JTAM (Jurnal Teori dan Aplikasi Matematika)
http://journal.ummat.ac.id/index.php/jtam
p-ISSN 2597-7512 | e-ISSN 2614-1175
Vol. 9, No. 4, October 2025, pp. 1309-1327

Monitoring PH of Shrimp Water using Progressive Max Chart

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Article History:
Received : 24-03-2025
Revised : 01-08-2025
Accepted : 06-08-2025
Online : 23-10-2025

Keywords:
Simultaneous Control Chart;
Progressive Max Chart;
Max Chart;
EWMA-Max Chart;
Water PH.

ABSTRACT

Control charts aim to reduce variability in the process and monitor for out-of-control processes. So far, the process of monitoring quality is usually carried out partially, namely monitoring the mean process and process variability. This approach is less effective and time-consuming because two separate charts must be created simultaneously. One alternative is to analyze both parameters simultaneously, such as through the Progressive Max Chart method (Mixed-Methods Research: Quantitative and Applied). The Progressive Max Chart is a control chart designed for monitoring both the mean and variability by considering the case of subgroup observations. This study uses a quantitative approach, combining primary data collection and simulations to generate findings through statistical analysis and quantifiable measurements. The purpose of this research is to compare methods such as the Progressive Max Chart, EWMA-Max, and Max Chart. The analysis results show that the Progressive Max Chart method performs better than the Max Chart and EWMA- Max Chart, both in terms of mean, variance, and mean-variance detection, for small shifts and large shifts. The control chart performance results provide optimal outcomes for monitoring out-of-control signals at subgroup sizes of n = 2, 3, 5. This is characterized by ARL_1 values that approach 1 more quickly. This method is applied to pH data from vannamei shrimp pond water located in Madura. The Progressive Max Chart method provides optimal results by maximizing the detection of in-control signals. Additionally, it is tested on synthesized data and demonstrates optimal performance in detecting both small and large shifts in mean, variance, and mean-variance.

Crossref
https://doi.org/10.31764/jtam.v9i4.30255

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A. INTRODUCTION

In statistical process control (SPC), the control chart is a quite popular technique to monitor the process efficacy (Knoth et al., 2021). A control chart is a tool used to determine whether a process is in statistical control. The ultimate goal of statistical process control is to reduce variability in the process. Although this method cannot completely reduce variability, control charts are able to effectively minimize variability (Syarifah Nazia et al., 2023). Control charts aim to the simplicity of understanding them, while at the same time being efficient in detecting defects in production processes (Qiu, 2018). The Max- K̄Faſ control chart is a technique used in monitoring process control by evaluating the stability of the process mean and variability using a single chart, and has the main advantage that using inspection attributes is easy to use and costs less than variable-type inspections that use true values. The Max-K̄Faſ control chart is an alternative control chart with variable-type inspection (Rifki et al., 2025). The development of the control chart into Max-EWMAMS by taking the maximum value between the two absolute values of the standard normal variable which is the estimator of the mean and variance of the process. Max EWMAMS charts generally detect mean shifts, variance increases and simultaneous changes in process mean and variability faster than the other two charts (Javaid et al., 2020). In the research (Javaid et al., 2020), it is proven that the proposed Max-HEWMA control diagram is more sensitive than the previous one, namely the AIB-Max EWMA control diagram. The Max-HEWMA control diagram is proven to be efficient so that it monitors shifts in mean and variance based on Average Run Length (ARL) and Standard Deviation of the Run Length (SDRL). EWMA-Max is superior to other existing simultaneous control charts in that it monitors small shifts in process parameters, it is also good at monitoring large shifts (Sanusi et al., 2020).

On the other hand, the performance of an EWMA chart, characterized by unknown parameters, was analyzed through the application of quantiles of the average run length (ARL) and the standard deviation of the average run length (SDAR) (Saleh et al., 2015). The standard deviation of the run length (SDRL), which incorporates both average run length (ARL) and run chain length, is applicable solely for the performance analysis of a single control chart. As the average run chain length approaches the setpoint ARL0, particularly under unknown parameters, the standard deviation of the average run length (SDARL) diminishes, indicating that the performance of the control chart is more aligned with a known situation. A control chart designed for repeated monitoring of lognormal processes was developed, with its performance analyzed using average run length (ARL) and standard deviation as statistical performance indicators (Khoo et al., 2015). Additionally, the efectiveness of the control chart's monitoring capabilities was verifed (Quinino et al., 2020).

Furthermore, the development of the Maximum Exponentially Weighted Moving Average-Max (EWMA-Max) control chart, an extension of the Max chart that applies the EWMA technique to its statistics, builds on comparisons showing that the Max-CUSUM chart outperformed the Max-MEWMA chart and the Alternate Variable Multivariate chart in detecting simultaneous small shifts in process mean and covariance, as well as in identifying mean shifts alone (Ajadi et al., 2021).

Aslam, M. (2016) proposed the EWMA–CUSUM charts for monitoring correlated data using the Average Run Length, extra quadratic loss, and relative Average Run Length as criteria to measure the efficiency with Shewhart, CUSUM, EWMA, Shewhart-CUSUM, and Shewhart-EWMA charts. The newly proposed control charts have efficiency in detecting better than the compared charts. In 2017, Osei-Aning (2017) proposed the CUSUM-EWMA chart to detect the change of variation in the process using the ARL, extra quadratic loss, and relative Average Run Length as criteria to measure the efficiency with Shewhart, EWMA and CUSUM charts. It was found that the CUSUM-EWMA chart had better efficiency for detection than the control charts of Shewhart, CUSUM-S2, S2-EWMA, CS-EWMA, floating T-S2, floating U-S2, classical EWMA, and CUSUM charts (Lu, 2017).

Previous research has compared two different methods for controlling water nutrient levels in non-circulating hydroponics based on the projected canopy area, using linear regression as the primary method, while hydroponics itself is a soilless cultivation technique that delivers nutrients to crops through a closed-loop irrigation system, typically submerging plant roots in the nutrient solution (Sulaiman et al., 2025). From this research, it has elaborated with Progressive Max Chart on the object of vannamei shrimp to test the quality of water PH media (Setyastuti et al., 2023).

Vannamei shrimp (*Litopenaeus vannamei*), a significant aquaculture species contributing to over 53% of total farmed crustacean production, has seen steadily rising cultivation and consumption globally (Kim et al., 2020). In Indonesia, ponds are among the most popular breeding grounds for this species. These man-made coastal systems, often utilizing brackish or seawater, are designed for cultivating aquatic animals like fish, shrimp, and shellfish. Vannamei shrimp farming is particularly favored due to its competitive pricing and adaptability to high-density, intensive production systems, which aim to maximize output. However, exceeding the pond's carrying capacity at a given biomass level can compromise shrimp survival, especially under excessively high stocking densities (Mustafa et al., 2023).

Based on the description above, this research will develop a Progressive Max chart- based control diagram. It is expected that this chart will be able to efficiently monitor the mean and variability simultaneously and have better performance. The performance of the developed control chart will be compared with several other simultaneous control charts such as EWMA-Max and Max-chart. Furthermore, the Progressive Max control chart will be applied to monitor water pH in vannamei shrimp ponds.

B. METHODS

1. Data Source and Data Structure

This research employs a quantitative research approach, utilizing both primary data collection and simulation methods. Quantitative research is a type of study that generates findings through statistical procedures or other quantifiable measurement techniques. The primary data was collected through direct observations conducted at vannamei shrimp ponds in Madura over a four-month period, from November 2022 to February 2023. Meanwhile, simulation data was generated using the R programming language, which allowed for computational modelling and analysis to supplement the empirical findings. By combining field observations with computational simulations, this research ensures a robust and comprehensive analysis of the subject matter.

2. Data Source

This research uses both primary data and simulated data. The simulated data were generated using the R programming package by creating data that represent an in-control process. These simulated data were used to evaluate the performance of the Progressive Max control chart. The primary data were collected from a vannamei shrimp pond in Madura between November 2022 and February 2023, consisting of 116 samples. To measure the pH level of the pond water, the process involved selecting pond Block A8 as the observation site, collecting water samples using a small plastic cup, immersing a pH meter (pH meter 10) into the cup, and pressing the meter's button to obtain the pH value, as shown in Table 1.

Table 1. Data Source

Samples	**pH of Water(X)**	
	1	**2**
1	X_{11}	X_{12}
2	X_{21}	X_{22}
3	X_{31}	X_{32}
⋮	⋮	⋮
i	X_{i1}	X_{i2}
⋮	⋮	⋮
m	X_{m1}	X_{m2}

3. Data Structure and Variables

The data structure in this study consists of pH level measurements of pond water, with m observation units and a subgroup size of [= 2 , representing measurements taken in the morning and at noon. The data structure used for the Progressive Max control chart is presented in Table 2.

Table 2. Research Data Structure

Samples	**pH of Water(X)**	
	1	**2**
1	X_{11}	X_{12}
2	X_{21}	X_{22}
3	X_{31}	X_{32}
⋮	⋮	⋮
i	X_{i1}	X_{i2}
⋮	⋮	⋮
m	X_{m1}	X_{m2}

Table 3. Research Variables

Variables	**Variable Name**	**Specification Limit**
X	Water pH	7.5-9

4. Progressive Mean Control Diagram

Progressive Mean control charts are control charts in monitoring the process mean by considering the case of individual observations. Besides conventional Westgard rules, methods like Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) assist laboratory professionals in identifying small shifts and trends, though Progressive Mean control charts are more effective than these Shewhart-type charts for monitoring process means in individual observations (Çubukçu, 2021). Progressive Mean control charts have excellent performance not only for small and medium shifts, but also show good performance for large shifts. If K_k, $k = 1,2,3, \dots, m$ is a sequence of independent and identically distributed observations, then the Progressive Mean control chart is defined as the cumulative mean over time (Riaz et al., 2020). Mathematically, Progressive Mean is defined as follows.

$$PM_i = \frac{\sum_{k=1}^{i} X_k}{i},\ i = 1,2,\dots,m \tag{1}$$

IF PM_i is an unbiased estimator of the sample mean μ_0, and the variance is $\frac{\sigma_0^2}{i}$, where μ_0 is the mean and σ_0^2 is the variance. The following are the control limits of the Progressive Mean.

$$LCL = \mu_0 - h\frac{\sigma_0}{\sqrt{i}} \tag{2}$$

$$CL = \mu_0 \tag{3}$$

$$UCL = \mu_0 + h\frac{\sigma_0}{\sqrt{i}} \tag{4}$$

5. Max Chart Control Diagram

If Max control chart-XS otherwise known as Max Chart is a univariate simultaneous control chart to monitor the mean and variability of the process in one chart. Given X is a particular characteristic of a process, is the process mean, and is the process standard deviation. Given $\bar{X}_i = \frac{(X_{i1}+\ldots+X_{in})}{n}$ is the mean of the *i-th* sample and $S_i{}^2 = \frac{\sum_{j=1}^{n}(X_{ij}-\bar{X})^2}{(n_i-1)}$ is the variance of the *i-th* sample. Thus, the simultaneous control diagram is formulated as follows:

$$U_i = \frac{(\bar{X}_i - \mu)}{\frac{s}{\sqrt{n}}}, \qquad i = 1,2,\ldots,m \tag{5}$$

U_i = transformation $\bar{X}_i$
n = subgroup size
m = number of subgroups

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n-1)S_i^2}{\sigma^2}; n-1\right)\right\} \tag{6}$$

where $\Phi(z) = P(Z \leq z)$ for $Z \sim N(0,1)$, the standard normal distribution, $\Phi^{-1}(.)$ is the inverse function of $\Phi(.)$, and $H(w,v) = P(W \leq w|v)$ for $W \sim \chi_v^2$, the chi-squared distribution with v degrees of freedom. U_i and V_i are transformations of $\bar{X}_i$ and S_i^2, when $a = 0$ and $b = 1$, $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$. In particular, the statistics for simultaneous control charts are defined as follows.

$$M_i = max(|U_i|, |V_i|), i = 1,2,\ldots m \tag{7}$$

U_i : monitors shifts in the process mean
I_i : monitors shifts in the process variability

The value of the test statistic M becomes large when the process shifts away from μ or when theprocess variability increases or decreases. On the other hand, the value of M shrinks when the process mean and variability remain close to their respective target values.

6. EWMA-Max Control Diagram

The EWMA-Max chart is an extended control chart of the Max chart by applying the EWMA technique to the Max chart statistics. Given $X_1, X_2, \ldots, X_i$ denote the quality characteristics of a process with μ denoting the process mean and σ denoting the process standard deviation where i denotes the sample number index. The EWMA statistic with smoothing constant λ is shown in the following equation.

$$Z_i = (1-\lambda)Z_{i-1} + \lambda X_i \tag{8}$$

with $0 < \lambda \leq 1$. When the process is in-control, the mean and variance of the EWMA statistic are.

$$E(Z_i) = \mu_0 \tag{9}$$

$$Var(Z_i) = \sigma^2 \frac{\lambda[1-(1-(1-\lambda)^{2i}]}{2-\lambda} \tag{10}$$

where μ_0 and σ^2 denote the *mean* and variance of target X_i, respectively. The *EWMA-Max* statistic is defined in the following equation.

$$G_i = (1-\lambda)G_{i-1} + \lambda M_i \tag{11}$$

Given that the initial value $G_0 = 1{,}128379$, and assuming that U_i and I_i are mutually independent, when ÿ = 0 and þ = 1, both U_i and I_i follow a standard normal distribution. $U_i \sim A(0,1)$ and $I_i \sim A(0,1)$. The upper control limit (UCL) on the huber function for the *EWMA- Max* control diagram is given as follows (Malik et al., 2024).

$$UCL = 1{,}128379 + 0{,}602810L\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{12}$$

7. ARL (Average Run Length)

ARL is the *mean of* several observation points until the first *out-of-control* signal is found. There are two types of ARL_0 and ARL_1 in performance assessment of control charts. The ARL_0 formula is defined as follows:

$$ARL_0 = \frac{1}{\alpha} \tag{13}$$

$$ARL_1 = \frac{1}{1-\beta} \tag{14}$$

α refers to a Type I error, which is the probability of signaling that the process is out of control when it is actually operating normally. A typical value for alpha, often used when applying 3σ control limit is 0.0027 . Meanwhile, ý refers to a Type II error, which is the probability of failing to detect that the process is out of control when a shift has actually occurred. A control chart is considered effective when it has a large value of ARL_0, representing the average number of samples before a false alarm, and a small value of ARL_1, indicating the average number of samples needed to detect a true shift in the process.

C. RESULT AND DISCUSSION

The control charts presented in this study are Progressive Max chart-based univariate control charts and Max chart control charts or conventional control charts. The first Max chart control diagram is to find the value of U and V which is the transformation of $\bar{X}$ and S, then find the value of M_i. The next control diagram is to carry out a performance evaluation comparison against the Progressive Max chart control diagram using the ARL criterion. Progressive Max chart and Max chart control diagrams are implemented through monitoring the water pH characteristics data of Vannamei shrimp ponds located in Madura, Indonesia.

1. Progressive Max Chart Statistics and Control Limits

The statistics in the Progressive Max chart control diagram proposed in this study are a development of the previous control diagram, namely the Max chart control diagram combined with the Progressive Mean control diagram from the research. This control diagram is expected to be able to carry out good performance for small, medium and large shifts. Here is the mathematics of the Progressive Max chart control diagram with the following equation.

$$\mathrm{PrM_i} = \frac{\sum_{k=1}^{i} \mathrm{M_k}}{\mathrm{i}}, \quad \mathrm{i} = 1,2,\ldots,\mathrm{m} \tag{15}$$

where $\mathrm{PrM_i}$ is the i-th Progressive Max moving mean of the $\mathrm{M_i}$ value., the data element moves from k to i,, with i being the number of subgroups from 1 to m, each containing no n data. The control limit in the Progressive Max chart control diagram study is the upper control limit or Upper Conttrol Limit (UCL) value found in the following equation.

$$\mathrm{UCL_i} = \mu_{\mathrm{prm}} + \mathrm{h}\frac{\sigma_{\mathrm{prm}}}{\sqrt{\mathrm{i}}} \tag{16}$$

where μ_{prm} is the mean of observations which is 1.128379 and σ_{prm} is the standard deviation of observations which is 0.60281, the value of h is a value designed to stop the ARL_0 or in-control process. ARL_0 is the mean of the number of observations that the plot is expected to first exit at the time of the in-control state.. The following ARL_0 table with the parameter h in getting the $ARL_0 \cong 370$ value on the Progressive Max chart control diagram.

Table 4. Parameter values of (h) so as to obtain the ARL values of ≅ 370 in subgroups (n = 2,3,5)

Subgroup (g)	h
2	1.12
3	1.122
5	1.124

2. Performance of Progressive Max, EWMA-Max, and Max chart control charts

a. Performance of Progressive Max Control Diagram based on the Number of Characteristics

The performance of this research was evaluated based on the ARL_1 values derived from simulated data $K \sim A(\mu_{shifa}, \sigma_{shifa})$, where the in-control average run length ARL_0 was set at approximately 370, corresponding to a Type I error rate α of 0.0027. The study assessed detection performance under different shift magnitudes, beginning at 0.25, across three scenarios: shifts in the mean with constant variability, shifts in variability with a stable mean, and simultaneous shifts in both mean and variance. The Progressive Max chart's effectiveness was examined using subgroup sizes of 2, 3, and 5. When evaluating mean shifts with constant variability, the results demonstrated varying ARL_1 values depending on the subgroup size. For a subgroup size of [= 2 the ARL_0 at zero shift was 366.520. A mean shift of 0.25 reduced ARL_1 to 180.080, while a shift of 0.5 led to a further decline to 24.627. At a larger shift of 2.5, ARL_1dropped to its minimum value of 1. Similarly, for [= 3 , a mean shift of 0.25 resulted in an $ARL_1$of 89.101, which decreased to 11.952 at a shift of 0.5 and eventually reached 1 at a shift of 2.5. The trend continued with [= 5 , where a shift of 0.25 yielded an ARL_1 of 41.523, a shift of 0.5 reduced it to 6.260, and a shift of 2.5 again brought it down to 1. These findings highlight how larger subgroup sizes improve detection sensitivity, as evidenced by the faster decline in ARL_1 values with increasing shift magnitudes. Performance evaluation on mean shift with constant variability in large-scale processes as shown in Table 5.

Table 5. ARL of Progressive Max Diagram for $n = 2,3,5$ based on mean shift

Shift	Progressive Max		
	n = 2	**n =3**	**n =5**
0	366.520	356.007	374.530
0.25	180.080	89.101	41.523
0.5	24.627	11.952	6.260
0.75	7.304	4.038	2.370
1	3.239	2.237	1.493
1.25	2.250	1.573	1.200
1.5	1.630	1.275	1.050
1.75	1.300	1.098	1.012
2	1.150	1.040	1.006
2.25	1.091	1.017	1.000
2.5	1.000	1.000	1.000

Figure 1 is a visualization of the ARL_1 values of the Progressive Max control chart with subgroup sizes of [= 2, [= 3, and [= 5 when the mean shifts and variability remains constant. The larger the subgroup size used, the better the performance of the Progressive Max control chart, as indicated by the decreasing ARL_1 values. When the mean shift is greater than 0.5, it can be observed that the ARL_1 values for different subgroup sizes are approximately 1, indicating that the Progressive Max control chart is effective for monitoring large process mean shifts.

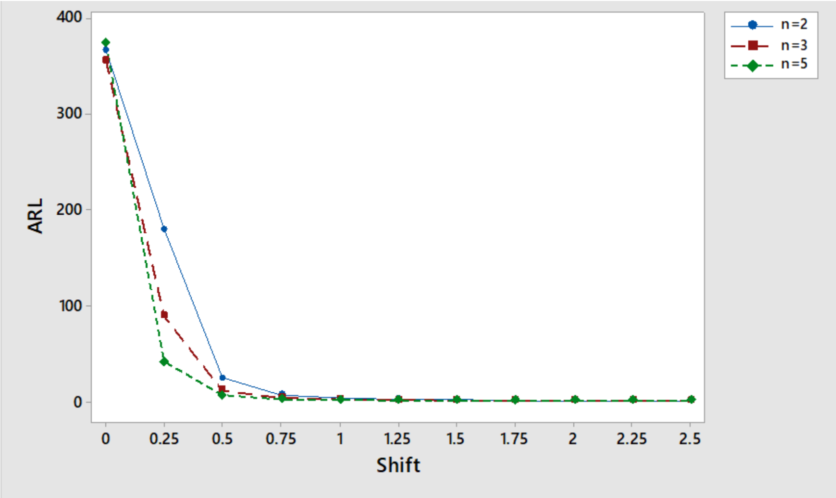

Figure 1. ARL of Progressive Max diagram monitoring mean shift based on the number of times n

b. Performance of Progressive Max, EWMA-Max, and Max chart control charts

The performance evaluation of this study was conducted based on ARL_1 values derived from simulated data following a normal distribution $K \sim A(\mu_{shifa}, \sigma_{shifa})$, with the in-control ARL_0 maintained at approximately 370 (α = 0.0027). The assessment examined large shifts starting from 0.25 under three distinct scenarios: processes experiencing mean shifts with constant variability, processes with stable means but shifting variability, and processes where both mean and variance exhibited shifts. The analysis focused particularly on evaluating the effectiveness of the Progressive Max control chart using subgroup sizes of [= 2, 3, and 5, with detailed results for mean shifts under constant variability presented in Table 6.

The comparative analysis revealed significant performance variations across different subgroup sizes. For the smallest subgroup size ([= 2), the Progressive Max chart demonstrated superior performance over both EWMA-Max and traditional Max charts, detecting a variance shift of 0.25 with an $ARL_1$of 98.253. When employing a moderate subgroup size ([= 3), the Progressive Max chart maintained its advantage, achieving an $ARL_1$of 12.671 for the same magnitude of variance shift. Most notably, with the largest subgroup size examined ([= 5), the Progressive Max chart exhibited its strongest performance, registering an ARL_1 of 4.251 for the 0.25 variance shift. These results consistently demonstrate the Progressive Max chart's enhanced sensitivity in detecting process variability shifts compared to alternative control chart methods, with its detection capability showing particular improvement as subgroup size increases. The findings underscore the Progressive Max chart's effectiveness as a statistical process control tool, especially in scenarios requiring prompt identification of out-of-control conditions in process variability.

Table 6. ARL_1 shift mean Progressive Max chart, EWMA-Max chart, and Max chart

mean	Shift			Progressive Max chart			EWMA-Max chart		Max chart
mean	n=2	n=3	n=5	n=2	n=3	n=5	n=2	n=3	n=5
0	366.52	356.007	374.53	378.378	382.05	336.317	349.248	341.116	348.477
0.5	24.627	11.952	6.260	68.917	39.732	19.441	133.091	90.432	51.043
1	3.239	2.237	1.493	10.946	6.944	4.208	25.663	13.94	6.030
1.5	1.630	1.275	1.050	4.691	3.304	2.275	7.175	3.727	1.786
2	1.150	1.040	1.006	2.955	2.191	1.596	2.819	1.621	1.133
2.5	1	1	1	2.140	1.687	1.185	1	1	1

3. Performance Evaluation Based on Variance Shift

a. Performance of Progressive Max Control Diagram Based on Numbers of Characteristics
Table 7 assessed performance through ARL_1 values generated from simulated data $K \sim A(\mu_{shifa}, \sigma_{shifa})$, maintaining an in-control ARL_1 of approximately 370 ($\alpha = 0.0027$). The evaluation examined substantial shifts beginning at 0.25, covering three key scenarios: mean shifts with stable variability, variability shifts with constant mean, and concurrent shifts in both mean and variance. The analysis focused on the Progressive Max chart's effectiveness using subgroup sizes of 2, 3, and 5. For mean shifts with constant variability, results demonstrated varying detection capabilities across subgroup sizes. The n=2 configuration showed gradual sensitivity improvement, with ARL_1 decreasing from 233.370 to 98.253 for shifts of 0.25 to 0.50 respectively, eventually reaching perfect detection at 2.50 shift. Larger subgroups exhibited enhanced performance, particularly n=5 which achieved rapid detection with ARL_1 values plunging from 34.802 to 4.251 for the same shift range. Similar patterns emerged in variability shift analysis, where larger subgroups consistently outperformed smaller ones in early anomaly detection. The Progressive Max chart demonstrated particular strength in identifying larger shifts (≥2.50) regardless of subgroup size, while showing graduated sensitivity to intermediate shifts based on subgroup dimensions. These findings highlight the chart's robust monitoring capabilities and the critical role of subgroup size selection in optimizing detection speed across different shift magnitudes and types, with larger subgroups generally providing superior performance in identifying out-of-control conditions.

Table 7. ARL of Progressive Max Diagram for n = 2,3,5 based on variance shift

Shift	Progressive Max		
	n = 2	n =3	n =5
0	366.520	356.007	374.530
0.25	180.080	89.101	41.523
0.5	24.627	11.952	6.260
0.75	7.304	4.038	2.370
1	3.239	2.237	1.493
1.25	2.250	1.573	1.200
1.75	1.300	1.098	1.012
2	1.150	1.040	1.006
2.25	1.091	1.017	1.000
2.5	1.000	1.000	1.000

Figure 2 visualizes the ARL_1values of the Progressive Max control chart when process variability shifts while the mean remains constant. The results demonstrate that larger subgroup sizes significantly improve the chart's performance, as evidenced by progressively smaller ARL_1 values. Notably, when variability shifts exceed 0.5, the ARL_1 values converge to approximately 1 across all subgroup sizes. This consistent performance at larger variability shifts confirms that the Progressive Max chart is particularly effective for monitoring substantial process variability changes, regardless of the subgroup dimension used. The visualization clearly shows the chart's enhanced detection capability with increased subgroup sizes while maintaining excellent sensitivity to detect major variability shifts in all configurations.

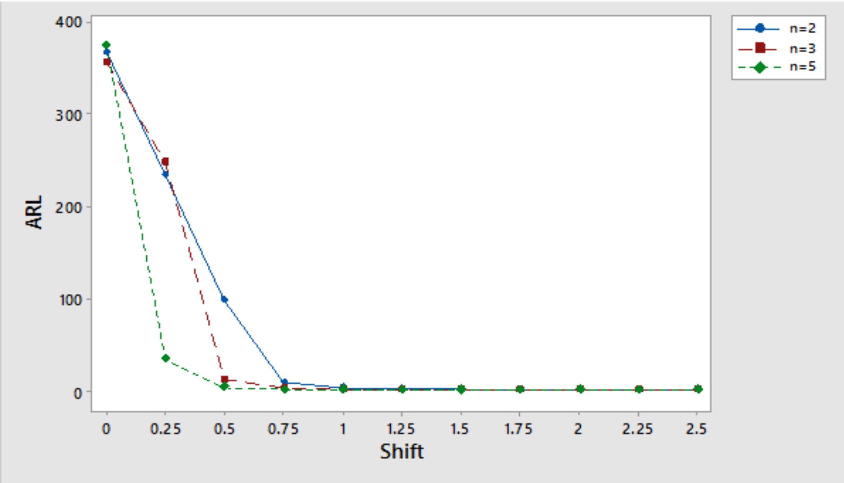

Figure 2. ARL of Progressive Max diagram monitoring variance shift based on the number of times n

b. Performance of Progressive Max, EWMA-Max, and Max chart control charts

The performance evaluation of this study was conducted based on ARL_1 values derived from simulated data following a normal distribution $K \sim A(\mu_{shifa}, \sigma_{shifa})$, with the in-control ARL_0 maintained at approximately 370 ($\alpha = 0.0027$). The assessment examined large shifts starting from 0.25 under three distinct scenarios: processes experiencing mean shifts with constant variability, processes with stable means but shifting variability, and processes where both mean and variance exhibited shifts. The analysis focused particularly on evaluating the effectiveness of the Progressive Max control chart using subgroup sizes of [= 2, 3, and 5, with detailed results for mean shifts under constant variability presented in Table 8.

The comparative analysis revealed significant performance variations across different subgroup sizes. For the smallest subgroup size ([= 2), the Progressive Max chart demonstrated superior performance over both EWMA-Max and traditional Max charts, detecting a variance shift of 0.25 with an $ARL_1$of 98.253. When employing a moderate subgroup size ([= 3), the Progressive Max chart maintained its advantage, achieving an $ARL_1$of 12.671 for the same magnitude of variance shift. Most notably, with the largest subgroup size examined ([= 5), the Progressive Max chart exhibited its strongest performance, registering an ARL_1 of 4.251 for the 0.25 variance shift. These results consistently demonstrate the Progressive Max chart's enhanced sensitivity in detecting process variability shifts compared to alternative control chart methods, with its detection capability showing particular improvement as subgroup size increases. The findings underscore the Progressive Max chart's effectiveness as a statistical process control tool, especially in scenarios requiring prompt identification of out-of-control conditions in process variability.

Table 8. ARL_1 shift variance Progressive Max chart, EWMA-Max chart, and Max chart

Shift Variance	Progressive Max chart			EWMA-Max chart			Max chart		
	n=2	n=3	n=5	n=2	n=3	n=5	n=2	n=3	n=5
0	366.520	356.007	374.530	378.378	382.050	336.317	349.248	341.116	348.477
0.5	98.253	12.671	4.251	155.603	41.622	13.908	195.334	109.991	43.205
1	4.266	1.918	1.171	15.679	6.426	3.305	54.594	16.977	4.379
1.5	1.673	1.085	1	5.538	6.426	1.958	17.181	4.126	1.350
2	1	1	1	3.395	3.132	1.417	6.066	1.643	1.002
2.5	1	1	1	2.497	2.151	1.030	2.636	1.038	1

4. Combined Performance Evaluation of Mean and Variance Shift

a. Performance of Progressive Max Control Diagram Based on Numbers of Characteristics

This research evaluates performance using ARL_1 metrics derived from simulated normally distributed data $K \sim A(\mu_{shif}a, \sigma_{shif}a)$, establishing a baseline ARL_0 of 370 (α=0.0027). The analysis investigates detection capabilities for substantial process deviations starting from 0.25σ across three fundamental scenarios: isolated mean shifts, isolated variability shifts, and concurrent mean-variance shifts. The Progressive Max chart's effectiveness was systematically tested with subgroup sizes of 2, 3, and 5 observations.

The comparative analysis reveals distinct detection patterns across subgroup configurations. For the smallest subgroup ([= 2), ARL_1 values transition from 180.080 (0.25σ shift) to 24.627 (0.5σ shift), demonstrating moderate sensitivity that improves significantly at larger shifts. Medium subgroups ([= 3) show enhanced performance, with ARL_1 decreasing from 89.101 to 11.952 for the same shift range. Optimal detection occurs with [= 5 subgroups, where ARL_1 drops sharply from 41.523 to 6.260, indicating superior sensitivity to moderate process changes. All configurations achieve perfect detection (ARL_1 = 1) at the maximum 2.5σ shift magnitude, confirming the chart's robustness for identifying major process disturbances regardless of subgroup size.

These findings highlight a crucial trade-off in statistical process control while larger subgroups ([= 5) offer substantially better detection of minor to moderate shifts ($0.25–0.5\sigma$), even smaller subgroups become equally effective for major process deviations ($\geq 2.5\sigma$). The results provide practical guidance for quality engineers, suggesting that subgroup size selection should be based on the specific detection requirements and expected shift magnitudes in the target process. The Progressive Max chart demonstrate consistent reliability across all tested scenarios, with its performance scaling predictably with subgroup size.

Table 9. ARL of Progressive Max Diagram for Shifts n=2,3,5 Based on Mean and Variance Shifts

Shift	Progressive Max		
	n = 2	**n = Ñ**	**n = 5**
0	366.52	356.007	374.53
0.25	160.498	70.867	15.264
0.5	13.484	5.296	2.467
0.75	3.489	1.998	1.296
1	1.965	1.362	1.055
1.25	1.395	1.134	1.005
1.5	1.202	1.018	1
1.75	1.074	1.001	1
2	1.013	1	1
2.25	1	1	1
2.5	1	1	1

Figure 3 presents the visualization of ARL_1values for the Progressive Max control chart when both mean and variability shifts occur. The results clearly demonstrate that larger subgroup sizes enhance the chart's detection performance, as reflected by progressively lower ARL_1 values. Notably, when both mean and variability shifts exceed 0.5σ, the ARL_1values converge to approximately 1 across all subgroup sizes. This consistent detection capability at larger shift magnitudes confirms the Progressive Max chart's effectiveness in monitoring significant simultaneous shifts in both process mean and variability. The visualization particularly highlights the chart's robust performance in detecting substantial process disturbances, regardless of the chosen subgroup dimension. The findings underscore that while larger subgroups ([= 5) offer superior sensitivity to smaller shifts, even moderate subgroup sizes ([= 2,3) become equally effective when monitoring more pronounced process deviations (≥ 0.5σ). This makes the Progressive Max chart a reliable choice for detecting major shifts in industrial processes where both location and dispersion parameters may change simultaneously.

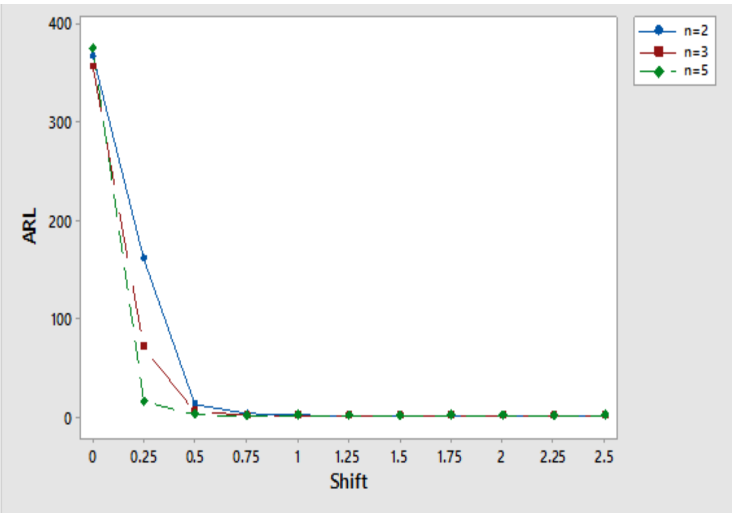

Figure 3. ARL Progressive Max Diagram Monitors the Shift of Mean and Variance Based on the Number of n

b. Performance of Progressive Max, EWMA-Max, and Max Chart Control Charts

The performance evaluation of this study was conducted based on ARL_1 obtained from simulated data following a normal distribution $K \sim A(\mu_{shifa}, \sigma_{shifa})$, with the in- control ARL_0 maintained at approximately 370 (α=0.0027). The assessment examined large shifts starting from 0.25 under three operational scenarios: processes with mean shifts under constant variability, stable means with shifting variability, and concurrent shifts in both mean and variance. The Progressive Max control chart's effectiveness was systematically evaluated using subgroup sizes of n=2, 3, and 5, with particular attention to mean shifts under constant variability as presented in Table 10.

The comparative analysis revealed distinct performance characteristics across different subgroup configurations. For the smallest subgroup size ([= 2), the Progressive Max chart demonstrated superior detection capability compared to both EWMA-Max and conventional Max charts, registering an ARL_1 of 13.484 for a combined mean and variance shift of 0.25. The performance advantage persisted with moderate subgroup sizes (n= 3), where the Progressive Max chart achieved a significantly lower ARL_1 of 5.296 for the same shift magnitude. Most notably, when employing the largest subgroup size ([= 5), the Progressive Max chart exhibited its strongest performance with an ARL_1 of just 2.467, confirming its enhanced sensitivity in detecting process deviations. These results consistently demonstrate the Progressive Max chart's superior performance in identifying out-of-control conditions, particularly when monitoring concurrent shifts in both process mean and variability.

Table 10. ARL_1 shift mean and variance Progressive Max chart, EWMA-Max chart, and Max chart

Shift mean and variance	Progressive Max chart			EWMA-Max chart			Max chart		
	n=2	n=3	n=5	n=2	n=3	n=5	n=2	n=3	n=5
0	366.520	356.007	374.53	378.378	382.050	336.317	349.248	341.116	348.477
0.5	13.484	5.296	2.467	68.917	17.897	8.094	101.934	58.303	26.509
1	1.965	1.362	1.055	7.010	4.161	2.689	18.436	8.232	2.959
1.5	1.202	1.018	1	3.501	2.459	1.831	5.276	2.209	1.139
2	1.013	1	1	2.420	2.033	1.260	2.232	1.165	1
2.5	1	1	1	1.962	1.506	1.009	1	1	1

5. Application Comparison Progressive Max chart and Conventional Control Diagrams.

Progressive Max consisting of subgroups including 2, 3 and 5 in monitoring synthesis data and water pH data in Vannamei shrimp ponds located in Madura, Indonesia.

a. Application to Synthesized Data

In this section, the Progressive Max control chart is applied to the synthesized data. There are random variables consisting of 3 subgroups and generated from the univariate normal distribution $N(\mu, \Sigma)$. Each random variable is generated as many as 100 samples and there are 3 scenarios. In Scenario I, there is a shift in the mean while the variance remains constant, where a total of 70% of the generated observations are used as in-control data defined as Dataset 1 and the other part is used as out-of-control data defined as Dataset 2 for 30% of the observations. Dataset 1 is generated with $X_{in} \sim N(0,$

1), while for large shifts dataset 2 is generated with X_{out}~N(3, 1), medium shifts with X_{out}~N(2, 1), and small shifts with X_{out}~N(1.5, 1). The monitor results through the application of Progressive Max control charts with scenarios I with large shifts are shown in Figure 4. Based on the performance evaluation results in Figure 4, it can be seen that the value of the subgroup size (n = 3) with scenario I provides optimal performance at various levels of mean shift with the data X_{in}~N(0, 1) to X_{out}~N(3, 1).

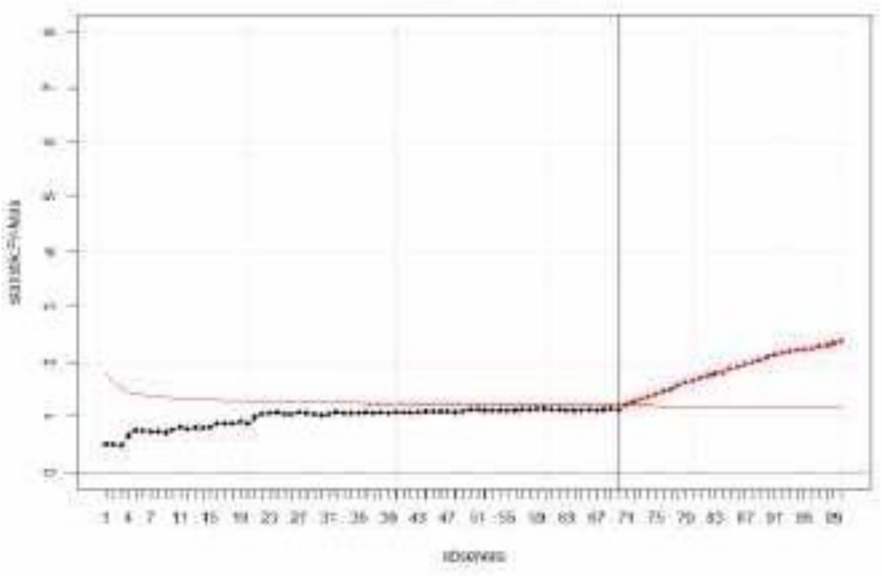

Figure 4. Progressive Max Control Diagram on size (n = 3) with scenario I (Mean shift)

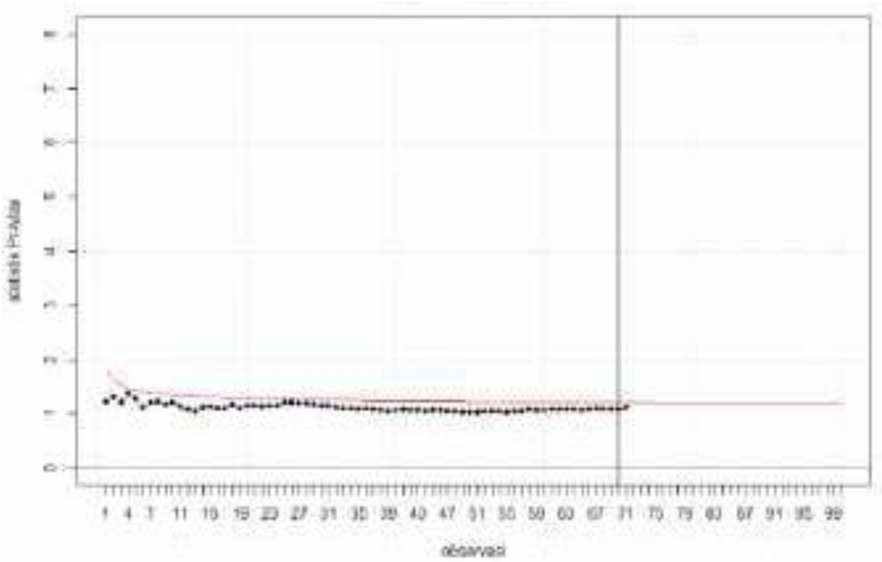

Figure 5. Progressive Max Control Diagram on size (n = 3) with scenario II (Variance shift)

In Scenario II, there is a shift in the variance while the mean remains constant where a total of 70% of the generated observations are used as in-control data defined as Dataset 1 and the other part is used as out-of-control data defined as Dataset 2 for 30% of the observations. Dataset 1 is generated with X_{in}~N(0, 1) , while for large shifts dataset 2 is generated with $X_{out} \sim N(0, 4)$, medium shifts with $X_{out} \sim N(0, 3)$, and small shifts with $X_{out} \sim N(0, 2.5)$. The monitor results through the application of Progressive Max control charts with scenarios II with large shifts are shown in Figure 5. Based on the performance evaluation results in Figure 5, it can be seen that the value of the subgroup size (n = 3) with scenario I provides optimal performance at various levels of variance shift with the data $X_{in} \sim N(0, 1)$ to $X_{out} \sim N(0, 4)$.

In Scenario III, there is a shift in the mean and variance where a total of 70% of the generated observations are used as in-control data defined as Dataset 1 and the other part is used as out-of-control data defined as Dataset 2 for 30% of the observations. Dataset 1 is generated with $X_{in} \sim N(0, 1)$, while for large shifts dataset 2 is generated with $X_{out} \sim N(3, 4)$, medium shifts with $X_{out} \sim N(2, 3)$, and small shifts with $X_{out} \sim N(1.5, 2.5)$. The monitor results through the application of Progressive Max control charts with scenarios III with large shifts are shown in Figure 6. Based on the performance evaluation results in Figure 6, it can be seen that the value of the subgroup size (n = 3) with scenario I provides optimal performance at various levels of mean and variance shift with the data $X_{in} \sim N(0, 1)$ to $X_{out} \sim N(3, 4)$.

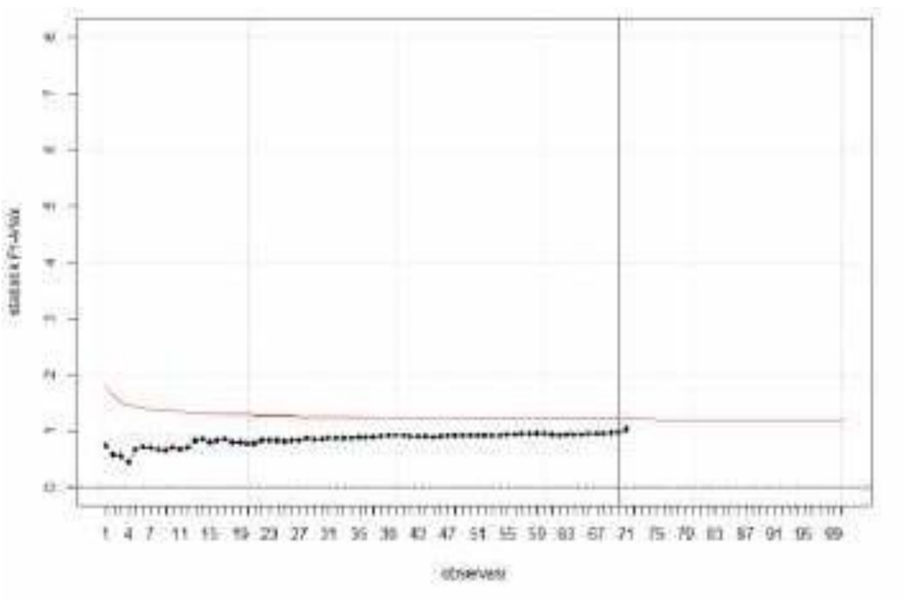

Figure 6. Progressive Max Control Diagram on size (n = 3) with scenario III (Mean and Variance shift)

b. Application to PH Data

In this section, the Progressive Max control diagram is applied to the pH data of Madura vannamei shrimp pond water. There is a random variable consisting of 2 subgroups. The monitor results through the application of Progressive Max control diagram in detecting out-of-control signals are shown in Figure 7. Based on the performance evaluation results in Figure 7, it can be seen that the ([= 2) subgroup size does not detect out-of-control signals on water pH data or data in an in-control state.

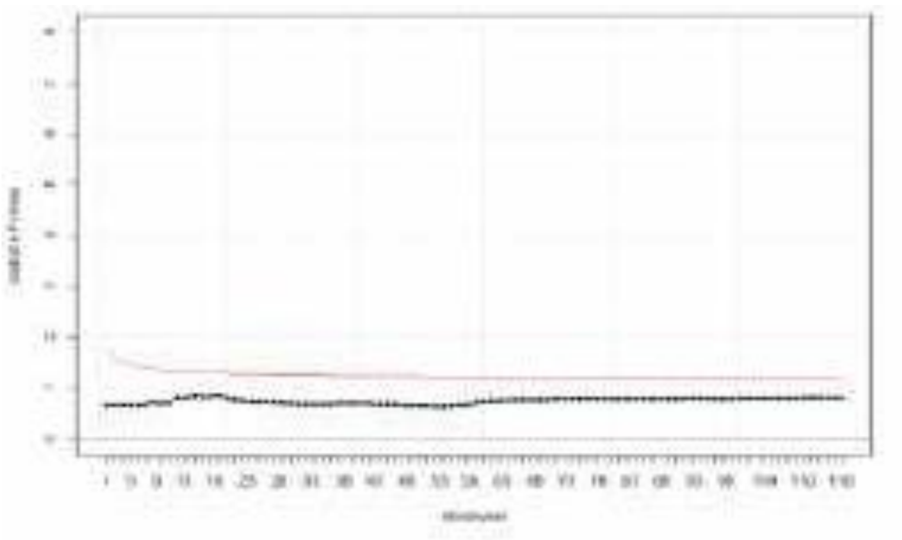

Figure 7. Application of Progressive Max Control Diagram on Water pH for (n = 2)

From all the research explained, it can be concluded that this research demonstrates that the Progressive Max control chart performs superiorly compared to EWMA-Max chart and Max chart in detecting process shifts, whether in mean shifts, variance shifts, or combinations of both, particularly for large shifts above 0.25. Its control statistics employ a cumulative mean that progressively incorporates all measurement data while maintaining historical data, with the upper control limit (UCL) calculated based on specific parameters. When applied to pH data of vannamei shrimp pond water, all three control charts (Progressive Max chart, EWMA-Max chart, and Max chart) showed in-control results, indicating that the process remained in a controlled state.

This research align with and build upon previous research by demonstrating the superiority of the Progressive Max control chart over other control charts in detecting process shifts. The researchers agree with previous research that highlighted the effectiveness of Max- based charts in monitoring mean and variability shifts. The Progressive Max chart's use of a cumulative mean that incorporates all measurement data while retaining historical data further refines the approach, supporting the trend in previous research toward more sensitive and efficient control charts. Additionally, the in-control results observed in the pH data of vannamei shrimp pond water are consistent with the practical applications noted in prior studies, reinforcing the reliability of these methods in real-world scenarios. Thus, the study's conclusions not only agree with but also advance the understanding of simultaneous control charts by introducing a more robust alternative.

D. CONCLUSION AND SUGGESTIONS

Progressive Max control charts work better than EWMA-Max and Max-chart control charts on small shifts in terms of mean, variance, and mean variance, and on large shifts in terms of mean, variance, and mean variance on Progressive Max control charts are also better than EWMA-Max and Max-chart control charts. This is characterized by the resulting ARL_1value getting closer to the value of 1 based on shifts in mean, variance, and mean variance. The application of the pH condition of vannamei shrimp pond water to the Progressive Max chart, EWMA-Max chart and Max chart control charts results in all in-control data or data in a controlled state.

ACKNOWLEDGEMENT

The authors express gratitude to CV Air Mas Jaya for providing data and supporting this research.

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