**JTAM (Jurnal Teori dan Aplikasi Matematika)**
http://journal.ummat.ac.id/index.php/jtam
p-ISSN 2597-7512 | e-ISSN 2614-1175
Vol. 9, No. 2, April 2025, pp. 615-628

# Estimation of Tail Value at Risk for Bivariate Portfolio using Gumbel Copula

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**Article History:**
Received : 27-02-2025
Revised : 12-04-2025
Accepted : 14-04-2025
Online : 29-04-2025

**Keywords:**
Archimedean;
Kendall's Tau;
Monte Carlo;
Simulation.

**ABSTRACT**

Investing in the stock market involves complex risks, especially under extreme and unpredictable conditions. While Value at Risk (VaR) is a widely used risk measure, it has limitations in capturing tail-end risks. This study employs Tail Value at Risk (TVaR) using the Gumbel Copula approach, which effectively models upper-tail dependence in return distributions—an aspect often overlooked by traditional linear correlation methods. This quantitative research utilizes copula-based Monte Carlo simulation. The data consists of daily closing prices of PT Adaro Energy Indonesia Tbk (ADRO) and PT Indo Tambangraya Megah Tbk (ITMG) from July 3, 2023, to July 30, 2024. The analysis begins with return calculation and tests for autocorrelation and homoskedasticity. The Gumbel Copula parameter is estimated using Kendall's Tau, resulting in a dependence parameter of 1.7791. Based on this, 1,000 simulations are conducted to generate new return data that reflect extreme dependencies between the two stocks. An optimal portfolio is constructed using the Mean-Variance Efficient Portfolio (MVEP) method, assigning weights of 31.61% to ADRO and 68.39% to ITMG. TVaR is then calculated from the simulated portfolio returns. The results show increasing TVaR values at higher confidence levels: 2.08%, 2.64%, 3.14%, and 4.11% for 80%, 90%, 95%, and 99%, respectively. These findings demonstrate that TVaR provides more accurate insights into potential losses in extreme market conditions, supporting investors in developing more informed and risk-sensitive portfolio strategies.

Crossref
https://doi.org/10.31764/jtam.v9i2.29952


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## A. INTRODUCTION

Investment is the process of allocating resources or funds in the present with the expectation of gaining future benefits (Salmah & Harahap, 2023). In making investment decisions, investors aim to achieve high returns. However, as the potential return increases, the associated risk also tends to rise, and vice versa (Bui et al., 2021). In Indonesia, the mining sector recorded the highest investment realization in the second quarter of 2024, amounting to IDR 32.1 trillion (BKPM, 2024). This figure illustrates a significant level of investor interest in the sector. Nevertheless, the potential for high returns in this sector is accompanied by significant investment risks.

Stocks are an alternative investment because they are considered capable of providing high returns. However, this potential also comes with a high level of risk. Risk can be interpreted as the possibility that the actual return will deviate from the expected return. To manage this, one effective strategy is portfolio formation. A portfolio is created by combining two or more stocks with the aim of optimizing returns while minimizing risk through diversification (ZongMing et al., 2020). Diversification helps reduce unsystematic risk, as the negative performance of one asset can potentially be offset by the positive performance of another (Markowitz, 1952). Before forming a portfolio, it is important for investors to estimate the level of risk associated with the assets involved. One widely used tool for measuring financial risk is Value at Risk (VaR). VaR estimates the potential maximum loss of an investment over a specified period and at a given confidence level, thus providing investors with a clearer picture of possible downside outcomes (Subekti et al., 2019;Jorion, 2007).

VaR is a statistical technique that estimates the maximum potential loss of an asset or portfolio within a specific time frame at a certain confidence level. This method can be implemented using various approaches, such as historical simulation, variance-covariance, and Monte Carlo simulation (Irsan & Sirait, 2020). Despite its popularity, VaR has limitations in estimating losses under extreme market conditions, as it does not account for the severity of losses beyond the VaR threshold (Sulistianingsih et al., 2022). To address this limitation, Tail Value at Risk (TVaR) is introduced as a more accurate measure for estimating the expected loss in the tail of the distribution (Klugman et al., 2019).

TVaR is a risk measurement technique that calculates the average loss exceeding the VaR threshold (Lu, 2023). In other words, while VaR provides a threshold indicating the maximum potential loss at a certain confidence level, TVaR measures the average of losses that go beyond this limit, offering a more comprehensive view of extreme risk. Traditional risk measurement methods often rely on assumptions such as linear relationships and normally distributed returns. However, financial return data tend to exhibit heavy tails and nonlinear dependencies, making standard models less accurate in capturing joint extreme losses (Danielsson 2011; Hotta et al., 2008). To overcome this, copula functions provide a flexible framework to model complex dependence structures by separating marginal distributions from their joint behavior, allowing for more accurate TVaR estimation (Sahamkhadam, 2021).

Copula is a flexible approach for modeling dependence between random variables because it does not rely on the assumption of normal distribution (Palaro & Hotta, 2006). This approach allows the separation of the dependence structure from the marginal distributions of each variable, making it applicable to various types of data, including complex financial data (Dewick & Liu, 2022). Unlike traditional methods that rely solely on linear correlation, copulas are capable of capturing non-linear relationships and extreme behaviors that often occur in financial markets. This ability makes copulas highly useful in risk modeling, especially when return distributions are asymmetric or have fat tails. Selecting the appropriate type of copula is an important step in the analysis process, as each type has different dependence characteristics. In practice, several copula families are commonly used, such as the Elliptical copula and the Archimedean copula. The Archimedean copula is often chosen in bivariate analysis due to its relatively simple functional form and includes several popular types, such as the Clayton, Gumbel, and Frank copulas (Nurrahmat et al., 2017).

One type of copula from the Archimedean family that is widely used in financial risk analysis is the Gumbel copula. This copula is effective in capturing upper tail dependence, making it suitable for modeling extreme events in financial markets. This characteristic is particularly relevant in volatile sectors such as mining, where high returns are often accompanied by high risks (Tinungki et al., 2023). However, the Gumbel copula has limitations, as it cannot capture lower tail dependence i006E the return distribution. This study aims to estimate the Tail Value at Risk (TVaR) of a stock portfolio consisting of two mining sector companies listed in the LQ45 index, namely PT Adaro Energy Indonesia Tbk (ADRO) and PT Indo Tambangraya Megah Tbk (ITMG), using the Gumbel copula.

## B. METHODS

This research is a quantitative study employing a Monte Carlo simulation approach based on the Gumbel copula to measure stock portfolio risk. The analysis is conducted using RStudio. The data used consists of the daily closing prices of two companies, from which daily returns are calculated. Autocorrelation is tested using the Autocorrelation Function (ACF) plot or the Ljung-Box test, while heteroscedasticity is examined using the ARCH-LM test. If heteroscedasticity is detected, the data may be filtered using the GARCH model to obtain more stable returns. Next, the dependence between stock returns is estimated using the Gumbel copula, with the copula parameter derived from Kendall's Tau. Based on this parameter, a Monte Carlo simulation is conducted $m$ times to generate $m$ sets of new return data that follow the dependence structure of the original data. Subsequently, the optimal portfolio weights are calculated once using the Mean-Variance Efficient Portfolio (MVEP) approach based on the original data. These weights are then used to calculate the portfolio return from each set of simulated data. From the $m$ portfolios formed through simulation, the Value at Risk (VaR) and Tail Value at Risk (TVaR) are calculated for each. The final step involves calculating the average VaR and TVaR from all simulations, interpret the results, and concluding the analysis.

### 1. Stock Return

Return is a key indicator used to measure the level of profit or loss from an investment. According to Handini et al., (2019). Return reflects the gain received by investors over a certain period and serves as an essential basis for investment decision-making, as it is closely related to risk and potential profit. This research utilizes two stocks listed in the LQ45 index, namely ADRO and ITMG, selected based on the following criteria: (1) listed in the LQ45 index, (2) categorized under the coal mining sector, and (3) possessing complete daily closing price data during the observation period. Stocks from other sectors or those lacking complete data were excluded from the analysis. The types of return used in this research include actual return and expected return. Actual return is calculated from historical stock price data, while expected return reflects the potential profit in the future. The actual value of stock return for each period is as follows (Miskolczi, 2017):

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

where $R_t$ is the return value in the period $t$, $S_t$ is the stock closing price in the period $t$, dan $S_{t-1}$ is the stock closing price in the period $t-1$. Then the value of expected return (average return) is as follows (Syuhada et al., 2024):

$$\bar{R} = \frac{1}{n}\sum_{t=1}^{n} R_t \tag{2}$$

where $\bar{R}$ is the expected return, $R_t$ is the stock return in the period $t$, dan $n$ is the amount of data.

## 2. Portfolio

An optimal portfolio is a portfolio in which investors choose from a variety of efficient portfolio alternatives that are tailored to the investor's ability to bear risk and expectations for return. This optimal portfolio is designed to maximize the expected profits while minimizing unwanted risks (Soeryana et al., 2017). The return of a portfolio is determined by the returns of the individual assets that make up the portfolio, taking into account the investment weight in each asset. Equations for calculating return the portfolio is as follows (Manurung et al., 2024):

$$Rp_t = \sum_{i=1}^{k} R_{i,t}\, w_i \tag{3}$$

where $Rp_t$ is the portfolio of return in the period $t$, $R_{i,t}$ is the return of stock-$i$ in the period $t$, $w_i$ is the weight of stock-$i$, and $k$ is the amount of stock.

## 3. Autocorrelation and Heteroscedasticity Test

Autocorrelation is a condition in which the values in a time series are interconnected with previous values in the same series. The purpose of this test is to see whether a correlation exists between the residual at time $t$ and the residual at time $t-1$ within a model. So in this research, the Ljung-Box Test will be used to see if there is an autocorrelation or not in the return stock closing price (Rönkkö et al., 2024). The Ljung-Box test is used to determine whether the residuals in a time series model are independent across different lags, based on the test statistic with the following hypotheses (Hiremath & Kumari, 2014):

Hypotheses:
$H_0 : \rho_1 = \rho_2 = \cdots = \rho_p = 0$ (no autocorrelation)
$H_1$ : there is at least one value $\rho_i \neq 0, i = 1,2, \ldots, p$ (there is an autocorrelation)

Test statistics:

$$Q = n(n+2)\sum_{l=1}^{p} \frac{\hat{\rho}_l^2}{n-l} \tag{4}$$

where $n$ is the amount of data, $p$ is the amount of lag, and $\hat{\rho}_l^2$ is the autocorrelation sample return on the lag-$l$.

Test criteria:
$H_0$ is rejected if $Q_m > X^2_{(\alpha,df)}$ or p-value $< \alpha$, meaning there is an autocorrelation. The Lagrange Multiplier (LM) test is a technique employed to identify the existence of heteroscedasticity (Hong, 1997). This test aims to identify the presence of an ARCH effect by using constants and squared residuals in a lag-$p$ residual regression. This forms the following model (Virginia et al., 2018):

$$a_t^2 = \beta_0 + \beta_1 a_{t-1}^2 + \beta_2 a_{t-2}^2 + \cdots + \beta_p a_{t-p}^2 + e_t, t = p + 1, \ldots, n \tag{5}$$

Hypothesis:
$H_0$: $\beta_1 = \beta_2 = \cdots = \beta_p = 0$ (no ARCH effect in residual up to the lag-$p$)
$H_1$: there is one value $\beta_l \neq 0, l = 1,2, \dots, p$ (there is an ARCH effect in the residual up to the lag-$p$)
Test Statistics:

$$LM = nR^2 \tag{6}$$

where $n$ is the amount of data and $R^2$ is the coefficient determination.
Test criteria:
$H_0$ is rejected if $LM > X^2_{(\alpha,m)}$ or p-value $< \alpha$, meaning there is an ARCH effect in the residual up to the lag-$p$

## 4. Monte Carlo Portfolio Simulation

Simulation Monte Carlo can be used to calculate VaR, both on single stock and portfolios. This method was first introduced by Boyle in 1977 for risk measurement. The basic principle of Monte Carlo is to simulate by generating random variables based on existing data, which are then used to estimate the value of VaR (Pasieczna, 2019). The simulation steps in Monte Carlo with Gumbel copula are as follows (Zuhra et al., 2014):

a. The Monte Carlo simulation was used to generate as much as $n$ data with $m$ times of repetition using the Gumbel copula parameter. The simulation is generated from the Gumbel copula which has been defined on a uniform distribution [0,1]; that is, $u_1$and $u_2$
b. The value of the bivariate return generation for $(R_{1,t}, R_{2,t})$, $t$ = 1, 2, ..., $k$ with $R_{1,t}$ = $F^{-1}$ $(u_1)$ and $R_{2,t}$ = $G^{-1}$ $(u_2)$. $F^{-1}$and $G^{-1}$consecutively expresses the inverse of the marginal distribution function.
c. Furthermore, calculate the portfolio return of the two stocks by the portfolio return $Rp_t = R_{1,t}w_1 + R_{2,t}w_2$, with $Rp_t$ is the portfolio returns in the period $t$, $R_{i,t}$ is the return of stock-$i$ in the period $t$, and $w_i$ is the weight of the stock-$i$.

## 5. Copula

The word "Copula" comes from Latin, meaning "relationship, bond, or link" (according to Cassell's Latin Dictionary). Copula was first used by Abe Sklar in 1959 in Sklar's Theorem. This theorem asserts that any multivariate joint distribution could be represented as a combination of one-dimensional marginal distributions linked by a copula (Nelsen, 2006). Durante and Sempi (2015) state that Copula can also be used to separate multivariate combined distributions into marginal distributions and functions that capture dependencies between variables. The Archimedean copula is often used to analyze bivariate cases, especially in modeling the dependence between two variables. The Function of an Archimedean copula in the case of bivariate can be written as follows (Nelsen, 2006):

$$C(u_1, u_2) = \emptyset^{-1}(\emptyset(u_1) + \emptyset(u_2)) \tag{7}$$

where $C(u_1, u_2)$ is the distribution function of the bivariate copula, $\emptyset(u)$ is the function of the copula generator, and $\emptyset^{-1}(u)$ is the inverse of the function of the copula generator.

The Gumbel copula is an Archimedean copula designed to model the dependency between two or more random variables, especially when the dependency is stronger in the tails. Generally, the generator function of the Gumbel copula is (Nurrahmat et al., 2017):

$$\emptyset(u) = (-\ln(u))^{\theta} \tag{8}$$

Suppose that $\emptyset(u) = y$, where the inverse of the generator function is $\emptyset^{-1}(y) = u$. Thus, Equation (8) can be written as $y = (-\ln(u))^{\theta}$. Next, the inverse of the Gumbel copula generator function is calculated.

$$y^{\frac{1}{\theta}} = (-\ln(u))$$

$$y^{\frac{1}{\theta}} = \ln\left(\frac{1}{u}\right)$$

$$e^{y^{\frac{1}{\theta}}} = e^{\ln\left(\frac{1}{u}\right)} = \frac{1}{u}$$

$$u = \frac{1}{e^{y^{\frac{1}{\theta}}}} = e^{-y^{\frac{1}{\theta}}}$$

$$\emptyset^{-1}(y) = e^{-y^{\frac{1}{\theta}}} = u$$

Then, substitute the generator functions of Gumbel copula $\emptyset(u_1)$ and $\emptyset(u_2)$ into Equation (7) to obtain the cumulative distribution function of Gumbel copula.

$$\begin{aligned} C(u_1, u_2) &= \emptyset^{-1}(\emptyset(u_1) + \emptyset(u_2)) \\ &= e^{\left(-((-\ln(u_1))^{\theta} + (-\ln(u_2))^{\theta})^{\frac{1}{\theta}}\right)} \end{aligned}$$

Thus, an equation is obtained from the distribution of the Gumbel copula as follows (Nurrahmat et al., 2017):

$$C^{Gu}(u_1, u_2; \theta) = exp\left(-\left[(-\ln(u_1))^{\theta} + (-\ln(u_2))^{\theta}\right]^{\frac{1}{\theta}}\right) \tag{9}$$

where $C^{Gu}(u_1, u_2)$ is the Gumbel copula distribution function and $\theta$ is the parameter of the Gumbel copula.

## 6. Value at Risk (VaR)

VaR is a method employed to evaluate the risk of a portfolio consisting of different assets, such as stocks and options. It measures the maximum potential loss within a given time period at a defined confidence level. As a quantitative measurement tool, VaR has become the standard in assessing how much risk a portfolio may face due to market fluctuations (Sulistianingsih et al., 2019). VaR at a $(1-\alpha)$ confidence level is defined as the fourth quantile within the return distribution. VaR can be calculated using the opportunity density function of return f (R), which describes the potential future yield of the asset, where $R$ is an individual asset or a portfolio. VaR in period $t$ with confidence level $(1-\alpha)$ is as follows (Handini et al., 2019):

$$VaR_{(1-\alpha)} = V_0 R^* \sqrt{t} \tag{10}$$

where $R^*$ is the quantification of the return distribution which is a critical value (*cut off value*) with a predetermined probability, $V_0$ is the initial investment, and $t$ is the period.

**7. Tail Value at Risk (TVaR)**

TVaR is a risk indicator that estimates the expected loss beyond a specified threshold of severity, offering a more comprehensive view of the risk associated with extreme losses (Molino & Sala, 2020). By focusing on the tail of the loss distribution, TVaR not only considers losses at a certain level of confidence, such as VaR, but also provides a summary of losses that exceeds that threshold. TVaR is the average loss that exceeds the quantity $100p\%$ with $p$ is the confidence level, where $0 < p < 1$. If $X$ is a random variable representing a loss, then the TVaR at the confidence level $(1-\alpha)$ is as follows (Klugman et al., 2019):

$$TVaR_{(1-\alpha)} = E[X | X \leq VaR_{(1-\alpha)}] \tag{11}$$

**8. Correlation of Kendall's Tau and Gumbel Copula Parameters**

Before estimating the Gumbel copula parameters, the first step that must be conducted is the mutual dependency test. This test aims to identify the dependence between the variables that will be used in the shared distribution modeling. The results of this test will be the basis for the estimation process of the Gumbel copula parameters. Kendall's Tau can be used to look for correlations from random samples of bivariate $X$ and $Y$. Kendall's Tau with $n$ samples is are as follows (Nelsen, 2006):

$$\tau = \frac{k-d}{\binom{n}{2}} = \frac{2(k-d)}{n(n-1)} \tag{12}$$

where $\tau$ is the correlation value of Kendall's Tau, $k$ is the number of concordant pairs, and $d$ is the number of discordant pairs. The Kendall's Tau correlation value is then tested for significance to assess whether the correlation between the variables is statistically correlated. The hypothesis for the Kendall’s Tau is as follows:

Hypothesis:

$H_0: \tau = 0$ ($X$ and $Y$ are not correlated)

$H_1: \tau \neq 0$ ($X$ and $Y$ are correlated)

Test statistics:

$$Z_{hit} = \sqrt{\frac{9n(n-1)}{2(2n+5)}} |\tau| \tag{13}$$

Test criteria:

$H_0$ is rejected if $Z_{hit} > Z_{\frac{\alpha}{2}}$ or p-value $< \alpha$

Then, the correlation value Kendall's Tau to Archimedean Copula can be written as follows (Najjari, 2018) :

$$\tau = 1 + 4\int_0^1 \frac{\emptyset(u)}{\emptyset'(u)}\,du \qquad (14)$$

where $\emptyset(u)$ is the function of the generator Gumbel copula and $\emptyset'(u)$ is the first derivative of the generator function Gumbel copula. Next, estimate the parameters Gumbel copula by the size of the dependency Kendall's Tau can be written as follows(Najiha et al., 2023):

$$\theta = \frac{1}{1-\tau}\,, 1 \le \theta \le \infty \qquad (15)$$

where $\theta$ is the Gumbel copula parameter and $\tau$ is the Kendall'*s* Tau correlation value.

### 9. Mean-Variance Efficient Portfolio (MVEP)

An optimal portfolio is the best portfolio choice that an investor takes from the various options in an efficient portfolio. One of the most widely used methods for constructing an optimal portfolio is the MVEP. This method aims to form a portfolio with the smallest variance among all possible portfolios that can be formed, resulting in a combination of assets with minimal risk without sacrificing the expected rate of return. Weighting MVEP of the portfolio can be written as follows (Daulay et al., 2022):

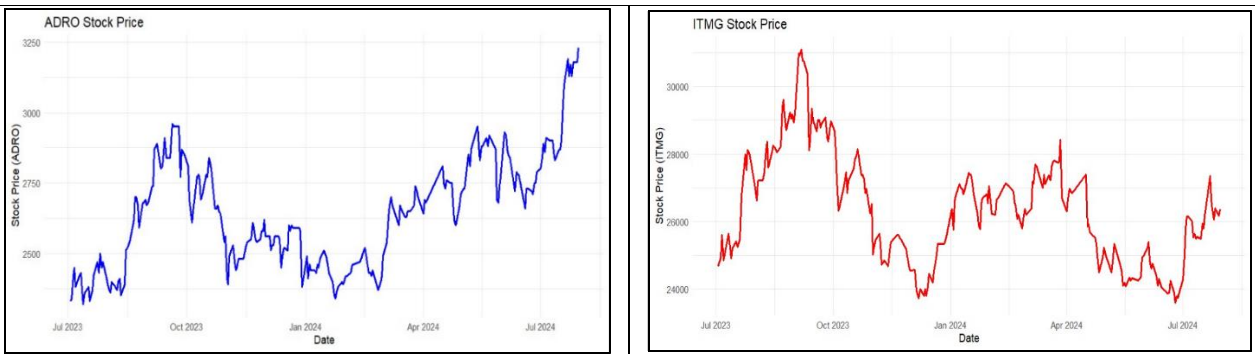
$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{1}_P}{\boldsymbol{1}_P{}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{1}_P} \qquad (16)$$

where $\boldsymbol{w}$ is the weight of the stock, $\boldsymbol{\Sigma}^{-1}$ is the inverse of the variance-covariance matrix, and $\boldsymbol{1}_P$ is a vector of units with dimensions $P \times 1$.

## C. RESULT AND DISCUSSION

### 1. Data Characteristics

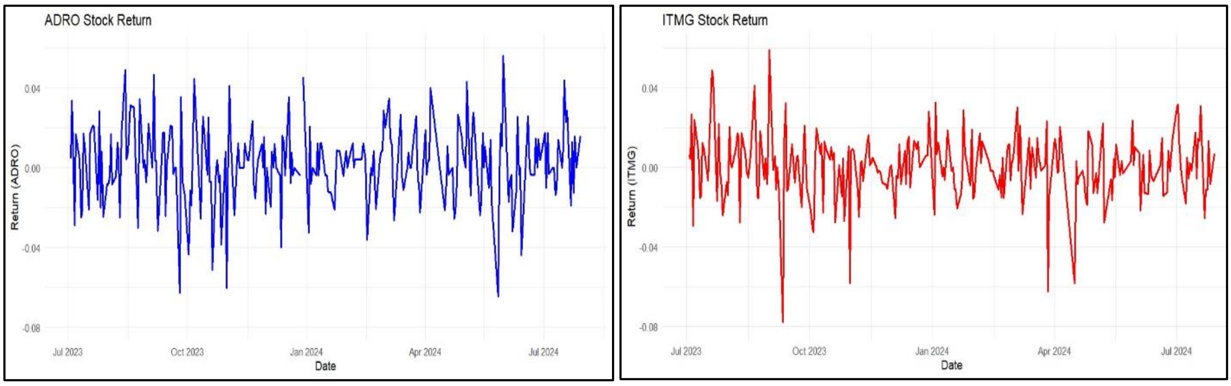
This research uses secondary data consisting of the daily closing prices of ADRO and ITMG stocks over the period from July 3, 2023, to July 30, 2024, covering 257 trading days. These data were obtained from Yahoo Finance. Figure 1 illustrates the movement of closing prices for both stocks.




(a) (b)

**Figure 1.** Closing Price Plot of ADRO and ITMG Stocks

In Figure 1, plot (a) shows that the price of ADRO shares tends to rise, albeit with some fluctuations. Meanwhile, plot (b) reveals that ITMG shares experienced a sharp decline at the end of 2023, followed by continued fluctuations until mid-2024, then showing a more stable trend compared to ADRO. The return of each stock is calculated using the logarithmic return formula, as shown in Equation (1), and the return plots are displayed in Figure 2.




a b

**Figure 2.** Plot ADRO and ITMG Stock Returns

## 2. Autocorrelation and Heteroscedasticity Test

To determine whether there is autocorrelation in the stock returns, this study applies the Ljung-Box test. This test was chosen because it is one of the most commonly used methods in time series analysis to detect autocorrelation at multiple lags. This is particularly important in the context of financial data, as the presence of autocorrelation can compromise the validity of the models used. As shown in Table 1, the p-values for both ADRO and ITMG at various lags are all greater than 0.05. Therefore, the null hypothesis of no autocorrelation is accepted. In this research, autocorrelation testing was conducted using the Ljung-Box test to obtain more precise results regarding the presence or absence of autocorrelation in stock returns. The results of the Ljung-Box test are presented in Table 1.

**Table 1.** Test of Ljung-Box of ADRO and ITMG Return Stocks

| Lag | p-value | |
|---|---|---|
| | ADRO | ITMG |
| 1 | 0.4910 | 0.8486 |
| 6 | 0.4561 | 0.0868 |
| 10 | 0.1439 | 0.1736 |
| 15 | 0.2558 | 0.1511 |
| 21 | 0.4070 | 0.0971 |

Table 1 indicates that the p-value of the return for ADRO and ITMG stocks are greater than $\alpha$ = 0.05, indicating that $H_0$ is accepted This suggests that there is no autocorrelation in the returns of ADRO and ITMG stocks. Next, the ARCH LM test is conducted to check for heteroscedasticity. The reason for using this test is its effectiveness in detecting changes in volatility over time, which the results of the test are presented in Table 2.

**Table 2.** ARCH LM Return Test of ADRO and ITMG Stocks

| Stock | p-value |
|---|---|
| ADRO | 0.7264 |
| ITMG | 0.3314 |

Based on the results in Table 2, both stocks have p-values above the 5% significance level. Thus, there is no significant evidence of heteroscedasticity in the return series of ADRO and ITMG. These results justify the use of standard time series models without the need for GARCH modeling.

### 3. Parameters of Gumbel Copula Using Kendall's Tau

The dependence between the returns of ADRO and ITMG stocks, Kendall's Tau is used due to its nonparametric nature and independence from the assumption of normal distribution. The correlation value of Kendall's Tau can be calculated by Equation (12). Using RStudio, at a significant level of α 5%, a correlation value τ of 0.4379 was obtained, and p-value $< 2{,}2 \times 10\text{^}(-16)$ .This indicates that $H_0$ was rejected, which means that the returns of ADRO and ITMG stocks significantly affect each other. The obtained correlation value is used to determine the parameter of the Gumbel copula (θ) by Equation (15) as follows.

$$\theta = \frac{1}{1-\tau} = \frac{1}{1-0.4379} = 1.7791$$

The Gumbel copula parameter value obtained is 1.7791 is utilized to generate new return data using Monte Carlo simulation.

### 4. Simulation Using Gumbel Copula Parameters

After obtaining the Gumbel copula parameter, the next step is to perform a Monte Carlo simulation to generate new return data that represents the potential future movements of stock returns. Next, form the Gumbel copula model using Equation (9) based on the $\theta$ parameter values that have been obtained previously:

$$C^{Gu}(u_1, u_2; \theta) = exp\left(-\left[-(ln(u_1\,))^{\theta} + (-\,ln(u_2)^{\theta}\,\right]^{\frac{1}{\theta}}\right)$$

$$C^{Gu}(u_1, u_2; 1.7791) = exp\left(-[-(ln(u_1\,))^{1.7791} + (-\,ln(u_2)^{1{,}7791}\,]^{\frac{1}{1.7791}}\right)$$

$$C^{Gu}(u_1, u_2; 1.7791) = exp(-[-(ln(u_1\,))^{1.7791} + (-\,ln(u_2)^{1{,}7791}\,]^{0.5621})$$

From the model, random variables were generated $u_1$ and $u_2$ through Monte Carlo simulations with the Gumbel copula parameter, where the variables were uniformly distributed [0.1]. This simulation produces 256 new return data pairs with $u_1$ and $u_2$ successively representing ADRO and ITMG returns which are then repeated as many as $m$ = 1000 times using RStudio software. This simulation is important because the joint distribution of ADRO and ITMG returns formed by the copula is complex and does not have an explicit functional form that is easy to analyze mathematically. The results of the first iteration of the simulation data are presented in Figure 3.

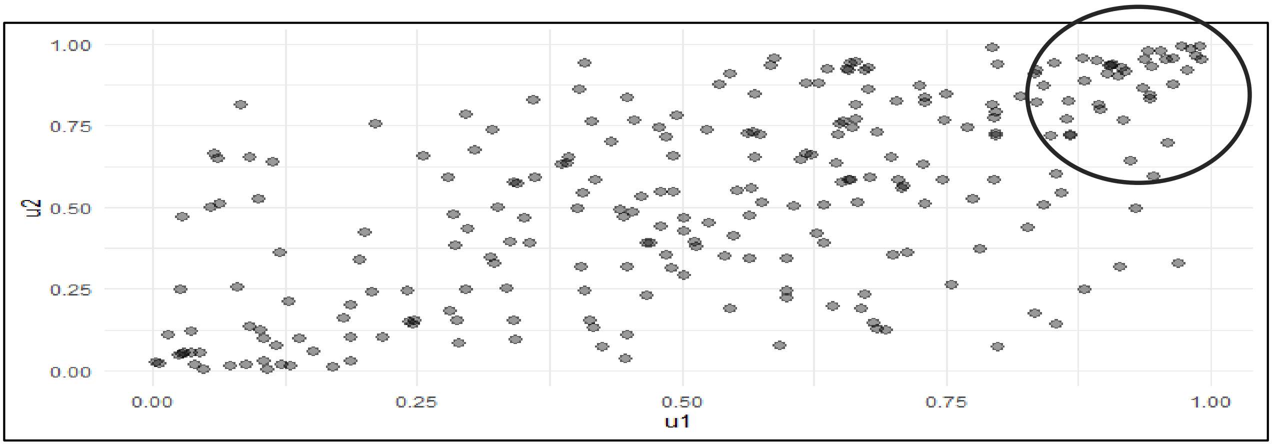



**Figure 3.** Scatter-Plot Generates New Return Data

Asymmetric correlation in this context refers to a stronger relationship between two variables in a specific part of the data distribution. As shown in Figure 3, the simulated data exhibit a stronger correlation in the upper tail, suggesting that during bullish market conditions, the returns of ADRO and ITMG tend to rise simultaneously. This pattern is important for risk modeling, as it indicates increased exposure during periods of positive market movements.

## 5. Portfolio Stock Weights with MVEP

The creation of a bivariate portfolio is achieved by combining the ADRO and ITMG stocks, with the weights determined using the MVEP method through Equation (16) as follows:

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \frac{\begin{bmatrix} 3583.5859 & -2284.7106 \\ -2284.7106 & 5095.2904 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 3583.5859 & -2284.7106 \\ -2284.7106 & 5095.2904 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \frac{\begin{bmatrix} 1298.875 \\ 2810.580 \end{bmatrix}}{[4109.455]} = \begin{bmatrix} 0.3161 \\ 0.6839 \end{bmatrix}$$

From this calculation, the weight of each stock is obtained, namely the weight of ADRO stock ($w_1$) of 0.3161 or 31.61% proportion to the portfolio and the weight of ITMG stock ($w_2$) of 0.6839 or 68.39% proportion to the portfolio. The greater allocation to ITMG indicates that this stock is considered to have a more stable return or a better risk-return profile compared to ADRO within the constructed portfolio. Therefore, for investors, this composition suggests that ITMG may be prioritized as a more dominant investment choice in terms of portfolio contribution and potential performance

## 6. Portfolio VaR and TVaR Calculation

The values of $u_1$ and $u_2$ obtained through the Monte Carlo simulation with the Gumbel copula parameter, then adjusted to the historical characteristics of the two stock returns. The simulated return values are subsequently utilized to compute the portfolio return based on Equation (2), with a weighting of 0.3161 for ADRO stock ($w_1$) and 0.6839 for ITMG stock ($w_2$).

After simulating the returns and computing portfolio returns using the estimated weights, VaR and TVaR are calculated at confidence levels of 80%, 90%, 95%, and 99%. These risk measures are estimated using Equations (10) and (11). The results are shown in Table 3.

**Table 3.** Estimation of VaR and TVaR of ADRO and ITMG Stock Portfolio

| | Confidence Level | | | |
|---|---|---|---|---|
| | 80% | 90% | 95% | 99% |
| Var | -1.22% | -1.88% | -2.43% | -3.49% |
| TVaR | -2.08% | -2.64% | -3.14% | -4.11% |

Based on Table 3, the negative values of VaR and TVaR indicate the maximum potential loss that an investor may face on the following day. For example, at a 90% confidence level, a VaR of 1.88% implies that losses are not expected to exceed this percentage in 90% of cases, while the average loss in the worst 10% of cases is estimated at 2.64% (TVaR). For an investment of IDR 100,000,000, the estimated losses exceeding VaR are IDR 2,080,000 (80%), IDR 2,640,000 (90%), IDR 3,140,000 (95%), and IDR 4,110,000 (99%).

These findings are consistent with Ardhitha et al. (2023), who found that TVaR values are consistently higher than VaR at the same confidence levels. The results also support Jorion (2007), who stated that TVaR provides a more conservative risk estimate, particularly under stressed market conditions. The use of the Gumbel copula in this study further supports this, as it captures upper tail dependence more effectively than the Gaussian copula, leading to more realistic estimates of extreme joint losses.

## D. CONCLUSION AND SUGGESTIONS

The estimation of Tail Value at Risk (TVaR) for the stock portfolio consisting of ADRO and ITMG using the Gumbel copula approach indicates potential daily losses ranging from 2.08% to 4.11% at confidence levels of 80% to 99%. The higher the confidence level, the greater the potential extreme loss that needs to be considered. These results are consistent with modern risk management approaches that emphasize the importance of measuring risk in the tail of the distribution, particularly under abnormal market conditions. Practically, these findings imply that long-term investors with low risk preferences may consider ITMG, which shows a more stable price trend, while ADRO may be more suitable for short-term investors with a higher tolerance for extreme fluctuations. Furthermore, comparing Archimedean and Elliptical copula families can be a strategic direction for future research, as each type of copula captures different dependence structures whether symmetric or asymmetric. Therefore, selecting the appropriate copula can improve the accuracy of portfolio risk modeling and provide a stronger foundation for investment decision-making based on comprehensive risk measurement.

## ACKNOWLEDGEMENT

The authors express their gratitude to the reviewers for their insightful feedback and recommendations, which have greatly contributed to enhancing the quality of this article.

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