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Portfolio Optimization for Rupiah Exchange Rate using Multidimensional Geometric Brownian Motion Model

Siti Masitah[1*], Retno Budiarti[1], I Gusti Putu Purnaba[1]
[1]Department of Mathematics, IPB University, Indonesia
sitimasitah@apps.ipb.ac.id

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ABSTRACT

Exchange rate fluctuations are critical in ensuring economic stability and shaping foreign investment, while foreign currencies serve as asset and wealth diversification instruments. This study aims to predict foreign exchange rates with a multidimensional geometric Brownian motion model and determine the optimal portfolio fund allocation with the Markowitz model using the Moore-Pendrose method. The multidimensional GBM model was employed for its ability to capture the volatility and interdependence among multiple currencies, making it more suitable for multi-asset portfolios than univariate models. The Markowitz model was used to determine the optimal asset allocation that achieves a specified expected return with minimal risk, while the Moore-Penrose method was applied to address matrix inversion challenges in high-dimensional data. Using data from 2023 to April 2024 on the Indonesian rupiah against the Singapore Dollar (SGD), Chinese Yuan (CNY), and Euro (EUR), this study finds that the multidimensional GBM model effectively forecasts exchange rate movements, as indicated by MAPE values below 10% for each currency. "The optimal portfolio yields a risk of 0.28% and an expected return of 0.009%, with asset allocations of 90.3% in SGD, 8.2% in CNY, and 1.5% in EUR. The dominance of SGD in the optimal portfolio suggests it was the most favorable investment option against the rupiah during the study period. This reflects Singapore's strong economic fundamentals and strategic position as a global financial hub. These findings provide valuable insights for investors and financial analysts seeking to manage currency risk and enhance returns through data-driven diversification strategies.

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A. INTRODUCTION

The exchange rate represents the price of one currency relative to another and serves as a key economic indicator, with its fluctuations significantly impacting various aspects of a country's economy (Eris et al., 2017). Exchange rate instability may arise from inflation, interest rates, money supply, and government intervention in the foreign exchange market (Carissa & Khoirudin, 2020). Such instabilities can adversely affect the value of investment portfolios, especially those involving foreign currencies. Exchange rate fluctuations can also disrupt trade balances and deter foreign investment due to heightened uncertainty. Consequently, managing currency risk becomes essential for both investors and policymakers.

Foreign currencies have become increasingly regarded as diversification and risk mitigation investment instruments. By investing in various foreign currencies, investors may achieve higher returns while managing risks more effectively (Li et al., 2019; Liu, 2011).

However, currency investments also entail risks of volatility and depreciation. To mitigate these risks, investors commonly construct portfolios to balance return and risk. Portfolio optimization is a critical issue in financial risk management, involving the strategic allocation of assets to achieve optimal performance within given constraints (Aksaraylı & Pala, 2018; Deng et al*.,* 2012). In this regard, Markowitz's (1952) foundational theory remains central to contemporary portfolio selection methods.

The Markowitz model employs a mean-variance (MV) framework, where the expected return is maximized for a given level of risk or vice versa (Kalayci et al., 2019). The model assumes that historical price data reflect future movements and that asset returns are correlated. The model quantifies returns and uses variance to measure risk (Huang & Yang, 2020; Li & Zhang, 2021). Investors can apply this method to identify efficient portfolios by minimizing risk within specified constraints (Grechuk & Zabarankin, 2014; Lv et al., 2016; Ramos et al., 2023). This framework remains a cornerstone of modern portfolio theory and is extensively cited in financial optimization research.

Accurate modeling of exchange rates is critical for supporting portfolio formation. One widely used approach is the Geometric Brownian Motion (GBM) model, which simulates asset price movements over time using stochastic differential equations. GBM has been applied in various contexts, including stock price prediction (Reddy & Clinton, 2016; Agustini et al., 2018; Suganthi & Jayalalitha, 2019; Brătian et al., 2021), palm oil price forecasting (Ibrahim et al., 2021), iron ore price modeling (Ramos et al., 2019), and gold derivatives pricing (Germansah et al., 2023). These applications highlight the versatility of GBM in financial modeling.

However, many previous studies focus on one-dimensional capital market analyses, modeling only a single security or asset. This approach overlooks the dynamic interdependencies among multiple assets commonly present in real-world portfolios. Therefore, this study employs a multidimensional version of the GBM model better to capture the joint behavior of multiple exchange rates. The multidimensional GBM extends GBM by incorporating correlations among several currencies, offering a more realistic framework for portfolio optimization. By accounting for interrelated asset movements, the model enables a more accurate estimation of risk and return in diversified currency portfolios.

The optimal portfolio in this study is constructed using the Markowitz model, while the Moore-Penrose pseudoinverse is applied to address covariance matrix singularity, which is common in high-dimensional financial data. Given its significant impact on Indonesia's economic growth (Silaban et al., 2023) the rupiah exchange rate is crucial in optimizing portfolios and managing currency risk. Accurate modeling enables investors to make more informed decisions. Thus, the integration of these methods is anticipated to offer practical benefits in financial planning and risk management.

This study aims to predict future exchange rates using a Multidimensional GBM model and determine optimal fund allocation across currencies using the Markowitz model with the Moore-Penrose method. The exchange rates used are based on Indonesia's foreign debt composition, focusing on three major creditor countries: Singapore, China, and France. The analysis aims to determine the real return and risk level for investors investing in foreign currencies from these key creditor nations. This study is expected to contribute to academic literature and practical investment strategies by providing an effective approach to managing exchange rate risk.

B. METHODS

This study employs a quantitative research design, utilizing both descriptive analysis and simulation-based methods to examine and forecast exchange rate movements. The data utilized in this study consist of the Indonesian rupiah (IDR) exchange rates against the Singapore Dollar (SGD), Chinese Yuan (CNY), and Euro (EUR) from January 2023 to April 2024. These data were obtained from the daily exchange rate statistical bulletin published by Bank Indonesia (BI). By applying mathematical modeling, specifically the Multidimensional Geometric Brownian Motion (GBM) model and the Markowitz portfolio optimization framework using the Moore-Penrose method, this study aims to simulate future exchange rate trends and identify optimal portfolio allocations. The methodology integrates statistical analysis, model simulation, and optimization to generate results that are both theoretically robust and practically relevant for financial decision-making. This study discusses the prediction of future exchange rates using a multidimensional GBM model and the allocation of funds to obtain an optimal portfolio using the Markowitz model with the Moore-Penrose method. The analysis procedure is conducted through the following stages:

1. Determine the rupiah exchange rate data against the Singapore dollar, the Chinese yuan, and the French Euro.
2. Determine the exchange rate return of each currency.
3. Conduct normality tests of return data for each currency exchange rate.
4. Select the rupiah exchange rate against foreign currencies that follow a normal distribution.
5. Calculate the expected return, volatility, and covariance of each rupiah exchange rate against foreign currencies that follow a normal distribution.
6. Simulate the rupiah exchange rate against foreign currencies using a multidimensional GBM model.
7. Calculate the Mean Absolute Percentage Error (MAPE) to evaluate the prediction accuracy of the multidimensional GBM model.
8. Calculate the optimal portfolio with the smallest risk using the Markowitz model for the rupiah exchange rate against foreign currencies. The steps are:
   a. Calculate the exchange rate correlation.
   b. Compute the variance-covariance matrix using the Pseudo-Inverse method.
   c. Construct a portfolio using the Pseudo-Inverse methods.

9. Construct an optimal portfolio with a Mean-Variance Efficient Frontier plot using the Pseudo-Inverse methods.
10. Interpret the results of the Pseudo-Inverse methods regarding the optimal portfolio outcomes.

In the mathematical modeling of multidimensional exchange rate movements, each exchange rate is assumed to follow a stochastic differential equation in the form of a Geometric Brownian Motion (GBM) model. Thus, the multidimensional exchange rate movement can be expressed as a stochastic differential equation. In general, the multidimensional stochastic differential equation for exchange rates is formulated as follows (Kloeden & Platen, 1992):

$$dS_t^i = S_t^i\left(\mu_i dt + \sum_{j=1}^{n} \sigma_{i,j}\, dW_t\right) \tag{1}$$

for $i = 1,2,..,n$ dimensions. Where μ is the expected return, $\sigma_{i,j}$ is the covariance of the-i asset and the-j asset, and W_t is the Wiener process or Brownian motion. The exact solution of multidimensional stochastic differential equation models is obtained by applying Ito's Lemma.
Ito's Lemma:
Suppose given a function G of X_t and t or written $G(X_t, t)$, which is a continuous and differentiable function. X_t is an Ito process defined as follows:

$$dX_t = a(X_t, t)dt + b(X_t, t)dW_t \tag{2}$$

where W_t is a standard Brownian motion process, and $G(X_t, t)$ has the following stochastic differential equation form:

$$dG(X_t, t) = \left(\frac{\partial G}{\partial t} + a(X_t, t)\frac{\partial G}{\partial X_t} + \frac{1}{2} b(X_t, t)^2 \frac{\partial^2 G}{\partial X_t^2}\right) dt + b(X_t, t)\frac{\partial G}{\partial X_t} dW_t \tag{3}$$

Based on the application of the Lemma Ito, the following solution will be obtained:

$$S_t^i = S_{t-1}^i \exp\left[\left(\mu_i - \frac{1}{2}\sum_{j=1}^{n}\left(\sigma_{i,j}\right)^2\right) dt + \sum_{j=1}^{n} \sigma_{i,j} W_t\right] \tag{4}$$

The solution of equation (4) is then called the multidimensional geometric Brownian motion model. This model represents that the exchange rate in the future period is always positive, with returns that are normally distributed.

C. RESULT AND DISCUSSION

1. Three Exchange Rate Movement Model

Suppose the exchange rates of IDR-SGD, IDR-CNY, and IDR-EUR are respectively expressed as $\mathrm{S}_t^1, \mathrm{S}_t^2$, and S_t^3. The expected return of the exchange rate per unit of time for each exchange rate above is symbolized *by* μ_1, μ_2, and μ_3, while the volatility of the exchange rate, represented as the standard deviation of the exchange rate, is respectively symbolized by σ_1, σ_2, and σ_3. The movement model of the three exchange rates formed is as follows:

$$dS_t^i = S_t^i\left(\mu_i dt + \sum_{j=1}^{3} \sigma_{i,j}\, dW_t\right) \tag{5}$$

for $i = 1,2,3$.

By applying Ito's Lemma, the following solution is obtained:

$$S_t^i = S_{t-1}^i \exp\left[\left(\mu_i - \frac{1}{2}\sum_{j=1}^{3}\left(\sigma_{i,j}\right)^2\right)dt + \sum_{j=1}^{3}\sigma_{i,j}W_t\right] \quad (6)$$

for $i = 1,2,3$ and $W_t \sim N(0, \sigma^2 t)$. This solution is called the three-dimensional geometric Brownian motion model and represents that future exchange rates are always positive. The three-dimensional PDS model in equation (6), which includes the expected exchange rate return, exchange rate variance (or standard deviation), and the correlation between exchange rates, provides the explicit solution derived using Ito's Lemma. This demonstrates that the exchange rate remains positive with normally distributed returns over time.

2. Multidimensional Geometric Brownian Motion Model Assumption Test

The rupiah exchange rate is assumed to follow a three-dimensional Geometric Brownian Motion (GBM) model, with exchange rate returns following a Brownian motion. To test this assumption, a normality test is conducted on each exchange rate return. The normality test used is the Kolmogorov-Smirnov test. The hypotheses in this test were:

H_0= rupiah exchange rate returns are normally distributed.

H_1 = rupiah exchange rate returns are not normally distributed

By using SPSS software, the following results were obtained:

Table 1. The result of the Normality Test

No	Exchange Rate	P-Value
1	SGD	0,66
2	CNY	0,200
3	EUR	0,200

Because all significance values ($p - value$) are greater than the significance level ($\alpha = 5\%$) for all currencies, H_0 is accepted, indicating that the exchange rate returns are normally distributed and meet the assumptions required for using the GBM model.

3. Simulation of Rupiah Exchange Rate with Multidimensional Geometric Brownian Motion Model

The three-dimensional geometric Brownian motion model obtained is an estimate of the movement of the rupiah exchange rate in a multidimensional exchange rate system. The model is used to formulate an estimate of the rupiah exchange rate in the future period through a simulation of the exchange rate movement. The results of the rupiah exchange rate simulation with the multidimensional geometric Brownian motion model are shown in Figure 1.

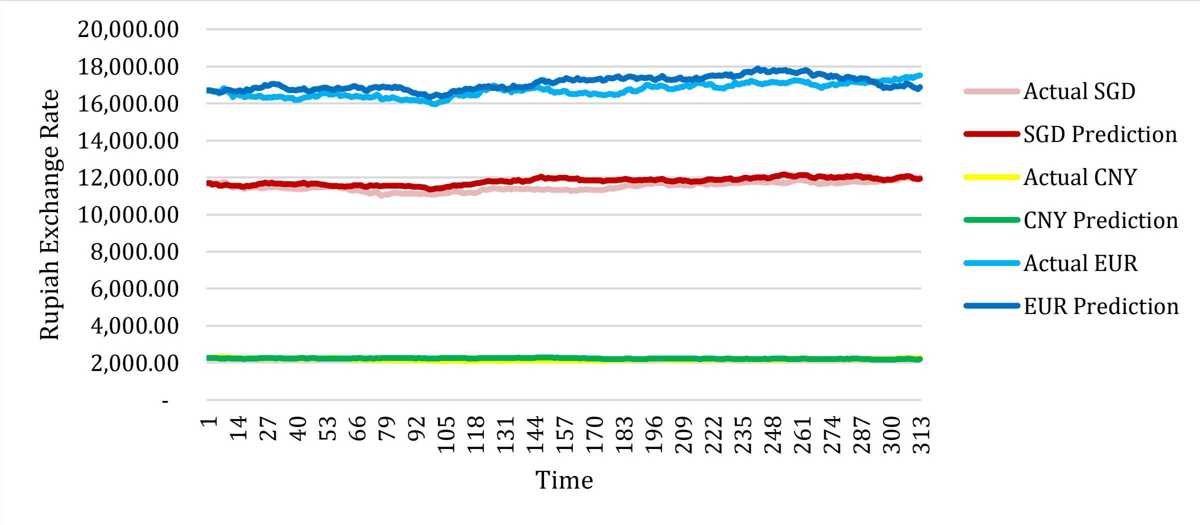

Figure 1. Simulation of rupiah exchange rate with multidimensional geometric Brownian motion model

The estimated movement of the rupiah exchange rate against SGD, CNY, and EUR, which is assumed to follow a three-dimensional geometric Brownian motion model, can be seen in Figure 1. The simulation results plot with the three-dimensional GBM model also shows the estimated movement of the three exchange rates with an observation period of 313 days. From the graph results in Figure 1, it is clear that the model simulation results are very close to the actual data. Each line almost overlaps, indicating that the model is highly representative of the actual data. To quantitatively ensure the model's accuracy, the Mean Absolute Percentage Error (MAPE) is calculated as a measure of prediction error.

Table 2. MAPE value for each exchange rate

No	Nilai Tukar	MAPE
1	SGD	2,57%
2	CNY	3,08%
3	EUR	2,64%

Table 2 presents the MAPE (Mean Absolute Percentage Error) values of the simulation results for each exchange rate, all of which are below 10%, indicating that the multidimensional Geometric Brownian Motion model exhibits a high level of accuracy in predicting actual movements.

4. Optimal Portfolio Calculation

There are three parameters involved in modelling the movement of the three exchange rates that follow the multidimensional geometric Brownian motion model, namely the expected returns, volatility, and exchange rate correlation coefficients. Based on this, the Markowitz model is used in the analysis of the formation of an optimal portfolio because it uses basic statistical measurements to form an optimal portfolio. In determining the optimal portfolio of the Markowitz model, the necessary components are first identified, including:

a. Expected Return

The expected return of the rupiah exchange rate is obtained by summing the returns of the rupiah exchange rate and then dividing by the number of transaction periods. The calculated expected return results are presented in the following Table 3.

Table 3. Expected return exchange rate

No	Exchange Rate	Expected Return
1	SGD	0,0000963
2	CNY	0,0000060
3	EUR	0,0001605

Table 3 presents the expected return values of the rupiah exchange rate against SGD, CNY, and EUR. The expected return for each exchange rate is positive, indicating that the rupiah exchange rate is expected to increase, which is favorable for foreign currency investments.

b. Standard Deviation

Standard deviation is a measure of exchange rate volatility that indicates the extent of fluctuations in the rupiah exchange rate. The standard deviation of each rupiah exchange rate is presented in Table 4.

Table 4. Standard deviation of rupiah exchange rate

No	Exchange Rate	Standard Deviation
1	SGD	0,002783
2	CNY	0,003485
3	EUR	0,003800

Table 4 contains the standard deviation of each rupiah exchange rate against SGD, CNY, and EUR. The rupiah exchange rate against EUR has a larger standard deviation compared to other rupiah exchange rates.

c. Correlation Matrix

The correlation matrix has a diagonal with a value of 1, while the other entries are correlation coefficients between rupiah exchange rates. The correlation values between rupiah exchange rates are obtained as follows Table 5.

Table 5. Correlation of rupiah exchange rates

No	Exchange Rate	Correlation		
		SGD	CNY	EUR
1	SGD	1	0,745834	0,744173
2	CNY	0,745834	1	0,424333
3	EUR	0,744173	0,424333	1

Table 5 shows that the correlation value between rupiah exchange rates is positive, meaning that the exchange rate movement is in the same direction. That is, when one rupiah exchange rate rises, the other exchange rate also tends to rise, and the same thing happens when the exchange rate falls. The correlation value between the rupiah exchange rate against SGD and CNY is greater than that of the others.

d. Variance Covariance Matrix

The variance-covariance matrix is a matrix whose main diagonal entries contain the variance of exchange rates, and the other entries contain the covariance between exchange rates. The variance-covariance matrix can be seen in Table 6.

Table 6. Variance covariance of rupiah exchange rates

No	Exchange Rate	Correlation		
		SGD	CNY	EUR
1	SGD	7,74434E-06	7,21063E-06	7,84391E-06
2	CNY	7,21063E-06	1,21469E-05	5,60152E-06
3	EUR	7,84391E-06	5,60152E-06	1,44385E-05

Table 6 provides the variance-covariance matrix, a key input for calculating optimal portfolio allocations using the Markowitz model. Next, a variance-covariance matrix will be created, and the determinant of the variance-covariance matrix will be calculated.

5. Application of Moore-Penrose Method in Determining Optimal Portfolio

The portfolio formation begins with calculating the expected return value for each rupiah exchange rate, and then the data is sorted from the smallest to the largest variance value. The expected return value for each rupiah exchange rate can be represented in the form of a matrix as follows:

$$\vec{\mu} = \begin{bmatrix} 0{,}0000963 \\ 0{,}000006 \\ 0{,}0001605 \end{bmatrix}$$

and a covariance matrix $\sum$ is formed with a matrix size of 3×3 as follows:

$$\sum = \begin{bmatrix} 0{,}0000077 & 0{,}0000072 & 0{,}0000078 \\ 0{,}0000072 & 0{,}0000121 & 0{,}0000056 \\ 0{,}0000078 & 0{,}0000056 & 0{,}0000144 \end{bmatrix}$$

Based on the covariance variance matrix above, we obtain $\det \sum = 2.46508 \times 10^{-16}$. The determinant of the variance-covariance matrix shows that the matrix is nearly singular. Therefore, the solution is to apply a pseudo-inverse matrix. The pseudo-inverse ($\sum^{+}$) is used to replace the inverse method ($\sum^{-}$), aiming to minimize errors in the inversion process. By replacing $\sum^{-}$ with $\sum^{+}$, the Markowitz model equation for determining portfolio allocation is obtained as follows:

$$\vec{w} = \frac{R_p A - B}{AC - B^2} \sum^{+} \vec{\mu} + \frac{C - R_p B}{AC - B^2} \sum^{+} \vec{1} \quad (7)$$

with

$$A = \vec{1}' \times \sum^{+} \times \vec{1} \quad (8)$$

$$B = \vec{1}' \times \sum^{+} \times \vec{\mu} \quad (9)$$

$$C = \vec{\mu}' \times \sum^{+} \times \vec{\mu} \quad (10)$$

Equations (7)-(10) outline the steps for applying the Moore-Penrose pseudoinverse to determine the optimal portfolio weights, addressing the near-singular nature of the covariance matrix. Here, $\sum^{+}$ denotes the pseudoinverse of the variance-covariance matrix, $\vec{\mu}$ represents the expected returns as a column vector, and R_p is the portfolio return. The R_p value is obtained from the highest expected return among several rupiah exchange rate returns. The initial value of $R_p = 0{,}0001604$, with the inverse of the variance-covariance matrix given as follows:

$$\sum^{+} = \begin{bmatrix} 579616{,}07737 & -243399{,}80853 & -219303{,}22748 \\ -243399{,}80853 & 202995{,}44031 & 52898{,}89172 \\ -219303{,}22748 & 52898{,}89172 & 167661{,}90144 \end{bmatrix}$$

After that, the values of A, B, and C are determined. The value of A is calculated using equation (8) as follows:

$$A = \vec{1}' \times \sum^{+} \times \vec{1}$$
$$A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 579616{,}07737 & -243399{,}80853 & -219303{,}22748 \\ -243399{,}80853 & 202995{,}44031 & 52898{,}89172 \\ -219303{,}22748 & 52898{,}89172 & 167661{,}90144 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$$A = 130665{,}13054$$

For the value of B, it is obtained as follows:

$$B = \vec{1}' \times \sum^{+} \times \vec{\mu}$$
$$B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 579616{,}07737 & -243399{,}80853 & -219303{,}22748 \\ -243399{,}80853 & 202995{,}44031 & 52898{,}89172 \\ -219303{,}22748 & 52898{,}89172 & 167661{,}90144 \end{bmatrix} \begin{bmatrix} 0{,}0000963 \\ 0{,}000006 \\ 0{,}0001605 \end{bmatrix}$$
$$B = 11{,}53553$$

Meanwhile, the value of C is obtained as follows:

$$C = \vec{\mu}' \times \sum^{+} \times \vec{\mu}$$
$$C = \begin{bmatrix} 0{,}0000963 & 0{,}000006 & 0{,}0001605 \end{bmatrix}$$
$$\begin{bmatrix} 579616{,}07737 & -243399{,}80853 & -219303{,}22748 \\ -243399{,}80853 & 202995{,}44031 & 52898{,}89172 \\ -219303{,}22748 & 52898{,}89172 & 167661{,}90144 \end{bmatrix} \begin{bmatrix} 0{,}0000963 \\ 0{,}000006 \\ 0{,}0001605 \end{bmatrix}$$
$$C = 0{,}00274$$

Next, by reducing the return value of the previous portfolio by 0.005% in the same way, several portfolio alternatives are obtained.

6. Portfolio Formation Using the Markowitz Model

According to Markowitz's portfolio theory, an investor can only allocate 100% of their funds. Below are the results of the rupiah exchange rate fund composition from 10 portfolios formed using the Moore-Penrose method, as shown in Table 7.

Table 7. Allocation of funds for each rupiah exchange rate

No	Composition (%)		
	SGD	CNY	EUR
1	95,00	0,29	4,71
2	94,67	0,85	4,48
3	94,13	1,75	4,12
4	93,59	2,65	3,76
5	93,05	3,56	3,39
6	92,51	4,46	3,03
7	91,97	5,37	2,66
8	91,43	6,28	2,29
9	90,90	7,17	1,93
10	90,31	8,17	1,52

Table 7 presents ten portfolio compositions, each reflecting a distinct risk–return trade-off. Based on the allocation of funds among the three currencies shown in Table 7, the corresponding expected portfolio return and risk are as shown in Table 8.

Table 8. Expected Portfolio Return and Portfolio Risk

No	$E_{(R_p)}$	σ_p
1	0,0000991	0,0027871
2	0,0000984	0,0027856
3	0,0000974	0,0027834
4	0,0000963	0,0027815
5	0,0000953	0,0027798
6	0,0000942	0,0027783
7	0,0000932	0,0027771
8	0,0000921	0,0027760
9	0,0000911	0,0027753
10	0,0000899	0,0027747

Based on Table 8, the values of expected return and portfolio risk for several rupiah exchange rates are presented. Furthermore, a plot of the proportions of 10 portfolios is generated based on the Markowitz model.

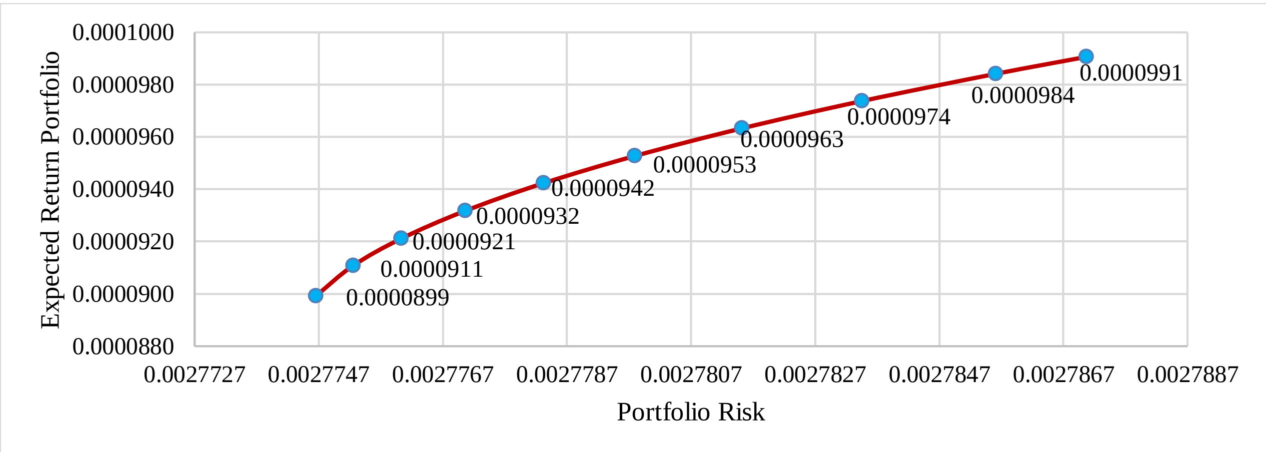

Figure 2. Plot Mean-Variance Efficient Frontier for Three Asset

Figure 2 presents a plot of the portfolios, indicated by blue dots, while the efficient frontier is represented by the red graph. As shown in Figure 2, all the portfolios lie along the red line, indicating that the 10 portfolios formed are efficient. One of the 10 efficient portfolios is categorized as the optimal portfolio with the smallest risk of the Markowitz model. The portfolio with the smallest risk is 0,0027747 or 0,27747%, with an expected return of 0,0000899 or 0,009%. Based on the expected return value, the portfolio proportions of the rupiah exchange rate are 0,9031, 0,0817, and 0,0152, respectively. The rupiah exchange rate against the Singapore Dollar holds the largest proportion of funds at 90.3%, while the Euro holds the smallest proportion at 1,5%.

This result highlights the effectiveness of the multidimensional GBM model in simulating exchange rates and the Markowitz model in minimizing risk through diversification. The Moore-Penrose method addresses near-singular covariance matrices, enhancing portfolio optimization. The optimal portfolio primarily favors the rupiah-Singapore Dollar exchange rate, reflecting Singapore's strong economic position and high ranking in global financial indices such as the GFCI. These findings provide valuable insights for managing currency risk and optimizing returns.

D. CONCLUSION AND SUGGESTIONS

The simulation results show that the multidimensional GBM model closely tracks actual rupiah exchange rate movements, with a MAPE value below 10%, indicating high predictive accuracy. However, a key limitation of the model is its assumption of constant volatility, which may not fully capture exchange rate dynamics under highly volatile market conditions, potentially affecting its predictive accuracy. In the Markowitz model, the covariance matrix has a determinant of 2.46508×10^{-16}, indicating near-singularity. This condition can compromise the stability of the resulting portfolio, as it becomes highly sensitive to small variations in the input data. To address this issue, the Moore-Penrose method was employed. The resulting optimal portfolio exhibited the lowest level of risk at 0.2775%, accompanied by an expected return of 0.009%. The portfolio allocations for each rupiah exchange rate are 90.3% for the Singapore Dollar, 8.2% for the Chinese Yuan, and 1.5% for the Euro, with the largest proportion assigned to the Singapore Dollar and the smallest to the Euro. Further research is recommended to explore more optimal approaches by applying alternative methods to datasets that result in singular covariance matrices.

REFERENCES

Agustini, W. F., Affianti, I. R., & Putri, E. R. M. (2018). Stock Price Prediction Using Geometric Brownian Motion. *Journal of Physics: Conference Series*, *974*(1), 9–12. https://doi.org/10.1088/1742-6596/974/1/012047

Aksaraylı, M., & Pala, O. (2018). A Polynomial Goal Programming Model for Portfolio Optimization Based on Entropy And Higher Moments. *Expert Systems with Applications*, *94*(1), 185–192. https://doi.org/10.1016/j.eswa.2017.10.056

Brătian, V., Acu, A. M., Oprean-stan, C., Dinga, E., & Ionescu, G. M. (2021). Efficient or Fractal Market Hypothesis? A Stock Indexes Modelling Using Geometric Brownian Motion and Geometric Fractional Brownian Motion. *Mathematics*, *9*(22), 11–20. https://doi.org/10.3390/math9222983

Carissa, N., & Khoirudin, R. (2020). The Factors Affecting The Rupiah Exchange Rate in Indonesia. *Jurnal Ekonomi Pembangunan*, *18*(1), 37–46. https://doi.org/10.29259/jep.v18i1.9826

Deng, G.-F., Lin, W.-T., & Lo, C.-C. (2012). Markowitz-Based Portfolio Selection With Cardinality Constraints Using Improved Particle Swarm Optimization. *Expert Systems with Applications*, *39*(4), 4558–4566. https://doi.org/10.1016/j.eswa.2011.09.129

Eris, I., Putro, T. S., & Kornita, S. E. (2017). Pengaruh Tingkat Suku Bunga Bi Rate, Jumlah Uang Beredar dan Neraca Pembayaran Terhadap Nilai Tukar Rupiah Tahun 2006-2015. *Jurnal Online Mahasiswa Fakultas Ekonomi Universitas Riau*, *4*(1), 393–404.

Germansah, G., Tjahjana, R. H., & Herdiana, R. (2023). Geometric Brownian Motion in Analyzing Seasonality of Gold Derivative Prices. *Eduvest - Journal of Universal Studies*, *3*(8), 1558–1572. https://doi.org/10.59188/eduvest.v3i8.892

Grechuk, B., & Zabarankin, M. (2014). Inverse Portfolio Problem With Mean-Deviation Model. *European Journal of Operational Research*, *234*(2), 481–490. https://doi.org/10.1016/j.ejor.2013.04.056

Huang, X., & Yang, T. (2020). How Does Background Risk Affect Portfolio Choice: An Analysis Based On Uncertain Mean-Variance Model With Background Risk. *Journal of Banking & Finance*, *111*(3), 2039–2054. https://doi.org/10.1016/j.jbankfin.2019.105726

Ibrahim, S. N. I., Misiran, M., & Laham, M. F. (2021). Geometric Fractional Brownian Motion Model For Commodity Market Simulation. *Alexandria Engineering Journal*, *60*(1), 955–962. https://doi.org/10.1016/j.aej.2020.10.023

Kalayci, C. B., Ertenlice, O., & Akbay, M. A. (2019). A Comprehensive Review Of Deterministic Models And Applications For Mean-Variance Portfolio Optimization. *Expert Systems with Applications*, *125*(1), 345–368. https://doi.org/10.1016/j.eswa.2019.02.011

Kloeden, P. ., & Platen, E. (1992). *Numerical Solution of Stochastic Differential Equation*. Springer-Verlag Berlin Heidelberg. https://doi.org/10.1007/978-3-662-12616-5

Li, B., Sun, Y., Aw, G., & Teo, K. L. (2019). Uncertain Portfolio Optimization Problem Under A Minimax Risk Measure. *In Applied Mathematical Modelling*, *76*(1), 274–281. https://doi.org/10.1016/j.apm.2019.06.019

Li, B., & Zhang, R. (2021). A New Mean-Variance-Entropy Model For Uncertain Portfolio Optimization With Liquidity Diversification. *Chaos, Solitons & Fractals*, *146*(1), 989–1003. https://doi.org/10.1016/j.chaos.2021.110842

Liu, S.-T. (2011). The Mean-Absolute Deviation Portfolio Selection Problem With Interval-Valued Returns. *Journal of Computational and Applied Mathematics*, *235*(14), 4149–4157. https://doi.org/10.1016/j.cam.2011.03.008

Lv, S., Wu, Z., & Yu, Z. (2016). Continuous-Time Mean-Variance Portfolio Selection With The Random Horizon in an Incomplete Market. *Automatica*, *69*(1), 176–180. https://doi.org/10.1016/j.automatica.2016.02.017

Ramos, A. L., Mazzinghy, D. B., Barbosa, V. da S. B., Oliveira, M. M., & da Silva, G. R. (2019). Evaluation Of An Iron Ore Price Forecast Using A Geometric Brownian Motion Model. *Revista Escola de Minas*, *72*(1), 9–15. https://doi.org/10.1590/0370-44672018720140

Ramos, H. P., Righi, M. B., Guedes, P. C., & Müller, F. M. (2023). A Comparison Of Risk Measures For Portfolio Optimization With Cardinality Constraints. *Expert Systems with Applications*, *228*(1), 6–12. https://doi.org/10.1016/j.eswa.2023.120412

Reddy, K., & Clinton, V. (2016). Simulating Stock Prices Using Geometric Brownian Motion: Evidence From Australian Companies. *Australasian Accounting, Business and Finance Journal*, *10*(3), 23–47. https://doi.org/10.14453/aabfj.v10i3.3

Silaban, S., Aadilah, H., & Matondang, K. (2023). Influence of Rupiah Exchange Rate on Indonesia's Economic Growth: Literature Study. *Journal of Business Management and Economic Development*, *1*(2), 123–131. https://doi.org/10.59653/jbmed.v1i02.48

Suganthi, K., & Jayalalitha, G. (2019). Geometric Brownian Motion in Stock Prices. *Journal of Physics: Conference Series*, *1377*(1). https://doi.org/10.1088/1742-6596/1377/1/012016