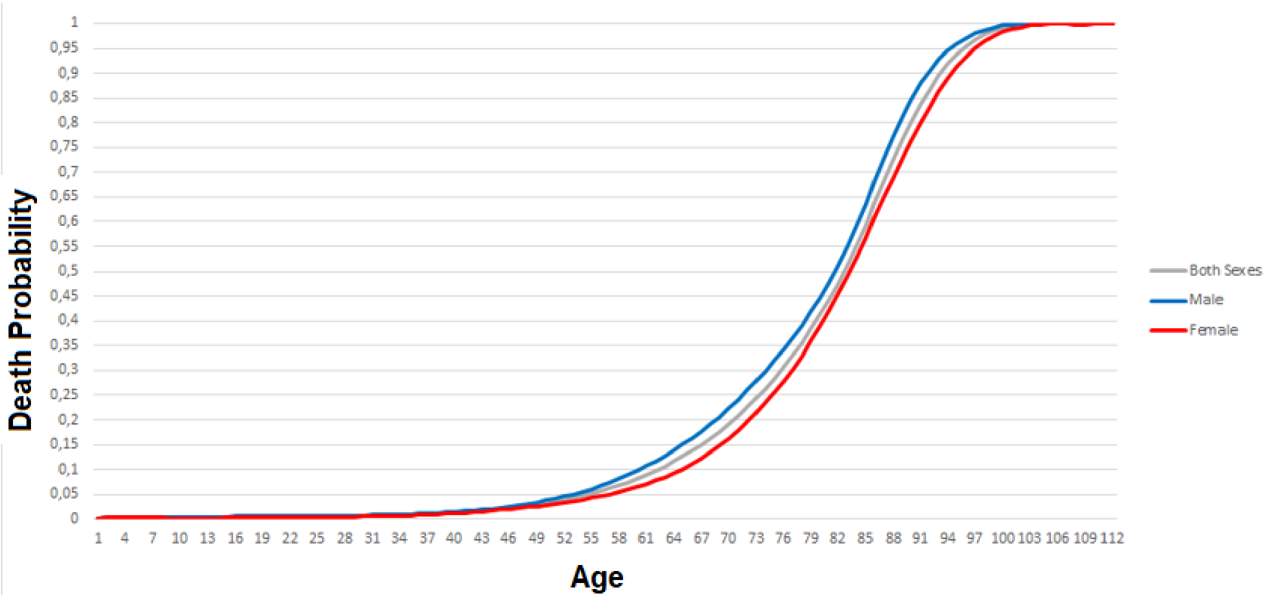

Figure 5. Graph of graduated Indonesian mortality rates based on 2019 WHO data.

The mortality risk should be increasing as a person gets older. However, based on the graduated results, there is inconsistency in this claim. The male mortality rates between ages 103 and 106 and the female mortality rates between ages 106 and 108 are decreasing. Therefore, the overall mortality rates are decreasing between ages 103 and 107. The decreasing values are between 0.0005 and 0.001 for each age. These fluctuations happened as there is exposure, data limitations as it is currently unavailable in the WHO website. Subsequently, the exposure data (w) were randomly generated, resulting in inconsistent results explained before. To remedy this fluctuation, it is crucial to request additional historical mortality data from WHO or conduct similar survey which will take time and resources. Afterwards, the latent Gaussian model is applied to the data, as proposed by (Alexopoulos et al., 2019). The Bayesian method is used to estimate the value of parameters and latent states of the model.

4. Mortality Rates Comparison Between WHO Data and TMI IV

The results of the Whittaker-Henderson graduation process for the mortality rates of Indonesian males and females based on the 2019 WHO data is compared with the TMI IV data. Figure 6 shows the comparison of mortality rates for the male population in Indonesia. According to the 2019 WHO Indonesia data, most men are expected to die between the ages of 91 and 92, with a mortality probability of 0.90661, and death is certain to occur between the ages of 109 and 110. In contrast, the TMI IV data indicates that the mortality rate for men aged 91 to 92 is only 0.17991. Furthermore, the probability of death between the ages of 110 and 111 is also very low, at 0.59244.

Figure 7 compares the female mortality rates between TMI IV and the graduated results from the 2019 WHO data. Similar to the male mortality rates, the female mortality rates show a significant difference. The WHO mortality rates show that a high mortality probability occurs when women are between the ages of 94 and 95 with a q_x value of 0.91361, and all women in Indonesia are expected to die between ages 110 and 111. In the TMI IV data, the mortality probability is notably low at older ages, with a q_x value of 0.58702 between ages 110 and 111. A study by (Lan, 2022) noted that young women tend to buy life insurance product compared to young men. This may lead to women having lower mortality rates that men.

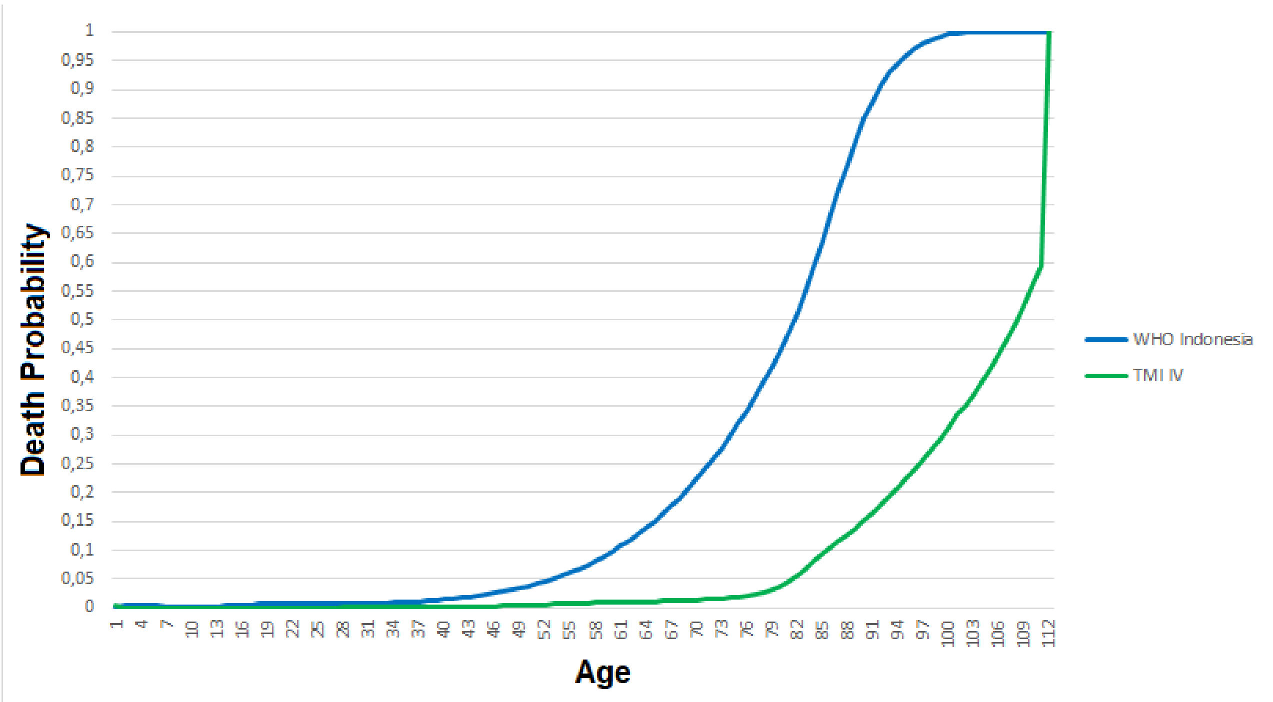

Figure 6. Comparison of male mortality rates between TMI IV and graduated results from WHO.

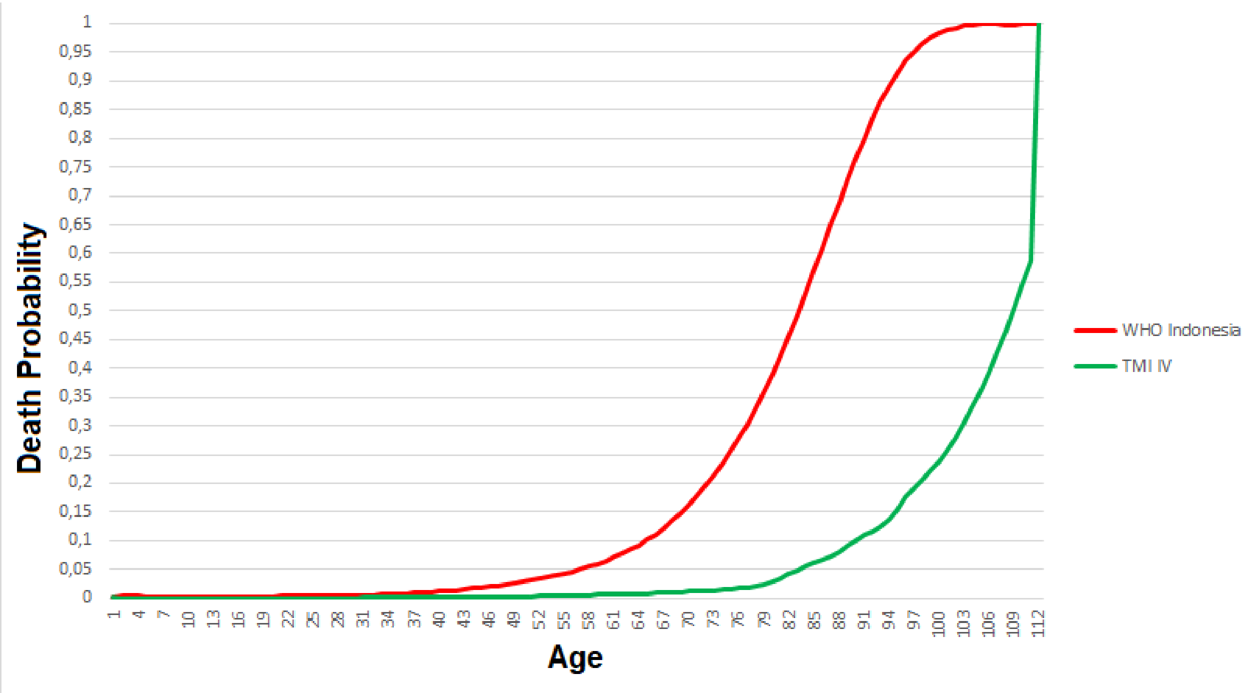

Figure 7. Comparison of female mortality rates between TMI IV and graduated results from WHO.

The significant difference in mortality rates between the 2019 WHO Indonesia data and the TMI IV data is due to the sources of data collection. The WHO Indonesia data is obtained from various household surveys, civil records, and healthcare facilities. Collecting data from these diverse sources results in a broader coverage of the population, including lower, middle, and upper socioeconomic groups, leading to higher mortality rates. Furthermore, the data was limited up to age 85. Therefore, the hike in mortality rates in these older ages were expected. In contrast, TMI IV data Suwondo et al. (2019) was sourced from 52 life insurance companies in Indonesia. This meant the data was limited to Indonesian residents with life insurance, who had been screened by insurance companies through underwriting processes, where most policyholders were from middle to upper socioeconomic groups (Cantiello et al., 2015). Additionally, these groups were typically more educated about the importance of adequate nutrition, health maintenance, and access to better healthcare. This results in significantly lower mortality rates for the TMI IV data. Due to the disparity in data collection, our research provided a clearer picture of the mortality rates for the Indonesian population. On average, older and uninsured people aged 70-110 were more prone to mortality risk compared to the insured ones. This is in line with a research by Woolhandler & Himmelstein (2017) which stated that the odds of dying for insured compared to uninsured lives were 0.71 to 0.97.

D. CONCLUSION AND SUGGESTIONS

Based on the study presented, the 2019 WHO data undergone interpolation and extrapolation processes. The mortality rates of each age bracket for ages 0-85 were extended to include individual ages from 0 to 110. The Whittaker-Henderson graduation method produces the best result among the hyperparameters with smoothing parameter $h = 100$ and differencing order $z = 3$. In comparison with TMI IV, there were several findings. First, the male mortality rates are higher than those of female which is aligned with the results from TMI IV.

Secondly, the mortality rates for older people were significantly higher than those of the younger generation for both genders. Thirdly, the uninsured people aged 70-110 were more prone to mortality risk compared to insured lives. The graduated results from WHO data could serve as a valuable reference for the government's UHC program, health policy development by evaluating it as an experience study of mortality, and economic projections. For future research, it is essential to collect exposure data from WHO to obtain more accurate results. To develop rates for future insurance products with high mortality risk, the fusion rates of TMI IV and WHO can be analyzed using credibility theory. To utilize the deep learning methods combined with the Lee-Carter or Cairns-Blake-Dowd mortality models following the processes described in the first section, it is also crucial to acquire the raw mortality data from several years collected from surveys instead of relying on the processed mortality rates table provided by WHO. These techniques help to identify trends contained in the historical data.

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