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Epistemological Obstacles in Solving PISA Adapted Problems on System of Linear Equations In Two Variables

Yunia Jumita Ningrum[1*], Dadan Dasari[1], Sufyani Prabawanto[1]
[1]Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia
yuniajumita@upi.edu

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ABSTRACT

This study aims to identify and analyze the epistemological obstacles encountered by junior high school students when solving PISA-based mathematical literacy problems on the topic of System of Linear Equations in Two Variables (SLETV), viewed from the perspective of PISA competency levels. The research seeks to contribute to the development of more effective mathematics instruction. This study is deemed essential because epistemological obstacles can hinder students' ability to apply mathematical concepts in real-world contexts an ability that is central to international assessments such as PISA. A qualitative approach was employed through Didactical Design Research (DDR), involving 23 ninth-grade students. Data were collected through a written test consisting of a mathematical literacy problem adapted from PISA items, and supported by interviews and classroom observations. The data were analyzed using qualitative descriptive methods, focusing on students' thinking processes and emerging error patterns. The findings reveal that students encounter various epistemological obstacles across all PISA competency levels (1b to 6), such as conceptual misconceptions, procedural errors, and difficulties in translating contextual information. At the lower levels (1b–3), students struggled to formulate basic algebraic models and perform arithmetic operations. At the higher levels (4–6), they experienced challenges in handling complex calculations, verifying solutions, and applying reasoning in abstract or multi-step situations. These obstacles stem from fragmented prior knowledge, limited exposure to contextual problems, and a lack of reflective habits. This study highlights the importance of integrating authentic real-world problems, providing systematic scaffolding, and fostering continuous self-verification practices in instructional design. The findings offer practical insights for developing targeted pedagogical interventions to enhance students' mathematical literacy and better prepare them to tackle SLETV problems in contexts similar to those featured in PISA.

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A. INTRODUCTION

Mathematical literacy is defined by the OECD and PISA as the ability to formulate, apply, and interpret mathematics in the context of real-world problems (Kabael & Baran, 2023; Manoy & Purbaningrum, 2021; OECD, 2022). Mathematical literacy has become increasingly crucial in the 21st century to bridge theoretical knowledge with practical application in daily life (Haara et al., 2017). It is measured by students' ability to formulate situations mathematically, apply mathematical concepts, and interpret and evaluate mathematical results. This not only supports problem-solving but also fosters critical and analytical thinking skills, which are vital in addressing global challenges and advancing educational innovation (She et al., 2018). Various studies have shown that learning approaches integrating real-life contexts, such as those adapted from PISA problems, contribute significantly to reducing psychological barriers, such as anxiety and low confidence in learning mathematics (Genc & Erbas, 2020). Additionally, the application of context-based tasks and realistic learning models has proven to enhance the impact of mathematics learning by linking theory to practice, thereby bringing mathematics learning closer to real-life issues and preparing students for life beyond the classroom (Dasaprawira et al., 2019; Oktiningrum et al., 2016; Sumirattana et al., 2017).

The PISA 2022 framework highlights the interconnectedness between mathematical reasoning, problem-solving (including mathematical modelling), mathematical content, real-life contexts, and 21st-century skills. Student performance in PISA is assessed through three core processes: (1) formulate, the ability to systematically formulate problems; (2) employ, the ability to apply mathematical concepts, facts, procedures, and reasoning to find solutions; and (3) interpret, the ability to interpret, apply, and evaluate mathematical outcomes (OECD, 2023). PISA 2022 places strong emphasis on linking abstract mathematical concepts to real-world situations, which lies at the heart of mathematical literacy. This is actualized through the integration of mathematical modelling as a central element of the assessment framework (Ikeda, 2015; Sosa-Nunez, 2022; Stacey, 2015). The primary focus of the PISA 2022 mathematics assessment is on mathematical reasoning, which encompasses three fundamental processes: (1) formulating real-world situations into mathematical representations, (2) employing mathematical concepts, facts, procedures, and reasoning, and (3) interpreting, applying, and evaluating mathematical results within their original contexts. PISA 2022 encompasses four contextual domains: personal, occupational, societal, and scientific. Students' competencies are classified into six levels, ranging from basic procedural skills (Levels 1–3) to complex and reflective problem solving (Levels 4–6) (OECD, 2023). The mathematical content in PISA 2022 is categorized into four main domains: Space and Shape, Change and Relationships, Uncertainty and Data, and Quantity (OECD, 2023).

The system of linear equations in two variables (SLETV) is a crucial topic in mathematical literacy, as it serves as a bridge between basic arithmetic and more advanced algebraic concepts (Andrews & Öhman, 2019; Zulfah, 2017). This topic falls under the Change and Relationships domain of the PISA 2022 framework, which emphasizes the ability to understand and represent relationships between variables in various forms(Thomson et al., 2013). In Indonesia's Merdeka Curriculum, SLETV is mandated to be taught in Phase D to develop students' abilities in solving contextual problems using multiple methods (Kemendikbudristek, 2022). Mathematical literacy within SLETV enables students to formulate and solve real-world problems, such as calculating the cost of essential goods, by accurately applying mathematical procedures (Amelia & Khotimah, 2025; Atuni et al., 2023; Thomson et al., 2013). A strong conceptual understanding of SLETV also enhances students' analytical abilities, which are vital for various practical situations (Assadi & Hibi, 2022; Rahmadiani et al., 2024).

Despite the significance of SLETV in promoting mathematical literacy, numerous studies have shown that both students and prospective teachers still encounter substantial challenges in understanding and applying its concepts. While mastery of solution techniques such as substitution and elimination can improve students' analytical skills, there remains a fragmentation in the application of mathematical literacy that impedes contextual problem-solving (Nugroho et al., 2024). Research by Amelia & Khotimah (2025) found that eighth-grade students' mathematical literacy skills in solving PISA-based SLETV problems varied across aspects such as communication, representation, problem-solving strategies, and reasoning. Similarly, Afriliziana & Kartini (2021) and Wahyuni et al. (2023) revealed that although conceptual understanding of SLETV was relatively strong, students still struggled with conceptual clarity, application of principles, mathematical representation, and contextual problem-solving. Consequently, continuous practice, the use of contextual problems, and the habitual reading and systematic writing of information are essential for enhancing students' comprehensive understanding and skills.

In mathematics education, learning obstacles refer to phenomena experienced by students when their learning outcomes are lower than previous achievements or when they face difficulties due to varying levels of prior knowledge (Sartika et al., 2024). Learning obstacles are generally classified into three types: epistemological, ontogenic, and didactical (Brousseau, 2002). Epistemological obstacles pertain to limitations in students' understanding that are effective only in specific contexts and may lead to difficulties in connecting, transferring, and interpreting knowledge when confronted with new situations (Maarif et al., 2020). Ontogenic obstacles arise when the level of instruction does not align with students' cognitive development either being too advanced to grasp or too basic to stimulate meaningful learning (Mahmud et al., 2023). Didactical obstacles result from inadequate delivery methods, the nature of the content, curriculum design, or instructional materials, which can cause confusion or misconceptions (Elisya et al., 2024; Rudi et al., 2022).

This study focuses on epistemological obstacles, which emerge from students' limited knowledge within specific contexts. These obstacles occur when conceptual understanding is partial and thus difficult to transfer to new situations (Mahmud et al., 2023; Sartika et al., 2024). Students' intuitive understandings that appear correct often contradict the actual structure of mathematical concepts (Murniasih et al., 2020). Such obstacles may hinder students' problem-solving processes related to SLETV, particularly in the context of mathematical literacy. Sumbandari et al. (2022) demonstrated that low abstraction ability, inadequate prerequisite knowledge, and insufficient didactical interventions contributed to the emergence of epistemological obstacles in the context of SLETV. These findings underscore the need for instructional approaches and learning materials that facilitate students' progression from concrete to abstract thinking. In line with Maarif et al. (2020), epistemological obstacles in solving SLETV are also attributed to incomplete understanding of the context, including conceptual, procedural, and operational aspects.

Based on the above discussion, it can be concluded that although SLETV plays a vital role in mathematical literacy, many students and prospective teachers still struggle to comprehend it, especially in literacy-based tasks such as those found in PISA assessments. Epistemological obstacles are among the key factors, as they limit the transfer and application of mathematical concepts to new contexts. Therefore, a thorough investigation is necessary to identify the specific forms of epistemological obstacles experienced by students when solving PISA-based mathematical literacy problems on the topic of SLETV. This study aims to contribute to the development of more effective and contextualized instruction, and to encourage the design of teaching strategies that bridge students' conceptual and applicative understanding of SLETV. In light of this background, the objective of this research is to identify and analyze epistemological obstacles encountered by students in solving PISA-oriented mathematical literacy problems related to the system of linear equations in two variables, as examined through the lens of PISA proficiency levels.

B. METHODS

1. Research Design and Participants

This study employed a descriptive qualitative approach, adopting the Didactical Design Research (DDR) methodology as developed by Suryadi (2013). In line with the initial phase of DDR, known as didactical situation analysis, the research was aimed at identifying students' learning obstacles in the topic of Systems of Linear Equations in Two Variables (SLETV) (Jannah et al., 2023).

The research was conducted at a public junior high school in North Bengkulu Regency and involved 23 ninth-grade students in the second semester of the 2024/2025 academic year who had previously studied the topic of SLETV. From this group, six students were purposively selected as the primary participants based on the following criteria: (1) they exhibited systematic error patterns in solving mathematical literacy problems related to SLETV, (2) their responses demonstrated varied forms of representation (graphical, symbolic, and verbal), and (3) they consented to participate in follow-up interviews. To maintain confidentiality, the six students were anonymized using the codes T1 through T6.

2. Data Collection Procedure

Data were collected in three sequential phases. First, a written test was administered in a classroom setting during a 90-minute session in April 2025, using mathematical literacy problems related to SLETV that were adapted from PISA items. Second, semi-structured interviews were conducted with the six selected students, lasting approximately 15–30 minutes each. The interviews aimed to explore students' conceptual understanding, problem-solving strategies, and potential epistemological obstacles. Third, observations were carried out during the test and interview sessions using observation sheets designed to document the alignment between researchers' predictions and students' actual responses (Sugiyono, 2019).

3. Instruments

Two types of instruments were used in this study: test and non-test instruments. The test instrument consisted of open-ended mathematical literacy questions adapted from PISA items. These items were designed to encompass the three mathematical literacy processes outlined by the OECD (2023): formulating (transforming real-world situations into mathematical models), employing (applying mathematical concepts and procedures), and interpreting (connecting mathematical results to problem contexts).

In addition, the test instrument was structured to align with the six proficiency levels established by the OECD (2023), ranging from Level 1 (basic procedural skills) to Level 6 (reflective critical thinking and complex problem-solving). The items were validated by two mathematics education lecturers and two mathematics teachers based on PISA's mathematical literacy indicators (OECD, 2023). The questions retained PISA's four contextual domains (personal, occupational, societal, and scientific)(OECD, 2023) while being adapted to the Indonesian context and national curriculum through consultations with mathematics teachers.

The non-test instruments included a semi-structured interview guide and an observation sheet. The interviews were designed to investigate students' reasoning, their use of representations, and their conceptual understanding, which may indicate the presence of epistemological obstacles. The observation sheet was used to document the alignment between the researchers' expectations and students' actual responses (Sugiyono, 2019).

4. Data Analysis

The data were analyzed thematically using a categorical approach to identify epistemological obstacles encountered by students in solving SLETV problems adapted from the 2022 PISA framework. The analytical procedure followed the three-step model proposed by Miles and Huberman: data reduction, data display, and conclusion drawing (Gusnardi & Muda, 2019). The students' test responses were examined to identify error patterns and solution strategies, while the interview and observation data were transcribed and coded to detect indicators of epistemological obstacles in students' understanding of symbolic representations, equation concepts, and mathematical modelling. Triangulation across instruments was employed to enhance the validity of the findings.

C. RESULT AND DISCUSSION

Question 1 (Level 1b)

This question involves calculating the number of chickens and goats owned by Pak Nyoman, who has 20 livestock with a total of 68 legs. In this question, students are asked to determine the number of chickens and goats based on the fact that each chicken has 2 legs and each goat has 4 legs. This question tests students' ability to formulate mathematical relationships, where they must set up a simple SPLDV to calculate the number of each type of livestock. The content area of this question relates to Change and Relationships, with the process of formulating mathematical formulas to solve the problem (Formulate). The context of this question is related to Occupational, where students are presented with a realistic situation in everyday life, such as in the farming world. The PISA competency level used for this question is Level 1b, which tests basic understanding of relationships between variables in a simple context, as shown in Figure 1.

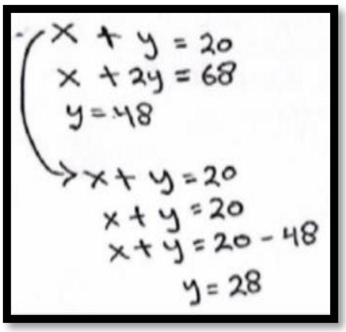

Figure 1. The answer sheet by T1

P : How did you create the SPLDV model for this question?

T1: The first equation is from the number of chickens and goats. Then, the second equation comes from the total number of legs of the livestock.

P : Then, how did you get y = 48?

T1: From 68 - 20, mam. So, y = 48.

P : Then why x + y = 20 - 48?

T1: I was working while I had a fever, mam. That's from the first equation, mam. The first equation is x + y = 20. So I just subtracted it, mam, and got y = 48. So, I got x + y = 20 - 48, and I got y = 28.

The analysis of student T1's responses reveals the presence of epistemological obstacles that are procedural, conceptual, and operational in nature. The student was able to identify two key equations, namely the total number of animals and the total number of legs. However, an error occurred in algebraic manipulation when determining the variable values. For instance, the student stated that y = 48 was derived from 68 − 20, which is a clear example of a procedural misconception, as it does not align with the logic of a system of linear equations in two variables (SPLDV). This type of error represents a procedural obstacle, where the student fails to apply mathematical operations correctly (Nurhayati & Retnowati, 2019). In addition, a conceptual obstacle was evident in the student's limited understanding of how mathematical equations are constructed from contextual situations (Prayitno & Widayanti, 2021; Untarti & Kusuma, 2019). The student's comment about being unwell during the test also indicates the presence of operational uncertainty, referring to a lack of confidence in the mathematical thinking process. Loh and Lim (2021) argue that such uncertainty, if unmanaged, may negatively affect academic performance. A high level of uncertainty has been found to correlate with lower learning achievement. Nevertheless, when deliberately and strategically integrated into learning, uncertainty has the potential to foster curiosity, enhance critical thinking, and develop problem-solving skills. However, if left unmanaged, uncertainty may also trigger negative affective responses that hinder the learning process.

The main findings at this level indicate that despite the simplicity of the problem context, the student still experienced substantial conceptual and procedural obstacles. This finding aligns with the studies by Bakar et al. (2019) , as well as Fatio et al. (2020), which highlight that students often struggle to transform contextual problems into correct SPLDV models. Students frequently find it difficult to translate real-world problems into mathematical models and perform accurate calculations. Therefore, although the question is categorized at PISA level 1b, which requires only basic understanding, these findings underscore that epistemological obstacles, both procedural and conceptual, remain significant challenges for students when modeling and solving SPLDV within real-world contexts. To address these epistemological obstacles, learning strategies based on the Didactical Design Research (DDR) approach can be directed toward the early identification of procedural misconceptions. For instance, by designing didactical situations that emphasize the validation of modeling logic and the systematic application of algebraic manipulation, teachers can support students in developing a more robust procedural understanding.

Question 2 (Level 2)

This problem focuses on calculating the age difference, the sum of ages, and the age ratio between Grisel and her mother in the year 2025. It is stated that five years from now, the age gap between them will be 22 years, and their total combined age is currently 56 years. Students are asked to determine the specific year in which Grisel's age will be exactly half of her mother's age. This task is designed to assess students' ability to formulate mathematical relationships involving changes over time and the comparative analysis of two individuals' ages. The cognitive process required in solving this problem is Formulating, which requires students to construct appropriate mathematical equations in order to find the year in question. The context of the problem is categorized as Personal, as it represents a familiar and relatable family scenario from everyday life. By situating the mathematical challenge within a personal setting, the task encourages students to apply their mathematical reasoning in meaningful, real-world contexts. This item falls under Level 2 complexity, requiring a deeper understanding of the relationships between variables and the application of time-based calculations through proportional reasoning. Students must interpret the given information, translate it into a mathematical model, and logically determine the solution based on the conditions provided, as shown in Figure 2.

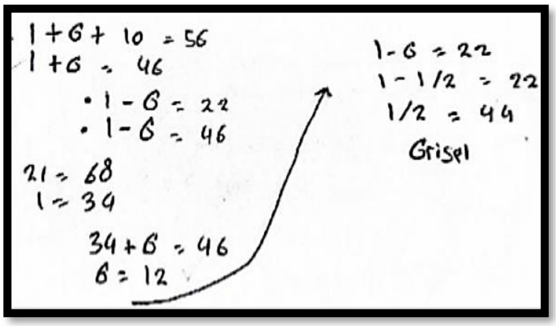

Figure 2. The answer sheet by T2

P : How did you solve Question 2? And how did you get the equations 1 and 2?
T2: From the question, mam. Then I just simplified it, mam. I got I = 32.
P : Why is this I - G = 46? When it should be I + G = 46?
T2: Oh, I made a mistake writing it, mam.
P : Okay, the correct answer is I = 34 and G = 12. Then how did you get I - G = 22 and then I - 1/2 = 22?
T2: I - G = 22 from equation 1, mam. Since the age difference between Ibu and Grisel is 22 years, I put G = 1/2 and then it became I - 1/2 = 22. Then both sides were multiplied by 2, resulting in 44. So Grisel's age is 44, mam.

Conceptual obstacles were evident in students' misunderstanding of the meaning of age difference and temporal relations. For example, subject T2 wrote the equation "I – G = 46" instead of the correct form "I + G = 46", indicating a fundamental misunderstanding of the linguistic meaning of the problem statement. This error exemplifies a symbolic representation misunderstanding, referring to the failure to interpret symbols or notations accurately within the context of systems of linear equations in two variables (Jupri & Drijvers, 2016; Pradini &

Winarsih, 2020). Procedural obstacles were reflected in students' lack of precision in constructing or simplifying equations. Although the student was able to identify the initial steps, a critical error occurred when G was substituted with ½ in the equation I−½=22, and then incorrectly multiplied by two to obtain Grisel=44. This reveals a weak procedural understanding of systems of linear equations, similar to the findings of Qetrani et al. (2021), who emphasized the importance of logically and systematically constructing and solving more complex equations. Additionally, operational obstacles were observed in basic algebraic manipulation errors, including incorrect substitution and unverified calculations. This suggests low verification and reflection skills in evaluating their own mathematical work.

These findings reinforce that at PISA Level 2, students are expected to formulate simple mathematical models and perform basic calculations in a sequential manner. However, epistemological obstacles such as misconceptions regarding mathematical symbols and inappropriate procedural applications hinder successful problem-solving. This is consistent with previous research indicating that students often struggle to connect the problem text with formal mathematical representations, particularly in real-life contexts (Jupri & Drijvers, 2016). From the perspective of Didactical Design Research, these results highlight the need for instructional designs that promote exploration of symbol meanings, the use of familiar real-life contexts, and gradual procedural training with immediate feedback. For instance, teachers can develop didactical situations that allow students to explore the distinction between “sum” and “difference” of ages through family-based contextual activities.

Question 3 (Level 3)

This problem concerns the calculation of expenses incurred by Olym to purchase notebooks and pencils, based on two receipt excerpts indicating the prices of the items bought. Students are asked to determine how many pencils and notebooks Olym can purchase with the money available, given the conditions that Olym must buy more than three notebooks and use any remaining money to buy pencils, with the constraint that all of Olym’s money is fully spent. This question assesses students’ ability to formulate mathematical relationships between the available funds, item prices, and quantities of goods that can be purchased, as well as to identify all possible purchase combinations within a limited budget. The cognitive process involved is Employing, where students must apply mathematical calculations to find the solution. The context of this problem is Social, reflecting a real-life everyday situation involving purchasing goods. This familiar setting encourages students to apply mathematical reasoning in practical scenarios. The problem is categorized as Level 3, which requires students to utilize skills in calculating and planning purchases based on multiple conditions and constraints, as shown in Figure 3.

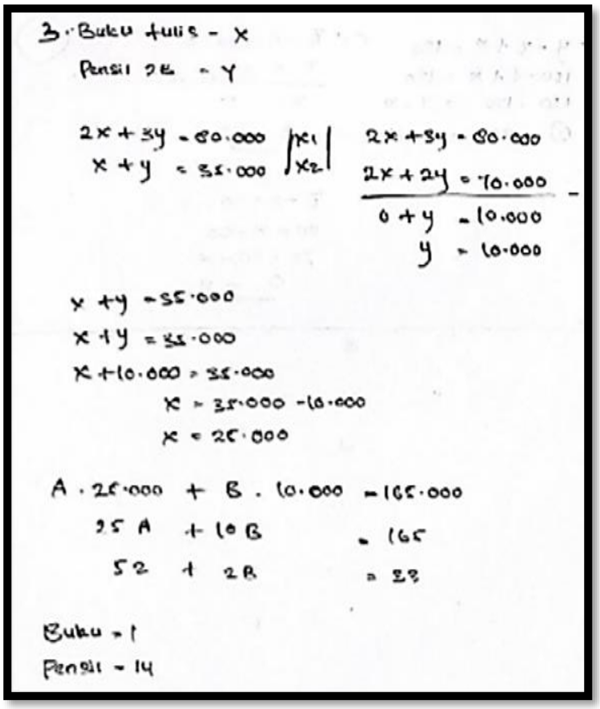

Figure 3. The answer sheet by T3

P : From the result y = 10,000 and x = 25,000, how did you get the answer that the notebook = 1 and pencil = 14? Try reading the question again.

T3: "If Olym wants to buy more than 3 notebooks, and the money must be fully used, how many total pencils and notebooks can be bought?" Oh, I see, mam, I made a mistake. The number of notebooks should be more than 3; it could be 4 or 5.

P : Okay, if you answer again, how many possibilities should there be?

T3: The total money is 165,000. If notebooks are 4, that means 4 x 25,000 = 100,000, then the remaining for pencils is 6, which means 6 x 10,000 = 60,000, and the remaining money is 5,000. If buying 5 notebooks, that means 5 x 25,000 = 125,000, and then 165,000 - 125,000 = 40,000, the remaining money for pencils is 4, so 4 x 10,000 = 40,000, and there is no remaining money. If buying 6 notebooks, it means 6 x 25,000 = 150,000, then the remaining for pencils is 1, which means 1 x 10,000 = 10,000, and the remaining change is 5,000. If buying 7 notebooks, it means 7 x 25,000 = 175,000, the money is not enough, mam.

P : So, how many possibilities are there?

T3: 4 notebooks and 6 pencils, 5 notebooks and 4 pencils, and 6 notebooks and 1 pencil. So there are 3 possibilities, mam.

Conceptual obstacles were evident when students failed to fully comprehend the constraints embedded in the problem conditions, particularly the requirements to "purchase more than three notebooks" and to "spend all the money." This issue was observable in the initial response of student T3, who answered with 1 notebook and 14 pencils, despite the question explicitly limiting such a solution. This error reflects a limited understanding of contextual constraints within mathematical modeling, as Qetrani et al. (2021) noted that students often struggle to connect mathematical results with contextual requirements. Procedural obstacles emerged when students did not develop a systematic calculation strategy to evaluate all possible combinations that meet the given conditions. Although the student was eventually able to identify three valid combinations (4 notebooks & 6 pencils, 5 notebooks & 4 pencils, 6 notebooks & 1 pencil), the approach employed was trial-and-error rather than a systematic strategy grounded in the structure of systems of linear equations in two variables (SLETV). This suggests a limited understanding of how to apply SLETV procedures to solve context-based problems (Edo & Tasik, 2022). Operational obstacles were relatively minimal at this level. Students demonstrated accurate arithmetic skills when performing price multiplication and money subtraction; however, minor misinterpretations were still present, such as assuming the number of combinations without rechecking whether all the problem constraints were satisfied.

These findings reinforce prior studies indicating that students at the intermediate PISA level often face challenges in evaluating solutions within complex and dynamic contexts (Edo & Tasik, 2022; Qetrani et al., 2021). The epistemological obstacles identified at this level highlight a gap between procedural application and contextual conceptual understanding. From a Didactical Design Research (DDR) perspective, these obstacles can be addressed through instructional designs that emphasize reflection on the meaning of obtained solutions, rather than focusing solely on computational results. Teachers need to incorporate context-based scaffolding strategies that guide students to validate their answers against the problem constraints and to explore multiple solution alternatives through classroom discussions. This approach may enhance the connectivity between mathematical models and real-world contexts, which is a core component of mathematical literacy assessment in PISA (OECD, 2023). In conclusion, the epistemological obstacles observed at PISA Level 3 underscore the importance of strengthening students' conceptual understanding and contextual modeling skills, ensuring they are not only capable of executing mathematical procedures but also of aligning solutions with complex real-world conditions and constraints.

Question 4 (Level 4)

This question involves calculating the total height of a building construction using two types of tiles, based on the information about the number and height of each type of tile. In this question, students are asked to calculate the total height of the two types of tiles used in two different construction diagrams. The first diagram consists of three tiles of the first type and two tiles of the second type, with a total height of 200 cm, while the second diagram consists of two tiles of the first type and one tile of the second type, with a total height of 130 cm. This question tests students' ability to formulate relationships between the number and height of tiles to calculate the height of each type of tile. The process involved is Formulate, where students must create a formula to calculate the height of tiles based on the given information. The context of this question is Scientific, depicting a situation in the construction engineering field. The PISA competency level for this question is Level 4, which requires students to handle more complex calculations and apply mathematical understanding in a technical context, as shown in Figure 4.

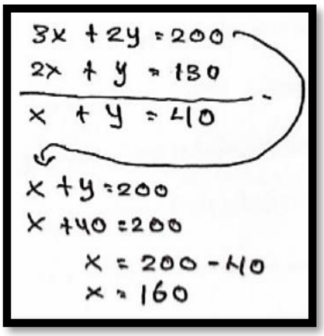

Figure 4. The answer sheet by T4

P : How did you solve this problem?
T4: I calculate for tile 1, then put it into equation 1. Same for tile 2.
P : How did you get the number 80 for equation 2? Two tiles of the first type + one tile of the second type equals 80?
T4: Oh yes, mam, I wrote it wrong. It should be 130.
P : Okay. You realize where the mistake is. If the equation is correct, how did you get y = 80?
T4: I just subtracted, mam, 200 - 80. So, y = 120. But it should be 70.
P : How did you get x + y = 200?
T4: From equation 1, mam. Then I subtracted 200 - 120. So, I got y = 80.

At Level 4, the identified epistemological obstacles can be classified into three categories: conceptual, procedural, and operational. Conceptual obstacles are evident in students' difficulties in accurately identifying the known and the unknown information, which often lead to errors in formulating mathematical models for instance, when constructing equations based on the number and height of tiles (Santoso et al., 2019; Wiyah & Nurjanah, 2021). Procedural obstacles arise when students improperly apply the steps in solving systems of linear equations, such as errors in algebraic manipulation and numerical subtraction (Pulungan & Suhendra, 2019; Purba & Nurlaelah, 2023). Operational obstacles are observed when students fail to logically or mathematically verify their obtained solutions, even when they are aware of inconsistencies in the values used (Ratu et al., 2024).

These findings reinforce previous studies indicating that while students often understand the general structure of systems of linear equations in two variables (SPLDV), they still struggle to consistently integrate contextual information into the mathematical model (Santoso et al., 2019). Although this item is categorized at Level 4, which requires more complex reasoning within a scientific-technical context, students did not demonstrate reflective abilities in evaluating their solution processes. In the context of Didactical Design Research, these results underscore the importance of designing learning activities that emphasize solution verification and symbolic meaning-making within real-world contexts. Didactical strategies such as the use of concrete building simulations or color coding in symbolic representations could serve as effective interventions to address the identified epistemological obstacles.

Question 5 (Level 5)

This question tests the students' ability to solve a mathematical problem involving an understanding of the changes and relationships between objects. The process involved is Formulate, based on the information provided in the diagram, which depicts the spatial relationships between a table, a cat, and a turtle in two different images. The context of this problem is Scientific Reasoning, requiring students to understand the relationship between objects in space and apply mathematical concepts related to height and distance. The PISA competency level for this question is Level 5, indicating that students are expected to use their skills to understand more complex relationships and apply their knowledge in a broader context, in this case, using the information provided to determine measurements that cannot be directly observed, as shown in Figure 5.

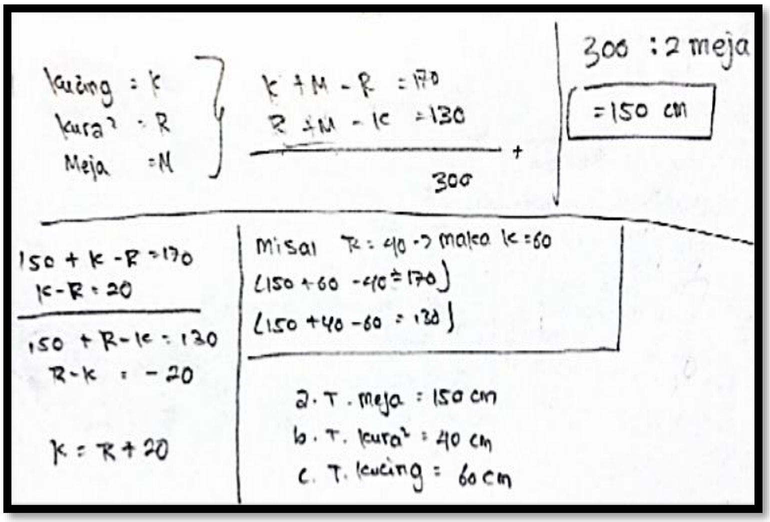

Figure 5. The answer sheet by T5

P : From these two equations, how did you solve it?

T4: This problem appeared on TikTok, mam. So, I just added these two equations together.

P : Then, R = 40, and K = 60? Why did you assume this?

T4: I tried it, mam. The first equation is K + M - R = 170. Since M = 150, it becomes 150 + 60 - 40 = 170, correct. The second equation is R + M - K = 130. I get 150 + 40 - 60 = 130, correct. So, the height of the table is 150 cm, the height of the turtle is 40 cm, and the height of the cat is 60 cm.

P : Try substituting the height of the turtle (R) = 20, the height of the cat (K) = 40, and the height of the table (M) = 150. Will these satisfy both equations?

T4: Yes, mam, both equations are satisfied.

P : So, how do you conclude? Which result is correct?

T4: So, the answer is uncertain, mam. Hahaha...

In the Level 5 item, the epistemological obstacles identified can be classified into two main categories: conceptual and procedural. Conceptual obstacles are evident in students' difficulties in understanding and formulating spatial relationships between objects based on the given visual information. Although students such as T4 were able to construct a system of linear equations in two variables (SPLDV) and apply substitution, they demonstrated a misunderstanding of the concept of solution coherence. When presented with alternative values, such as R = 20 and K = 40, the student still considered both equations to be "satisfied" without recognizing the underlying logical inconsistency. This indicates a weakness in higher-order mathematical thinking, particularly in validating solutions, which requires consistent use of deductive reasoning. Procedural obstacles also emerged when students adopted a trial-and-error strategy without a solid mathematical foundation, as illustrated by the statement: “This problem appeared on TikTok, ma’am. So, I just added these two equations together.” This finding supports the study by Pradini & Winarsih (2020), which revealed that students commonly struggle with formulating and applying equations within word problem contexts.

In addition, difficulties in comprehending abstract spatial relationships and thoroughly verifying solutions serve as indicators of operational obstacles. These challenges become more complex when students are not only expected to construct mathematical models but also to meaningfully connect symbolic representations with their contextual interpretations in a reflective manner. As explained by Pathuddin et al. (2024), solving SPLDV problems requires precision in transforming contextual information into appropriate mathematical forms. In the context of non-routine PISA-like problems that demand scientific reasoning, students encounter difficulties in applying mathematical knowledge to interpret and evaluate outcomes. This is in line with the findings of Putri & Wutsqa (2019), who noted that challenges in overcoming epistemological obstacles often arise when students are confronted with complex real-world problems. Therefore, from the perspective of Didactical Design Research, the teaching of SPLDV should incorporate activities that emphasize solution validation and the meaningful use of visual representations to foster students’ reflective reasoning in addressing similar problems.

Question 6 (Level 6)

This question tests students' ability to plan a lunch menu in a school canteen, taking into account the budget and calorie requirements for students. In this question, students are asked to determine how many portions of rice and vegetables, and rice and chicken, can be prepared by the cook with a limited budget of Rp240.000 and a total calorie requirement of 7.500 calories. Students are also asked to analyze whether changes in the cost of ingredients could affect the number of servings that can be prepared to meet both the budget and calorie needs. The process involved is Formulate, with the ability to manage multiple variables in calculations and use the available information to achieve an optimal solution. The context of this problem is Occupational, reflecting real-life conditions in budgeting and resource management in a school canteen. The PISA competency level for this question is Level 6, which requires deep understanding of the relationships between variables and the solution of highly complex problems, as shown in Figure 6.

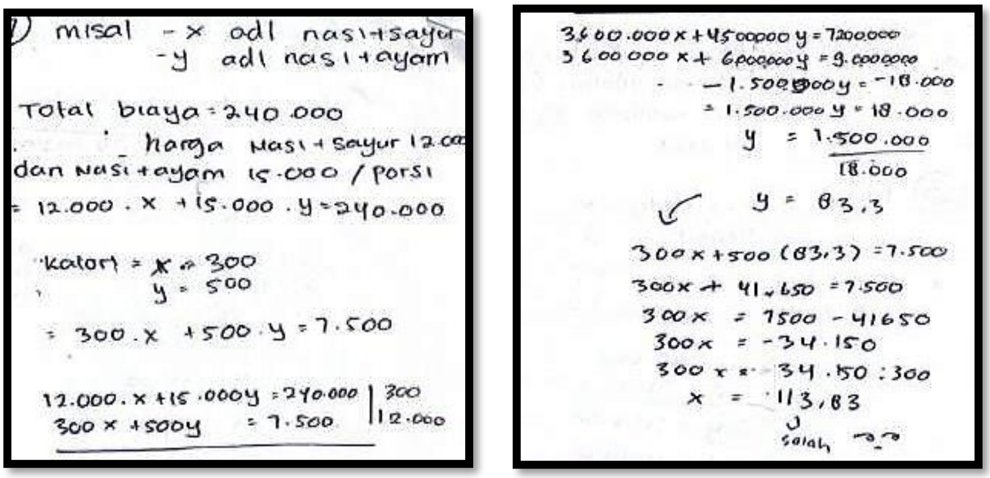

Figure 6. The answer sheet by T6

P : How did you solve this problem? This involves x = rice + chicken and y = rice + vegetables. Please check again the multiplication in your SPLDV model.

T6: Oh yes, mam, I missed a 0. It should be 72.000.000, but I wrote 7.200.000.

P : Then where did the -18,000 come from?

T6: I just subtracted, mam, 7.200.000 minus 90.000, but I forgot to write the 0, mam.

P : Assuming this answer is correct, does the operation 1.500.000y = 18.000 hold true? Can it be simplified to y = 1.500.000 / 18.000?

T6: Oh, yes, mam, I also made a mistake in simplifying it. I flipped it around. It should be y = 18.000.000/1.500.000. Then the result is 12.

At Level 6, students encountered complex epistemological obstacles encompassing conceptual, procedural, and operational aspects. Conceptual obstacles were evident in students' difficulties in understanding the relationships between variables and interpreting the meaning of mathematical models within real-world contexts, such as in planning a canteen menu under budget constraints and caloric requirements. Although some students were able to identify two variables and construct a system of linear equations in two variables (SPLDV), they struggled to interpret the magnitude of numerical values and relate the final solution to the contextual problem. Operational obstacles emerged when students made errors in manipulating large numbers, such as misplacing zeros or incorrectly simplifying fractional expressions. For instance, student T6 mistakenly simplified the expression y = 1,500,000 / 18,000 by inverting the result. Procedural obstacles were also observed in students' inaccurate calculations during subtraction and multiplication involving large numbers, as well as their failure to verify the correctness of final answers.

These findings align with those of Santoso et al. (2019), who stated that errors in solving SPLDV problems are generally due to a limited understanding of problem meaning, insufficient conceptual comprehension, and carelessness in procedural application up to the final solution stage. In addition, Sidik et al. (2021) found that students rarely practice solving context-based problems in daily mathematics instruction, resulting in unfamiliarity with contextual mathematical modeling approaches. These findings suggest that the epistemological obstacles at Level 6 can be addressed through instructional strategies grounded in Didactical Design

Research. Such strategies emphasize the connection between mathematical models and real-world contexts, practice in verifying results, and structured procedures for operating with large numbers. By providing students with more systematic and reflective experiences through well-designed didactical approaches, teachers can support the development of deeper mathematical literacy, enabling students to better navigate complex challenges such as those presented in the PISA assessment.

D. CONCLUSION AND SUGGESTIONS

This study identified conceptual, procedural, and operational epistemological obstacles encountered by students in solving PISA-adapted problems on the topic of systems of linear equations in two variables (SLETV) across various competency levels. The findings reveal that student errors occur not only in complex problems but also in simpler tasks, particularly in understanding context, constructing mathematical models, and verifying solutions. These results extend the application of the Didactical Design Research (DDR) framework by emphasizing the need for instructional designs that support early identification of misconceptions and promote contextual understanding of SLETV.

The practical implications of this study include the need to strengthen PISA-based contextual exercises, integrate conceptual reflection into SLETV learning, and employ didactical strategies that focus on solution validation and the connection between mathematical models and real-world contexts. This study is limited by the relatively small sample size and the homogeneous school context; therefore, future research should involve more diverse and larger populations. Overall, this study offers a significant contribution to the development of mathematical literacy through more reflective, contextual, and targeted DDR-based instructional strategies.

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