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Construction of Mortality Table for Credit Life Insurance using Whittaker-Henderson Graduation Method

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ABSTRACT

Indonesian Mortality Table III (2011) and Indonesian Mortality Table IV (2019) are still used as a reference for determining life insurance premiums, one of which is determining Credit Life Insurance premium rates. With different and more specific population and risk characteristics, it is necessary to have Mortality Table that reflects the characteristics of the Credit Life Insurance. This study is a quantitative applied study that aims to construct a specialized mortality table reflecting the unique characteristics of Credit Life insurance. First, the crude mortality rates were calculated using Microsoft Excel based on Credit Life Insurance portfolio data obtained from a life insurance company having a fairly large portfolio, during the period from 2017 to 2023. The crude mortality results were then adjusted using a smoothing technique of Whittaker-Henderson method assisted with the R program and Microsoft Excel. After obtaining the smoothed mortality rates, an extrapolation was carried out using Gompertz model assisted with the R program and Microsoft Excel to obtain the mortality rate for ages between 75 and 100 years. The extrapolated results are subsequently compared with the Indonesia Mortality Table III (IMT-III) and Indonesia Mortality Table IV (IMT-IV) to assess the consistency of mortality patterns. The main contribution of this study is the development of a more representative mortality table based on empirical data from credit life insurance portfolios, an area that has not been extensively explored. The findings of this study are expected to improve the accuracy of premium pricing, technical reserve estimation, and risk management for life insurance companies offering credit life insurance products.

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A. INTRODUCTION

In recent years, life insurance industry in Indonesia has faced significant challenges for credit life insurance products. The need for these products causes the growth of banking credit in Indonesia. As these products involve insuring a large number of borrowers, life insurance companies must implement risk selection processes to assess mortality risks and determine appropriate premiums. If the risk is inaccurately estimated, companies face potential financial instability. Therefore, accurate mortality estimates are essential to sustain the long-term viability of credit life insurance portfolios so that the premium set can be sufficiently met in the future. However, the mortality tables currently used by most life insurance companies in Indonesia are still based on general population data, such as the Indonesian Mortality Table III (IMT-III) and Indonesian Mortality Table IV (IMT-IV). These tables are not tailored to the specific risk profiles of credit life insurance, which differ in terms of insured population characteristics, loan duration, and age distribution (Suwondo et al., 2019). Consequently, the use of generic mortality tables may lead to biased estimations of death rates and inaccurate pricing.

In 2024, the Healthcare and Social Security Agency issued the 2023 Indonesian Population Mortality Table. This issuance came from the insured experience of social insurance which is different from commercial insurance (Mutia et al., 2022). The former applies no underwriting process with membership that is mandatory, while the latter has an underwriting process. Therefore, the mortality table will be different if it is applied by life insurance. Besides that, this table certainly cannot specifically estimate the level of death risk faced by the insurance company. Constructing mortality table in accordance with the characteristics of the insured population is still a challenge, especially in credit life insurance which has more specific risks than other traditional life insurances. The available data often has limitations, such as small sample sizes or uneven distribution between age groups, so it requires the right alignment method (Kostaki & Zafeiris, 2019).

To overcome the aforementioned limitations, constructing mortality table for credit life insurance uses a smoothing/graduation method, which has been the choice among actuaries for a long time. One of the methods is Whittaker-Henderson smoothing method (Weinert, 2007; Whittaker, 1922). This is effective in handling high data fluctuations by considering the balance between goodness-of-fit to observation data and smoothness of the mortality curve, resulting in more stable and reliable estimates (Biessy, 2024; Nocon & Scott, 2012). Previous studies were conducted to produce a more accurate and smoothed Indonesian National Mortality Table by applying the Whittaker-Henderson method to reduce random noise fluctuations in raw mortality rates using general population data (Setiady & Kusnadi, 2024). Meanwhile, its application in the context of credit life insurance is still rarely found in the academic literature. Therefore, this study focuses on the construction of mortality table specifically for credit life insurance products using the Whittaker-Henderson method which is expected to accommodate the unique characteristics of the available data. The characteristics of credit life insurance mortality are considered sufficient to be described by using data from one of the insurance companies that has a large amount portfolio for credit life insurance.

In addition to Whittaker-Henderson method, there is an extrapolation technique, which is used to project and obtain the mortality rate for ages between 75 to 100 years. This technique was conducted because in credit life insurance the entry age of participants is usually limited, which is only available up to the age of 75 years, as a requirement to obtain a loan. Therefore, data for participants above 75 years has limitations (Richmond & Roehner, 2016; Shklovskii, 2005).

The main objective of this study is to construct a specialized mortality table that reflects the characteristic of the Credit Life Insurance based on the insured experiences for credit life insurance using the Whittaker-Henderson method. This study does not only contribute to the academic aspect by expanding the application of the Whittaker-Henderson method in actuarial field, but also has practical implications for the life insurance industry in improving the accuracy and reliability of premium pricing, technical reserve estimation, and risk management for life insurance companies offering credit life insurance products, it can help maintain the company’s financial stability.

B. RESEARCH METHODS

This study was conducted in several stages. First, it began by collecting credit life insurance portfolio data, namely inforce data and claim data, which was obtained from one of the life insurance companies with a fairly large amount of credit life insurance portfolio during the period 2017 to 2023. The inforce data, which would be used as exposure, included information on the number of active participants each year, while claim data was the occurrence of death claims that happened in the same period. This data collection aimed at providing a comprehensive and representative database which would then be used for further analysis.

After collecting data, the next step was to determine the age of the insured in the study period for inforce/exposure data and at the time of death for claim data. Age calculation used the last birthday approach. This process was prominent to ensure that the analysis was carried out consistently and accurately by considering the age factor as a key variable in determining the mortality rate (Lee & Carter, 1992).

Then, in each period, the inforce/exposure data and claim data were compared to calculate the probability of death or crude/initial mortality rate. This rate was calculated by comparing the number of death claims with the number of inforce/exposure policies at the beginning of the study period for each age group (Booth & Tickle, 2008). This process was carried out for each year in the study period, namely from 2017 to 2023. Furthermore, the crude/initial mortality rate that was obtained from each year was averaged to produce an average mortality. This average calculation aimed at reducing annual fluctuations and providing a more stable and reliable estimate for further analysis (Haberman & Renshaw, 2011).

The next stage was graduation process using the Whittaker-Henderson method to smooth the averaged mortality rate. This method was a mathematical technique designed to reduce fluctuations or “noise” that might be present in mortality data (Cox et al., 1971). These fluctuations were frequently caused by random variability in the data, which can interfere with the analysis and produce inaccurate estimates. This method could produce a smoother and more realistic mortality curve, which reflected the general trend in mortality rates without being affected by unwanted variations. This graduation or smoothing process involved the application of a mathematical formula that considers existing mortality values and combines them with a smoother approach. In this way, more stable estimates for each age group could be obtained (Weinert, 2007; Whittaker, 1922) ,

The principle of the Whittaker-Henderson method is to combine two main criteria: Fidelity to data and Regularity or smoothness. The former means the extent to which smoothing approaches the original data, while the latter means the extent to which the smoothing results form a smooth curve without sharp waves. This method was formulated through a penalized least squares approach, where the model would minimize an objective function that combines both criteria. In this case, the penalty means giving a mathematical effect to the smoothing results that are too volatile (not smooth). It means that the sharper the change between ages, the greater the penalty imposed. Hence, the model is encouraged to produce a smoother and more reasonable curve. The function can be written as follows Equation 1:

$$M = \sum_{x=0}^{n} w_x \left(q_x - \hat{q}_x \right)^2 + \lambda \sum_{x=0}^{n-z} \left(\Delta^2 \hat{q}_x \right)^2 \tag{1}$$

The first component of the M function, namely $\sum w_x \ (q_x - \hat{q}_x)^2$, is called the fidelity component, aiming at maintaining the closeness of the smoothing results to the original data. The second component, $\lambda \ \sum_{x=0}^{n-z} (\Delta^2 \ \hat{q}_x \ )^2$, is the regularity component functioning to minimize the roughness of the smoothing curve. The selection of the parameter value of λ played an essential role in determining the smoothing results. A too small value can cause the curve to follow the noise of the original data, while a too large value will smooth the curve excessively until it loses important structures. With this approach, the Whittaker-Henderson method is able to produce a mortality curve that is not only smooth, but also still represents the basic pattern of the original data effectively. To determine the most optimal λ value in producing a smooth mortality rate estimate but still reflects the basic pattern of empirical data. For evaluation purposes, the calculation of the prediction error size was carried out using the Mean Squared Error (MSE) between the empirical value and the smoothing result value of each λ. MSE measured the average squared difference between the original value and the model result value, the smaller the MSE value, the better the smoothing result was in representing the data without overfitting (Hodson, 2022). It can be written in Equation 2.

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y_i})^2 \tag{2}$$

where y_i observed value (q_x actual), $\hat{y_i}$ predicted value (q_x smoothing result), n numbers of data points. In addition to using the MSE approach, based on, the selection of smoothing parameters can also be based on information criteria, such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). These two criteria provide an assessment of the model by considering the quality of data presentation and model complexity simultaneously.

AIC and BIC are designed to balance goodness-of-fit and model parsimony by penalizing the number of parameters in the model. Too complex models for example, with too small smoothing parameter values, resulting in a curve that is very close to the original data can experience overfitting, so that their predictive performance decreases on new data(Burnham & Anderson, 2002). Therefore, models with lower AIC or BIC values are considered more optimal because they are able to represent the data well without being too complex

In the context of smoothing mortality rates using the Whittaker-Henderson approach, AIC and BIC could be calculated based on the prediction error and the number of effective parameters (degrees of freedom) used in the smoothing process. Through this approach, the selection of the optimal λ value did not only depend on minimizing errors, but also considered information efficiency and overall model simplicity. It can be written **Equation 3**, and **Equation 4**.

$$AIC = n \,.\ln\left(\frac{RSS}{n}\right) + 2k \tag{3}$$

$$BIC = n \,.\ln\left(\frac{RSS}{n}\right) + k.\ln(n) \tag{4}$$

with the following $RSS = \sum_{i=1}^{n} (y_i - \hat{y_i})^2$ = Residual sum of squares, n the numbers of observation and k is effective degree of freedom (number of parameters used in smoothing).

After the graduation process using the Whittaker-Henderson method, the next step was extrapolation to obtain the mortality rate q_x for ages over 75 years. At this stage, the Gompertz model was used as one of the most commonly used models in mortality studies. Gompertz's Law states that in humans over the age of 35 years, the mortality rate increases exponentially with a doubling time of about 10 years with a pattern that remains valid until the age of 106 years (Gompertz, 1833; Olshansky & Carnes, 1997). The Gompertz model is formulated as follows Equation 5:

$$q_x = A.e^{B.x} \tag{5}$$

where q_x is the mortality rate at age x, A dan B are parameters determined based on existing data and e is the base of the natural logarithm (approximately 2.71828).

This model allows us to design the exponential increase in the Mortality Rate with the increasing age. One of the advantages of the Gompertz model is its ability to describe the pattern of increasing risk of death that often occurs in older populations. In many studies, the mortality rate in the elderly group shows a sharp increase, and the Gompertz model can capture this phenomenon well (Missov et al., 2015). Extrapolation apply because in this study empirical data was limited to the age of 75 years where generally banks or other financial institutions provided a maximum limit for providing credit. Besides that, many participants generally have completed their credit before they reach old age, so that their insurance coverage has also finished.

Extrapolation using the Gompertz model is important in the context of credit life insurance because insurance companies need to understand the risks associated with older customers. With a more accurate estimate of the mortality rate, insurance companies can set more appropriate premiums, manage risks more effectively, and design insurance products that are more in line with customer needs. Overall, the use of the Gompertz model as an extrapolation of the mortality rate above the age of 75 provided a reliable approach to support better decision making in the life insurance industry.

C. RESULTS AND DISCUSSION

1. Data Description

This study used credit life insurance data, which was obtained from one of the Life Insurance Companies that has a fairly large amount of data. Data collection was carried out in the period 2017 to 2023. However, the inforce/exposure data and claim data were the data at the end of each year during the study period. The following is the amount of data used, as shown in Table 1.

Table 1. Inforce/Exposure Data and Claim Data

Year	Inforce Data	Claim Data
2017	3,884,623	7,782
2018	4,706,508	8,414
2019	3,015,468	6,223
2020	1,411,258	5,332
2021	1,152,848	5,151
2022	916,771	6,150
2023	725,569	2,517

From data Table 1, the age of participants for inforce/exposure data was calculated in accordance with the study period using the Last Birthday approach, so that the result was an integer age. The age of participants for claim data was calculated based on the age of death in the study period also using the Last Birthday approach. Furthermore, in each year, participant data was grouped by age to obtain l_x or the number of participants alive at age x from an initial hypothesis group. Since the data position was at the end of the year, to obtain the number of participants alive at the beginning of the year was to add the number of inforce/exposure participants with claim participants in accordance with the age in the study period. It can be see in Table 2, Table 3, and Table 4.

Table 2. End of Year Inforce/Exposure Data

Age	l_x						
	2017	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**
18	27	41	8	1	0	0	0
19	129	127	41	5	1	0	0
20	359	311	83	15	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
74	144	208	413	185	176	286	158
73	441	932	671	386	299	305	280
75	104	45	23	11	8	163	8
Total	3,884,623	4,691,283	2,999,556	1,411,258	1,152,848	916,771	725,569

Table 3. Claim Data

Age	d_x						
	2017	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**
18	1	0	0	0	0	0	0
19	1	0	1	0	0	0	0
20	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
74	10	13	21	20	18	9	2
73	8	4	2	5	6	6	1
75	6	3	1	0	0	0	0
Total	7,782	8,414	6,223	5,332	5,155	6,150	2,517

Table 4. Early Year Inforce/Exposure Data

Age	l_x						
	2017	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**
18	28	41	8	1	0	0	0
19	130	127	42	5	1	0	0
20	359	311	83	15	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
74	154	221	434	205	194	295	160
73	449	936	673	391	305	311	281
75	110	48	24	11	8	163	8
Total	3,892,405	4,699,697	3,005,779	1,416,590	1,158,003	922,921	728,086

2. Mortality Rate Calculation

After obtaining data on the number of participants who were alive (l_x) and the number of participants who died (d_x), then by comparing d_x and l_x for each year, the probability of death at age x (q_x) would be obtained. After that, the q_x value at each age during 2017 to 2023 was averaged to produce the average q_x. It is called mortality data. The mortality data value (q_x) was still a rough and irregular pattern as in general mortality tables in Table 5, and Figure 1.

Table 5. Early Year Inforce/Exposure Data

Age	q_x							
	2017	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**Average**
18	0.03571	0	0	0	0	0	0	0.00510
19	0.00769	0	0.02381	0	0	0	0	0.00450
20	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
73	0.02217	0.01376	0.03035	0.04926	0.05678	0.02866	0.00709	0.02972
74	0.05263	0.01887	0.00482	0.02632	0.03297	0.02055	0.00629	0.02321
75	0.05455	0.06250	0.04167	0	0	0	0	0.02267

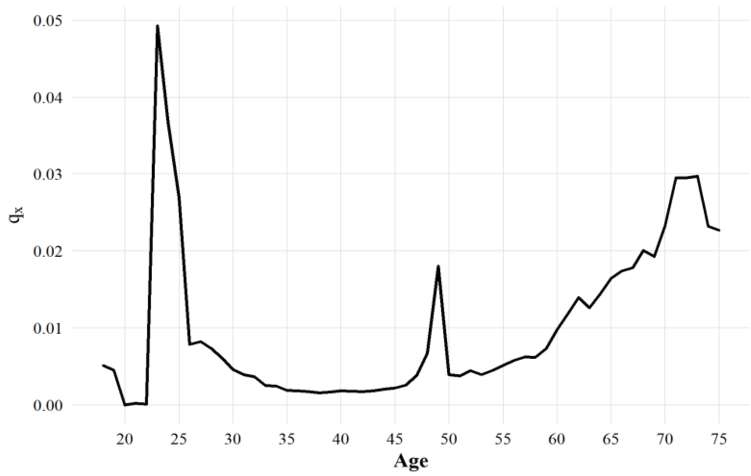

Figure 1. Research-based mortality data

3. Mortality Rate of Smoothing Results

After obtaining the mortality estimate (q_x) for each age calculated using death claim data and the number of exposures in each age group during the study period, as known as mortality data, this value then became the initial basis for further analysis to carry out the graduation or smoothing process to obtain a more stable mortality value and in accordance with the expected pattern. The data processing and visualization process were carried out assisted with the R program to ensure the accuracy of the calculations and support visual interpretation of mortality patterns in Figure 2, and Figure 3.

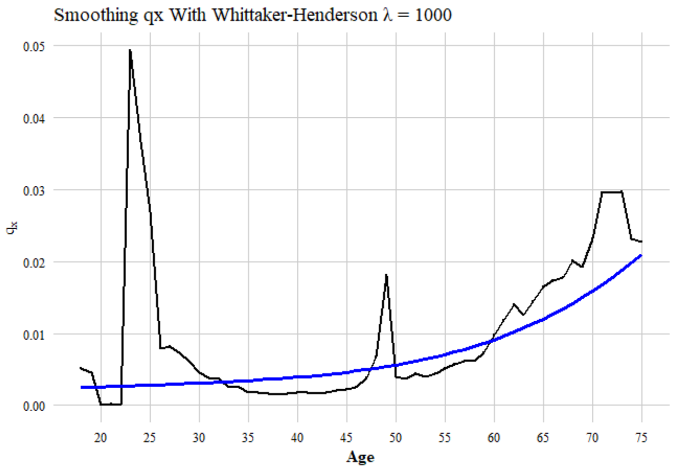

Figure 2. Mortality smoothing with Whittaker-Henderson λ = 1000

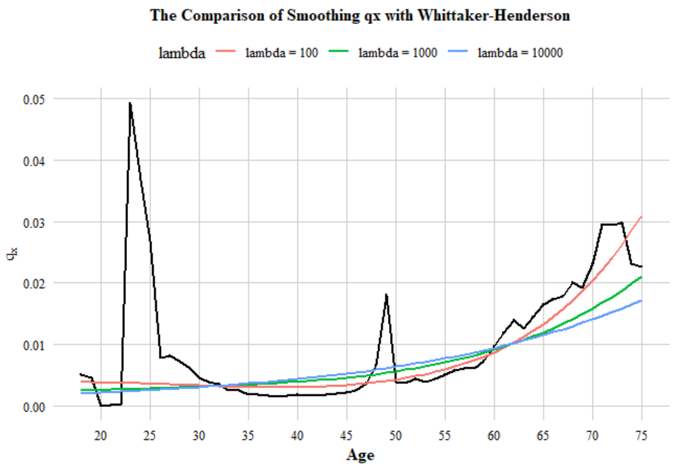

Figure 3. Mortality smoothing with Whittaker-Henderson λ = 100, λ = 1000 and λ = 10.000

4. Consideration for Determining λ

After the smoothing process using the Whittaker-Henderson method with various lambda values as smoothing parameters, the next stage was to determine the most optimal λ value in producing a smooth mortality rate estimate but still reflects the basic pattern of empirical data carried out using the Mean Squared Error (MSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), as shown in Table 6 and Table 7.

Table 6. MSE of each λ value

Lambda (λ)	MSE
100	0.000074177
500	0.000080268
1,000	0.000084472
5,000	0.000090974
10,000	0.000092138

Table 7. AIC/BIC of each λ value

Lambda (λ)	AIC	BIC
100	-439.52547	-324.140662
500	-434.948063	-319.563255
1,000	-431.98737	-316.602561
5,000	-427.686435	-312.301626
10,000	-426.949235	-311.564426

Based on the results of the smoothing analysis of mortality rates using the Whittaker-Henderson method with various values of the smoothness parameter (λ) in Table 7, it was found that the selection of the λ = 100 value provided the most optimal results in balancing between the accuracy of data representation and the smoothness of the curve. Objective evaluation was carried out using the Mean Squared Error (MSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) criteria. The λ = 100 value produced the lowest MSE compared to other λ values, and had the smallest AIC and BIC values. This shows that the model with λ = 100 provided the best fit to the empirical data without causing excessive complexity. By considering the aspects of goodness-of-fit and model parsimony, the Whittaker-Henderson method with λ = 100 was chosen as the most representative smoothing approach to form a stable, smooth mortality curve that still reflects the basic characteristics of the observed mortality data.

5. Extrapolation of Age Above 75 Years

After the smoothing process of mortality rate using Whittaker-Henderson approach with optimum smoothing parameter (λ = 100), extrapolation process was needed to expand the age range to 100 years, considering the available empirical data was limited to age 75 years. For this purpose, Gompertz model was used as a common parametric approach in elderly mortality analysis. Gompertz model states that the probability of death at age x, namely q_x, follows an exponential function in the form $q_x = A.e^{B.x}$ where A and B are model parameters estimated based on smoothed q_x data in the age range of 60 to 75 years. Estimation of parameters A and B was carried out using a non-linear regression approach through the non-linear least squares (NLS) method. This process was conducted using the R software by utilizing the nls() function. The initial values of the A and B parameters in Gompertz modeling are determined heuristically based on the mortality data scale and common practices in nonlinear modeling (Forfar et al., 1988) The initial parameter values were set as A =0.0001 and B =0.1. These starting values were chosen based on general characteristics of mortality rates, where A is assumed to be small due to the relatively low mortality at younger ages, and B is set around 0.1 to reflect the exponential increase in mortality risk with age.

The selection of age range of 60 to 75 years was based on the assumption that the structure of increase in risk of death at that age was quite stable and representative of the exponential pattern assumed in Gompertz model. Once the model parameters were obtained, the function was used to extrapolate the q_x value at ages between 76 to 100 years. To maintain the validity of the probability of death, the extrapolated q_x value is limited to not exceed 1. The extrapolation results were then combined with the smoothed q_x data at ages between 18 to 75 years, constructing a complete mortality table up to age 100 years, which would be used as the basis for constructing the final mortality table. The use of mortality for the elderly up to age 100 years was rare for credit life insurance cases because loans for elderly participants were rarely provided by lending institutions. However, to assess the suitability and validity of smoothed mortality using the Whittaker-Henderson method, extrapolation was carried out up to age 100 years. It can be see in Figure 4.

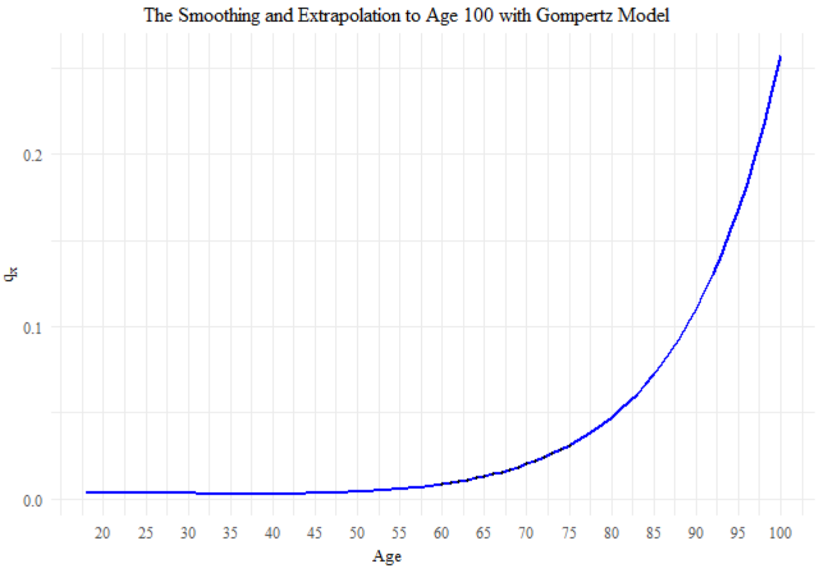

Figure 4. Mortality extrapolation

6. The Comparison of IMT-III (2011) and IMT-IV (2019)

To assess the suitability and validity of the smoothing results of the mortality rate using the Whittaker-Henderson method, a comparison was carried out with two official references, namely the 2011 Indonesian Mortality Table (IMT-III) and the 2019 Indonesian Mortality Table (IMT-IV) issued by the Indonesian Life Insurance Association (AAJI/*Asosiasi Asuransi Jiwa Indonesia*) and the Indonesian Actuarial Association (PAI/*Persatuan Aktuaris Indonesia*). This comparison aimed at examining whether the smoothing results with the parameter λ = 100 showed consistency in mortality patterns with reference tables that have been widely used in actuarial practice in Indonesia for determining premium rates, reserving, and risk assessment (Macdonald et al., 2018). The approaches used in this comparison included visualization of the q_x curve of the smoothing results against IMT-III and IMT-IV and examination of the suitability of the general increasing mortality trend in adulthood and the elderly (Ramadhan et al., 2025). The results of this study revealed that the smoothing results were not only consistent with empirical data, but also within the actuarially acceptable range compared to industry standards. This indicated that the Whittaker-Henderson approach with λ = 100 provided a representative and reliable estimate of the mortality rate for further analysis, as shown in Figure 5.

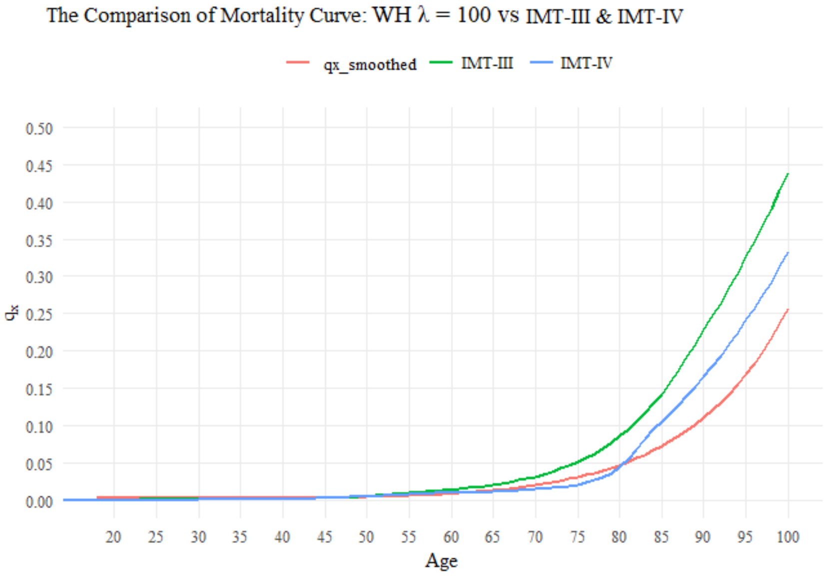

Figure 5. The Comparison of q_x smoothing with IMT-III and IMT-IV

From Figure 5, A comparison between the mortality table developed in this study and the IMT-III (2009) and IMT-IV (2019) shows significant differences in mortality rates. These differences indicate that the national standard tables do not fully capture the actual risk profile of credit life insurance policyholders. One of the main contributing factors to this differences is the unique characteristics of the insured population in credit life insurance products, who are typically active borrowers with diverse socio-economic backgrounds, varying health conditions, and subject to relatively lenient or even incomplete risk underwriting processes. Additionally, differences in data collection methods, data quality, and observation periods also influence these differences. The data used in this study is based on the actual experience of insurance companies from 2017 to 2023, which is relatively more up-to-date compared to the historical data used in the construction of TMI-III (2011) and TMI-IV (2019).

The result of this study indicate that mortality patterns in credit life insurance have different characteristics compared to mortality patterns in the general population. This finding offers a novel contribution and complements previous studies, which generally have not specifically constructed mortality tables based on empirical data from credit life insurance portfolios. By adopting data-driven approach, this study is expected to enhance the accuracy of premium rate setting, technical reserve estimation, and risk management for life insurance companies offering credit life products. Overall, the results underscore the importance of developing more representative and contextually appropriate mortality tables that align with the unique characteristics of the insurance portfolios being analyzed.

D. CONCLUSIONS & RECOMENDATION

This study constructed an empirical mortality table based on actual data from a credit life insurance portfolio covering the period 2017–2023. Key findings show that mortality rates in the productive age group are consistently higher compared to IMT-III (2009) and IMT-IV (2019). This difference reflects a mismatch between the actual risk profile of credit life insurance policyholders and the mortality assumptions commonly used in the industry. Consequently, the continued use of such standard mortality tables may lead to an underestimation of mortality risk, resulting in inaccurate premium pricing and insufficient technical reserve calculations.

The scientific contribution of this research consists of two main aspects. First, from a methodological perspective, the study applies a smoothing technique using the Whittaker-Henderson method to produce mortality rate estimates that are both stable and representative of empirical data. This approach has proven effective in capturing the distinct mortality patterns within credit life insurance portfolios. Second, from an applied perspective, the results of this study provide a more accurate basis for life insurance companies in designing premiums, setting reserves, and managing risks more appropriately for credit life insurance products.

Based on these findings, a short-term recommendation is for life insurers to consider adopting empirically derived mortality tables, particularly for credit life products in order to improve pricing accuracy and reserve adequacy. Furthermore, there is a need to re-evaluate the mortality assumptions currently employed in industry practice, including the potential development of more specialized national reference tables tailored to products with specific risk characteristics. For long-term development, this study recommends external validation of the proposed mortality table using data from multiple companies, to ensure broader generalizability. In addition, future research should explore dynamic models that incorporate time-varying factors and individual characteristics, such as survival analysis or machine learning approaches, to better capture changes in mortality risk patterns and adapt to the evolving profile of the insured population.

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