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Analysis of Online Game Addiction with Crowley-Martin Incident Rate Function

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ABSTRACT

This study aims to build and analyze a new mathematical model of online game addiction with the Crowley-Martin type incidence rate function approach. This research is categorized as a theoretical-quantitative study using mathematical modeling as its primary approach. The research instruments used include symbolic computation, simulation software, and parameter estimation techniques derived from literature. Stability analysis is conducted through Jacobian linearization, the Routh-Hurwitz criterion, and the Next Generation Matrix method to calculate the basic reproduction number. Optimal control is formulated using Pontryagin's Minimum Principle with two strategies: parental guidance and counseling therapy. Data analysis combines analytical techniques in stability and control theory with numerical simulations to evaluate the system. The results show that: The addiction-free fixed point T_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$, the addiction fixed point T^* is locally asymptotically stable if $\mathcal{R}_0 > 1$. Numerical simulations demonstrate that combined control strategies effectively reduce the number of exposed and addicted individuals.

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A. INTRODUCTION

In recent years, online gaming has grown rapidly around the world, and Indonesia is no exception. According to a report by the Ministry of Communication and Information Technology, the number of gamers in Indonesia has reached more than 170 million people across various platforms, showing significant growth in recent years (Kementerian Komunikasi dan Informatika, 2022). Data from the Indonesian Internet Service Providers Association (APJII) in 2024 noted that internet penetration in Indonesia has reached 79.5%, with the majority of users coming from Generation Z and millennials (Asosiasi Penyelenggara Jasa Internet Indonesia, 2024). This high internet access has contributed to the increasing number of online game players in the country. Factors driving this increase include the ease of internet access, the availability of affordable mobile devices, and the development of increasingly sophisticated and attractive game technology (Paulus et al., 2018).

Despite its popularity, online gaming addiction has adverse health effects. Individuals with gaming addiction tend to experience emotional disturbances such as depression, anxiety, and social isolation, and exhibit symptoms similar to substance addiction such as mood swings and uncontrollable impulsive behavior (Anderson et al., 2016; Gros et al., 2020; Jeong et al., 2020).

Excessive gaming activity has also been linked to sleep disturbances, decreased academic performance, and physical health problems such as muscle and joint pain, obesity, visual impairment, and even the risk of heart problems due to lack of rest (Djannah et al., 2021; King et al., 2020). In addition, increased aggression and impulsivity, especially from exposure to violent content in games, can exacerbate dependence and create a cycle of maladaptive behavior that is difficult to break, amplifying vulnerability to stress and social relationship dysfunction (Gentile et al., 2011; Jeong et al., 2020; Kuss et al., 2018).

In controlling the negative impact of online gaming addiction, it is important to understand the dynamics of the spread of online gaming addiction (Kuss & Griffiths, 2012). The spread of online gaming addiction can be simplified with a mathematical model approach (Guo & Li, 2020; Li & Guo, 2022). Mathematical models make it possible to analyze the spread of online gaming habits in the population. One relevant approach is a model that groups the population into categories. This model is inspired by epidemiology, where the spread of disease is analogous to the spread of gaming habits.

Research on mathematical models of online game addiction has existed before. Li developed a mathematical model of online game addiction by dividing the population into four compartments (susceptible, addicted, professional, and quit) and applied Pontryagin's maximum principle to formulate optimal control strategies through education and treatment (Li & Guo, 2019). Viriyapong examined a mathematical model to investigate the dynamics of online game addiction in children and adolescents in Thailand (Viriyapong & Sookpiam, 2019). Guo developed and analyzed a two-stage mathematical model of online game addiction and determined the optimal control strategy to minimize the number of addicts (Guo & Li, 2020). Seno developed a mathematical model based on a system of differential equations to describe the population dynamics of online game addiction, considering the influence of social interaction as a major factor in the transition from moderate use to addiction, and evaluated the impact of intervention effectiveness and timing on the stability of the addict population (Seno, 2021). Li discusses the development and analysis of mathematical models of online game addiction with positive and negative media influences and optimal control strategies to significantly reduce the level of addiction (Li & Guo, 2021). Guo developed a dynamic model of gaming addiction with optimal control to minimize the prevalence of addiction through family education, isolation, and treatment (Guo & Li, 2022). Li discusses the development and analysis of a mathematical model to describe the dynamics of online gaming addiction, specifically considering the phenomenon of incomplete recovery and using data in China (Li & Guo, 2022).

The key to this model lies in the incidence rate function, which describes the interaction between groups such as susceptible and addicted. In previous studies, it is common to use a linear function $f(S, I) = \beta SI$ to measure the transmission rate of online gaming habits (Guo & Li, 2022; Li & Guo, 2022; Viriyapong & Sookpiam, 2019). This function illustrates that the transmission rate increases as the number of susceptible and addicted people increases. However, in the case of the transmission of online gaming habits, an increase in the number of susceptible or addicted does not necessarily make the transmission rate increase. Therefore, in this study, the Crowley-Martin function (Kumar & Nilam, 2019) is used, which considers the social barrier factor in the spread of addiction. Unlike the bilinear function, this function is more realistic as not everyone will be affected even if many of their friends play games and it takes into account limiting factors, such as limited social scope or awareness of the negative effects of gaming.

This study aims to analyze the stability and optimal control of the online game addiction model to identify the conditions under which addiction can be suppressed or eliminated. Through the optimal control approach, this study also seeks to find intervention strategies with minimal cost but effective in reducing the number of addicted individuals. The control strategies used in this study include parental guidance and counseling therapy. Parental guidance plays an important role because the family is the first environment that shapes children's behavior. Through supervision, good communication, and limited play time, parents can prevent the development of addictive behaviors early on. Meanwhile, counseling therapy is aimed at providing psychological support to individuals who have shown symptoms of addiction, by helping them understand the root of the problem, develop healthy coping mechanisms, and rebuild motivation to live a more balanced life. To assess the effectiveness of both strategies, numerical simulations were conducted to predict population dynamics over a period of time and evaluate the impact of the interventions.

B. METHODS

This study begins by developing a new model related to the dynamics of online game addiction in the form of a system of nonlinear differential equations. The model is formed into five compartments: Susceptible (S), Exposed (E), Addiction (I), Permanent Quitter (R), and Not Interested in Game Online (W). S represents individuals who are potentially exposed to online game addiction. Dynamically, S can move to E if it experiences influence from I through Martin's Crowley function. E is a group that has exposure to online gaming and its social role is to act as a bridge between S and I. Group I are individuals who are addicted to online games and can transmit addiction through social interactions. R are individuals who have completely stopped playing online games. W represents individuals who have no interest in online gaming and are socially immune to the influence of addiction.

The next step in this research method is to analyze the equilibrium point and local stability of the system. Fixed points are determined by finding stationary solutions of the model system, then analyzed for stability through linearization using the Jacobian matrix, as well as the application of the Routh-Hurwitz criterion to evaluate the eigenvalues of the system. In addition, the basic reproduction number (R_0) was calculated using The Next Generation Matrix method (Van Den Driessche & Watmough, 2002), to assess the potential spread of addiction in the population. An optimal control formulation was developed using Pontryagin's Minimum Principle (Ali et al., 2016; Boscain et al., 2021; Guo & Li, 2022), involving two intervention strategies: intensity of parental guidance to vulnerable individuals, and counseling therapy for addicted individuals. The objective function is designed to minimize the number of addicted individuals and reduce the cost of implementing the control. A Hamiltonian function is formed to obtain the optimal condition used to determine the best form of control.

This research is categorized as a theoretical-quantitative study with the main approach being mathematical modeling. The research instruments used include symbolic computing, simulation software, and parameter estimation techniques taken from previous literature studies. The data analysis technique applied is a combination of analytical methods in stability and control theory, as well as numerical simulation approaches to evaluate system behavior under various intervention scenarios. Simulations were conducted to visualize the impact of implementing parental guidance strategies and counseling therapy on reducing the number of E and I individuals. The results of numerical simulation are used to evaluate the effectiveness of control strategies in reducing the prevalence of addiction at minimum cost.

C. RESULT AND DISCUSSION

1. Mathematical Model

In the formation of the cigarette addiction spread model, the population is divided into 5 groups, namely: Susceptible (S), Exposed (E), Addiction (I), Permanent Quitter (R), and Not Interested in Online Game (W). In simplifying the model for the spread of online game addiction, the following assumptions are given: 1). S states the group of individuals who are susceptible to online game addiction. 2). E states the group of people exposed to less exposure to games (Li & Guo, 2022). 3). I states that the group is addicted to online games (Li & Guo, 2022) and can transmit addiction to other individuals. 4). R states the group of individuals who permanently stop playing online games and cannot become susceptible again. 5). W states that the group of individuals who permanently stop playing online games and cannot become susceptible again. 6). The transmission rate uses the Crowley-Martin function $f(S,I) = \frac{\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)}$. The Crowley-Martin function shows that transmission does not increase infinitely even if the number of addicted and susceptible individuals increases. Based on these assumptions, the mathematical model of the spread of online game addiction is:

$$\frac{dS(t)}{dt} = C - f(S,I) - (\mu+\delta)S(t)$$
$$\frac{dE(t)}{dt} = f(S,I) - (\mu+\theta)E(t)$$
$$\frac{dI(t)}{dt} = \theta E(t) - (\mu+\omega)I(t) \quad (1)$$
$$\frac{dQ(t)}{dt} = \omega I(t) - (\mu+\gamma)Q(t)$$
$$\frac{dW(t)}{dt} = \delta S(t) + \gamma Q(t) - \mu W(t)$$

where $S(t) \geq 0,\ E(t) \geq 0, I(t) \geq 0,\ Q(t) \geq 0,\ W(t) \geq 0$ dan $N(t) = S(t) + E(t) + I(t) + Q(t) + W(t)$ is the total population. The parameter description of the system (1) is described in the following Table 1.

Table 1. Parameters of The System (1)

Parameter	Description	Unit
C	Number of new susceptible individuals	Person/year
α_1	The inhibiting factors of susceptible	1/ year
α_2	The inhibiting factor of addiction	1/ year
β	Rate of contact effectiveness	1/(person × year)
θ	Transfer rate from E to I	1/ year
ω	Transfer rate from I to Q	1/ year

Parameter	Description	Unit
γ	Transfer rate from Q to W	1/ year
δ	Transfer rate from S to W	1/ year
μ	natural death rate	1/ year

The solution area of the spread of online game addiction in the system (1) is nonnegative and finite for all time. This can be shown based on the following lemma 1.

Lemma 1. The set $\Omega = \left\{(S(t), E(t), I(t), Q(t), W(t)) \in \mathbb{R}_+^5 : 0 \leq S(t) + E(t) + I(t) + Q(t) + W(t) \leq \frac{C}{\mu} + N_0\right\}$ is a non-negative, finite solution of the system where N_0 is the total population at time t_0.

Proof. Suppose $N(t) = S(t) + E(t) + I(t) + Q(t) + W(t)$, based on system (1), $\frac{dN}{dt} = C - \mu N$ is obtained so that $N(t) = \frac{C}{\mu}(1 - e^{-\mu t}) + N_0 e^{-\mu t}$. Since $0 < e^{-\mu t} \leq 1$ for every $t \geq 0$, $N \leq \frac{C}{\mu} + N_0$ is obtained. Since $S(t), I(t), C(t), R(t)$, dan $D(t)$ non-negative, for every $t \geq 0$, $0 \leq S(t) + E(t) + I(t) + Q(t) + W(t) \leq \frac{C}{\mu} + N_0$. This shows that the solution of the system is nonnegative and finite.

2. Fixed Point and Basic Reproduction Number

a. Fixed Point without Control

The fixed point of the system is obtained by $\frac{dS(t)}{dt} = \frac{dE(t)}{dt} = \frac{dI(t)}{dt} = \frac{dR(t)}{dt} = \frac{dW(t)}{dt} = 0$, so two fixed points are obtained, namely the fixed point without addiction (T_0) and addiction (T^*). Fixed point $T_0(S_0, E_0, I_0, Q_0, W_0) = T_0\left(\frac{C}{\mu+\delta}, 0,0,0, \frac{\delta C}{\mu(\mu+\delta)}\right)$ and $T^*(S^*, E^*, I^*, Q^*, W^*)$

where

$$S^* = \frac{\alpha_2 \theta C + (\mu + \theta)(\mu + \omega)}{\left(\theta\beta + \alpha_2\theta(\mu + \delta) - \alpha_1(\mu + \theta)(\mu + \omega)\right)}$$

$$I^* = \frac{\theta\left(\theta\beta C - (\mu + \delta + \alpha_1 C)(\mu + \theta)(\mu + \omega)\right)}{(\mu + \theta)(\mu + \omega)\left(\theta\beta + \alpha_2\theta(\mu + \delta) - \alpha_1(\mu + \theta)(\mu + \omega)\right)}$$

$$E^* = \frac{(\mu + \omega)I^*}{\theta}, Q^* = \frac{\omega I^*}{\mu + \gamma}, W^* = \frac{\delta S^* + \gamma Q^*}{\mu},$$

b. Basic Reproduction Number

From system (1), the infection class equation is rewritten, namely

$\frac{dE(t)}{dt} = \frac{\beta S(t) I(t)}{1 + \alpha_1 S(t) + \alpha_2 I(t)} - (\mu + \theta)E(t), \frac{dI(t)}{dt} = \theta E(t) - (\mu + \omega)I(t)$

Based on the system (1), the functions $\mathcal{F}_i$ and $\mathcal{V}_i$ are obtained as follows

$$\mathcal{F}_i = \begin{bmatrix} \frac{\beta S I}{1 + \alpha_1 S + \alpha_2 I} \\ 0 \end{bmatrix}, \mathcal{V}_i = \begin{bmatrix} (\mu + \theta)E \\ -\theta E + (\mu + \omega)I \end{bmatrix}$$

The Jacobi matrices of functions $\mathcal{F}_i$ and $\mathcal{V}_i$ evaluated at T_0 are

$$M = \begin{bmatrix} 0 & \dfrac{\beta C}{(\mu+\delta+\alpha_1 C)} \\ 0 & 0 \end{bmatrix}, N = \begin{bmatrix} (\mu+\theta) & 0 \\ -\theta & (\mu+\omega) \end{bmatrix}$$

The basic reproduction number is obtained from the dominant eigenvalue of the MN^{-1} matrix, i.e. $\mathcal{R}_0 = \frac{\beta\theta C}{(\mu+\delta+\alpha_1 C)(\mu+\theta)(\mu+\omega)}$.

3. Stability

Model (1) is difficult to solve analytically, so this model is linearized by the Taylor series, the Jacobi matrix is obtained as follows.

$$J = \begin{bmatrix} -\left(\frac{\beta I(1+\alpha_2 I)}{(1+\alpha_1 S+\alpha_2 I)^2}+\mu+\delta\right) & 0 & -\frac{\beta S(1+\alpha_1 S)}{(1+\alpha_1 S+\alpha_2 I)^2} & 0 & 0 \\ \frac{\beta I(1+\alpha_2 I)}{(1+\alpha_1 S+\alpha_2 I)^2} & -(\mu+\theta) & \frac{\beta S(1+\alpha_1 S)}{(1+\alpha_1 S+\alpha_2 I)^2} & 0 & 0 \\ 0 & 0 & -(\mu+\omega) & 0 & 0 \\ 0 & \theta & \omega & -(\mu+\gamma) & 0 \\ \delta & 0 & 0 & \gamma & -\mu \end{bmatrix} \quad (2)$$

The stability of the fixed point is analyzed as follows according to the theorem.

Theorem 1. The non-addicted fixed point T_0 of the system (1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof. The characteristic equation of (2) evaluated at the fixed point T_0 is

$$(\lambda+\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\delta)(\lambda^2+\zeta_1\lambda+\zeta_2) = 0 \quad (3)$$

with $\zeta_1 = (\mu+\theta)+(\mu+\omega) > 0, \zeta_2 = (\mu+\theta)(\mu+\omega) - \frac{\beta\theta C}{\mu+\delta+\alpha_1 C}$.

Some roots of equation (3) are λ_1=$-\mu$, λ_2=$-(\mu+\gamma)$, λ_3=$-(\mu+\delta)$, and the other roots can be determined by the Routh-Hurwitz criterion. If $R_0 < 1$ means $(\mu+\theta)(\mu+\omega) - \frac{\beta\theta C}{(\mu+\delta+\alpha_1 C)} > 0$, k_2>0 are obtained. Based on this explanation, it is clear that $\zeta_1, \zeta_2 > 0$. Because $\zeta_1, \zeta_2 > 0$ then based on the Routh-Hurwitz criterion (Brauer & Castillo-Chavez, 2011) obtained $\lambda_{4,5} < 0$. This shows that Re(λ_i)< 0 for i=1,2,3,4,5 if $R_0 < 1$, so the stability of the fixed point without addiction is locally asymptotically stable if $R_0 < 1$. **Theorem proved**.

Theorem 2. The addicted fixed point T^* of the system (1) is locally asymptotically stable if $R_0 > 1$.

Proof. The characteristic equation of (2) evaluated at the fixed point T^* is

$$(\lambda+\mu)(\lambda+\mu+\gamma)(\lambda^3+\zeta_1\lambda^2+\zeta_2\lambda+\zeta_3) = 0 \quad (4)$$

with

$$a = \frac{\beta I^*(1+\alpha_2 I^*)}{(1+\alpha_1 S^*+\alpha_2 I^*)^2}+\mu+\delta, b = (\mu+\theta) > 0, c = (\mu+\omega) > 0, d = \frac{\beta S^*(1+\alpha_1 S^*)}{(1+\alpha_1 S^*+\alpha_2 I^*)^2},$$

$$e = \frac{\beta I^*(1+\alpha_2 I^*)}{(1+\alpha_1 S^*+\alpha_2 I^*)^2}, \zeta_1 = a+b+c, \zeta_2 = ab+ac+bc-d\theta, \zeta_3 = abc-ad\theta.$$

Some roots of equation (4) are λ_1=$-\mu$, λ_2=$-(\mu+\gamma)$, and the other roots can be determined by the Routh-Hurwitz criterion. If $R_0 > 1$ means $\theta\beta - \alpha_1(\mu+\theta)(\mu+\omega) > 0$ and $\theta\beta C - (\mu+\delta+\alpha_1 C)(\mu+\theta)(\mu+\omega) > 0$, we get $S^* > 0$, $I^* > 0, a > 0, d > 0, e > 0$, and $\zeta_1 = a + b + c > 0$. Since $bc - d\theta = \frac{\beta\theta S^*}{1+\alpha_1 S^*+\alpha_2 I^*}\left(\frac{\alpha_1 S^*+\alpha_2 I^*}{1+\alpha_1 S^*+\alpha_2 S^*}\right) + \frac{\alpha_1\beta\theta S^{*2}}{1+\alpha_1 S^*+\alpha_2 I^*} > 0$ then
$\zeta_2 = ab + ac + bc - d\theta > 0$ and $\zeta_3 = a(bc - d\theta) > 0$. Next,
$\zeta_1\zeta_2 = (b+c)(ab+ac+bc-d\theta) + a(ab+ac) + a(bc-d\theta) > a(bc-d\theta) = \zeta_3$

Since $\zeta_1, \zeta_2, \zeta_3 > 0$ and$\zeta_1\zeta_2 > \zeta_3$, $Re(\lambda_{4,5})$ is negative according to the Routh-Hurwitz criterion (Brauer & Castillo-Chavez, 2011). This shows that $Re(\lambda_i) < 0$ for $i = 1,2,3,4,5$ if $\mathcal{R}_0 > 1$, so the stability of the fixed point T^* is locally asymptotically stable if $\mathcal{R}_0 > 1$. **Theorem proved.**

4. Optimal Control

The optimal control of the system is solved to minimize the number of individuals playing online games with parental guidance (u_1) and counseling therapy (u_2) strategies. With the addition of control strategies, the system becomes

$$\begin{aligned}
\frac{dS(t)}{dt} &= C - \frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)} - (\mu+\delta)S(t) \\
\frac{dE(t)}{dt} &= \frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)} - (\mu+\theta)E(t) \\
\frac{dI(t)}{dt} &= \theta E(t) - (\mu+\omega+u_2)I(t) \\
\frac{dQ(t)}{dt} &= (\omega+u_2)I(t) - (\mu+\gamma)Q(t) \\
\frac{dW(t)}{dt} &= \delta S(t) + \gamma Q(t) - \mu W(t)
\end{aligned}$$

The minimized objective function is defined as follows

$$J = min\left(\int_{t_0}^{t_f}\left[A_1 E(t) + A_2 I(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t)\right]dt\right)$$

Where the optimal control is at $0 \le u_1 \le 1, 0 \le u_2 \le 1, 0 \le t \le t_f$ with t_f the end time of control, coefficient A_1 is the objective weight value to reduce the Exposed population (E), coefficient A_2 is the objective weight value to reduce the addicted population (I), coefficient B_1 is the objective weight value of the control function u_1, coefficient B_2 is the objective weight value of the control function u_2. Thus, the equation presents a list of control functions $(u_1^*, u_2^*) \in U$. In these equations, an optimization problem is given, and then u^* is determined that satisfies $J(u^*) = \min\{J(u): u \in U\}$. The search for u^* using the Pontryagin minimum principle is to determine the Hamiltonian function of the objective function. The general form of the Hamiltonian function is

$$H(t,x,u,\lambda) = f(t,x,u) + \lambda^T(t)g(t,x,u)$$

with

$$f(t,x,u) = A_1E(t) + A_2I(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t)$$

and the Lagrange multiplier of the equation

$$\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5]^T$$

then the Hamiltonian function can be decomposed

$$\begin{aligned} H = A_1E(t) + A_2I(t) + \frac{B_1}{2}u_1^2(t) \\ &+ \frac{B_2}{2}u_2^2(t) + \lambda_1\left(C - \frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)}\right. \\ &\left. - (\mu+\delta)S(t)\right) + \lambda_2\left(\frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)} - (\mu+\theta)E(t)\right) + \lambda_3\big(\theta E(t) \\ &- (\mu+\omega+u_2)I(t)\big) + \lambda_4\big((\omega+u_2)I(t) - (\mu+\gamma)Q(t)\big) + \lambda_5\big(\delta S(t) + \gamma Q(t) \\ &- \mu W(t)\big) \end{aligned}$$

The state equation is obtained

$$\begin{aligned} \frac{\partial H}{\partial \lambda_1} &= C - \frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)} - (\mu+\delta)S(t) \\ \frac{\partial H}{\partial \lambda_2} &= \frac{(1-u_1)\beta S(t)I(t)}{1+\alpha_1 S(t)+\alpha_2 I(t)} - (\mu+\theta)E(t) \\ \frac{\partial H}{\partial \lambda_3} &= \theta E(t) - (\mu+\omega+u_2)I(t) \\ \frac{\partial H}{\partial \lambda_4} &= (\omega+u_2)I(t) - (\mu+\gamma)Q(t) \\ \frac{\partial H}{\partial \lambda_5} &= \delta S(t) + \gamma Q(t) - \mu W(t) \end{aligned}$$

The costate equation is obtained

$$\begin{aligned} \dot{\lambda}_1 &= -\frac{\partial H}{\partial S} = \left(\frac{(1-u_1)\beta I(1+\alpha_2 I)}{(1+\alpha_1 S+\alpha_2 I)^2}\right)(\lambda_1-\lambda_2) + (\mu+\delta)\lambda_1 - \delta\lambda_5 \\ \dot{\lambda}_2 &= -\frac{\partial H}{\partial E} = -A_1 + (\mu+\theta)\lambda_2 - \theta\lambda_3 \\ \dot{\lambda}_3 &= -\frac{\partial H}{\partial I} = -A_2 + \left(\frac{(1-u_1)\beta S(1+\alpha_1 S)}{(1+\alpha_1 S+\alpha_2 I)^2}\right)(\lambda_1-\lambda_2) + (\mu+\omega+u_2)\lambda_3 - (\omega+u_2)\lambda_4 \\ \dot{\lambda}_4 &= -\frac{\partial H}{\partial Q} = (\mu+\gamma)\lambda_4 - \gamma\lambda_5 \end{aligned}$$

$$\dot{\lambda}_5 = -\frac{\partial H}{\partial W} = \mu\lambda_5$$

The optimal forms of u_1 and u_2 obtained using the stationary condition are given by

$$u_1^* = min\left\{max\left\{0, \frac{(\lambda_2-\lambda_1)\beta S(t)I(t)}{B_1(1+\alpha_1 S(t)+\alpha_2 I(t))}\right\}, 1\right\}, u_2^* = min\left\{max\left\{0, \frac{(\lambda_3-\lambda_4)I}{B_2}\right\}, 1\right\}$$

5. Numerical Simulation

In performing numerical simulations on the system (1), it is assumed that $S(0)$=10 thousand, $E(0)$=8 thousand, $I(0)$=6 thousand, $Q(0)$=5 thousand, and $W(0)$=20 thousand. The number of new susceptible individuals (C) is 0.5 thousand, as shown in Table 2.

Table 2. Parameter of system (1)

Parameter	Value	Unit	Reference
C	0.50	Person/Years	Estimate
α_1	0.12	1/Years	Estimate
α_2	0.12	1/Years	Estimate
θ	0.1316	1/Years	(Li & Guo, 2022)
ω	0.0404	1/Years	(Li & Guo, 2022)
γ	0.15	1/Years	Estimate
δ	0.12	1/Years	Estimate
μ	0.0143	1/Years	Estimate

a. Numerical simulation of fixed point stability

Using the parameter values in Table 2 and β = 0.01, the values of R_0=0.42433 and T_0 =(7.46268, 0, 0, 0, 63.96588) are obtained. The dynamics of each compartment are shown in the Figure 1 below.

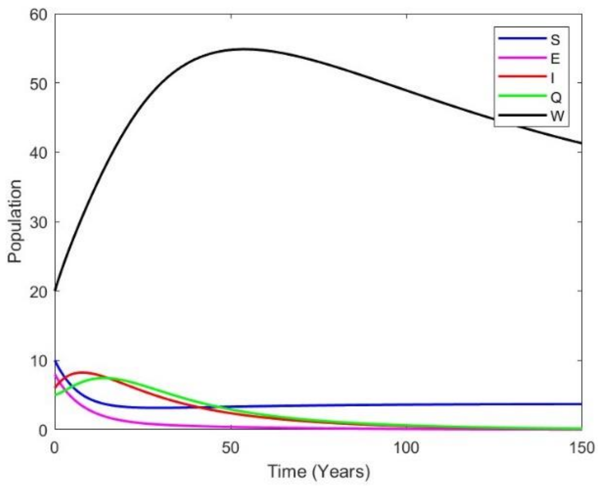

Figure 1. Solution curve of system (1) with $\mathcal{R}_0 < 1$.

In Figure 1, it can be seen that at the beginning of the simulation, all compartments experience population fluctuations that reflect temporary disturbances due to initial imbalances. However, as time passes, each compartment shows a tendency to converge towards a fixed point. In particular, the number of individuals in the addiction compartment decreases gradually until it approaches zero. This suggests that the addicted population can be eliminated from the system in the long run. This dynamic behavior is in line with the results of the local stability analysis as stated in Theorem 1. Using the parameter values in Table 2 and β = 0.1, the values of R_0=4.24336 and $T^* = (1.65525, 5.34475, 12.92957, 3.18509, 48.31389)$ are obtained. The dynamics of each compartment are shown in the Figure 2 below.

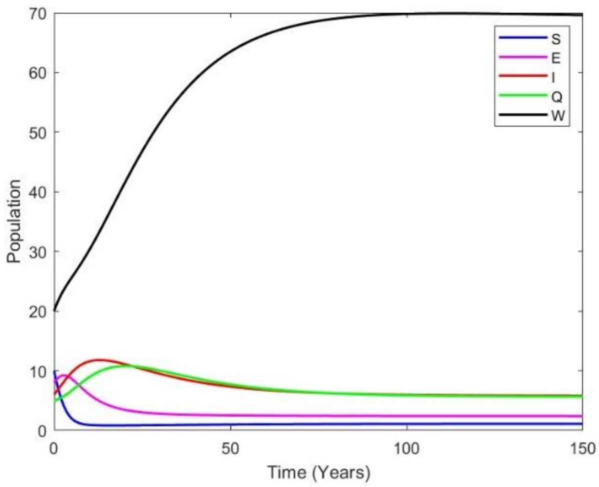

Figure 2. Solution curve of system (1) with $\mathcal{R}_0 > 1$.

Figure 2 shows that at the beginning of the simulation, all compartments experience fluctuations due to the uneven initial distribution of individuals. Over time, the system moves towards an equilibrium point. That is, although the number of individuals in each compartment changes, the population eventually reaches a steady state. Specifically, the number of individuals in the addiction compartment does not vanish, but remains at a relatively constant number. This suggests that despite interventions or behavioral changes, online gaming addiction will persist in the long term. This finding is in line with the analytical result in Theorem 2, which proves the local asymptotic stability of the endemic equilibrium point.

b. Numerical simulation of optimal control

In this section, numerical simulations are performed to demonstrate the effectiveness of the control. Parameters such as Table 2 and β=0.1 were used. Strategies are carried out with parental guidance and counseling therapy. The weight values are assumed to be $A_1 = A_2 = 1;\ B_1 = 2;\ B_2 = 3$.

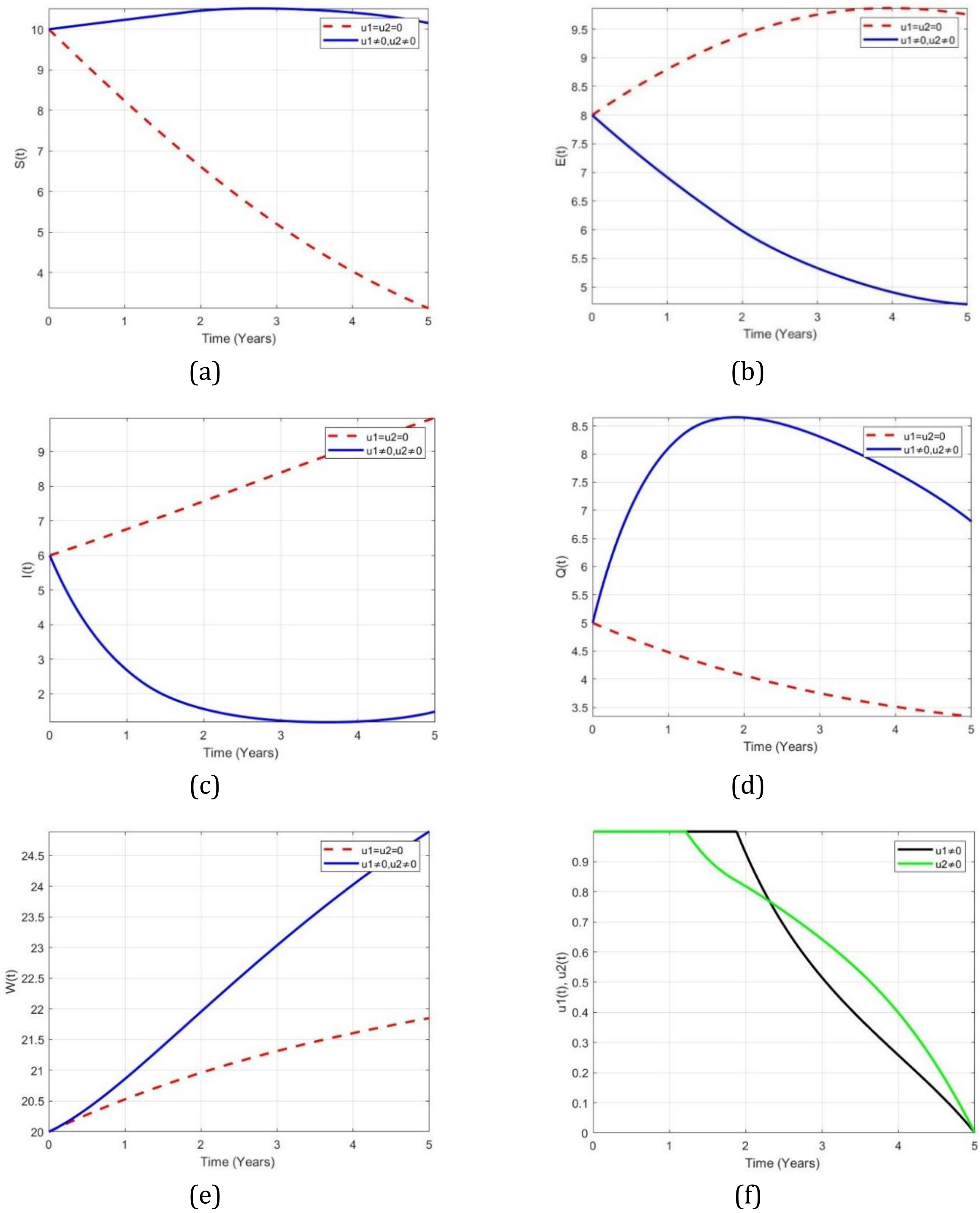

Figure 3. Comparison of system solutions with and without control

This strategy applies two interventions, namely parental guidance and counseling therapy. The simulation results in Figure 3 show that the number of individuals in the exposed and addicted groups decreased significantly compared to the no-intervention condition. This decrease is stable and sustained throughout the simulation period. This proves that the combination of parental guidance and counseling therapy is effective in reducing the prevalence of online game addiction at a manageable intervention cost. The importance of a close parent-child relationship as a protective factor that significantly reduces pathological symptoms of gaming addiction (Choo et al., 2015). Meanwhile, the effectiveness of counseling therapy, specifically cognitive behavioral therapy (CBT), has been empirically supported by (Stevens et al., 2019), who found that CBT significantly reduced online gaming addiction.

D. CONCLUSION AND SUGGESTIONS

This study develops a mathematical model of online game addiction using the Crowley-Martin incidence function that represents social interaction more realistically. Theoretical analysis shows that the addiction-free equilibrium point will be stable if $R_0 < 1$ meaning that the addiction will disappear from the population. Conversely, if $R_0 > 1$ then addiction will remain stably present. Simulation results support this finding. When beta = 0.01, which reflects the low influence of addiction, the number of addicts decreases to zero. However, when beta = 0.1, reflecting strong social influence, addiction persists and stabilizes. This shows that beta affects online game addiction in society.

The optimal control problem is formulated using Pontryagin's Minimum Principle by combining two intervention strategies: parental guidance and counseling therapy. Simulation results show that the combined control strategy significantly reduces the number of exposed and addicted individuals compared to the uncontrolled case. This shows that combining parental supervision and counseling therapy is efficient in minimizing the number of online game addictions with minimum cost.

These findings provide practical implications for policymakers, educators, and mental health practitioners. The model suggests that addiction reduction efforts should prioritize strengthening family-based interventions and professional counseling access, especially during early exposure phases. However, the model has limitations, such as assuming constant parameter values and homogeneous population behavior. Future development can integrate time-dependent parameters, stochastic elements, or age-structured populations to enhance realism. Despite these limitations, the current model offers a valuable framework for designing targeted and cost-effective strategies to address online gaming addiction in youth populations.

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