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Teaching Central Tendency Meaningfully: A Context-Based Trajectory using Water pH Levels

Allen Marga Retta[1], Ratu Ima Indra Putri[1*], Zulkardi[1], Ely Susanti[1]
[1]Department of Mathematics Education, University Sriwijaya, Indonesia
ratuilma@unsri.ac.id

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ABSTRACT

This study aims to design a contextual learning trajectory using water pH as an anchor to help student's understand central tendency concepts, including mean, median, and mode. The research employed a design research approach within a qualitative descriptive framework, consisting of three main stages: preliminary design, design experiment, and retrospective analysis. In the preliminary design stage, a Hypothetical Learning Trajectory (HLT) was developed and aligned with the learning objectives, ensuring that the tasks were suitable for fostering students' understanding of the central tendency concepts. The design experiment involved data collection from a pilot experiment with 6 students and teaching experiment with 29 eighth-grade students at the junior high school. Data was gathered using instruments, including student worksheets, video recordings, field notes, and test. The data collected from students' responses were analyzed to assess their understanding of the concepts. In the retrospective analysis stage, the Actual Learning Trajectory (ALT) was compared with the predicted HLT to identify any discrepancies and assess the effectiveness of the learning activities. The results showed that integrating the PMRI approach with Project-Based Learning (PjBL) successfully engaged students and contributed to their conceptual development, reinforcing the importance of using contextual problems such as water pH in teaching statistical concepts.

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A. INTRODUCTION

Statistics education equips students with essential skills in logical reasoning, data analysis, and evidence-based decision-making. Through learning statistics, students develop the ability to collect, organize, interpret, and analyze data systematically, skills indispensable both in academic settings and everyday life (MZ, 2024; Utomo, 2021). In today's data-driven world, where information is frequently presented through charts, graphs, and tables, critically interpreting statistical information is vital for informed decision-making (Yusuf et al., 2017). Therefore, statistics education not only enhances mastery of mathematical concepts but also fosters critical thinking and analytical skills necessary for the 21st-century (Hikayat et al., 2020). This study focuses on the concept of measures of central tendency, which include the mean, median, and mode. These concepts are foundational in descriptive statistics and essential for interpreting data sets effectively (Cazorla et al., 2023). The mean, or arithmetic average, is calculated by dividing the total of the data values by the number of items in the data set (Fauzan et al., 2018). It is widely used because it takes into account all data points and is sensitive to changes in the data, but it can be affected by extreme values, or outliers. In contrast, the median is the middle value that splits an ordered data set into two equal halves, making it less sensitive to outliers, and the mode represents the value that occurs most frequently in the data set (Cazorla et al., 2023). The mode is particularly useful for identifying the most common category or value in categorical data, but it may not always exist or may have multiple values. Proficiency in these concepts is essential for advancing to more complex statistical reasoning.

However, research consistently shows that students struggle to grasp these fundamental concepts. Many instructional practices rely heavily on procedural approaches, focusing on formula application without fostering deep conceptual understanding (Andriani & Fauzan, 2019; Landtblom, 2023; Rahayu & Putri, 2021). As a result, students frequently encounter difficulty applying their knowledge to novel or real-world situations. Various research findings indicate that many students still face difficulties in solving problems related to measures of central tendency, such as the mean, median, and mode (Guntur et al., 2025; Monica et al., 2024; Saidi & Siew, 2018; Whitaker et al., 2015). For instance, Dewi reported that 80% of students made errors calculating averages, and 83% struggled with data analysis tasks. Common misconceptions include incorrect median determination, often due to unsorted data, and misinterpreting the mode (Dewi et al., 2020). Common misconceptions include selecting the median from unsorted data and misidentifying the mode (Fauzan et al., 2018; Koparan, 2015; Yusuf et al., 2017). If this error is not immediately resolved, it will have a negative impact on the quality of student learning outcomes in statistics.

To address these learning challenges, this study adopts the Indonesian Realistic Mathematics Education approach called PMRI, a localized adaptation of the Dutch Realistic Mathematics Education (RME) framework (Zulkardi et al., 2020). PMRI is an approach that begins with the context of everyday life (Mubharokh et al., 2022; Rahayu & Putri, 2021). PMRI promotes meaningful learning by situating mathematical concepts in real-life contexts. In addition, incorporating real-life contexts can enhance student engagement and stimulate their curiosity, encouraging them to explore statistical concepts more deeply (Hung & Thuy Dieu, 2024). It is guided by six core principles: the reality principle, activity principle, level principle, interactivity principle, guidance principle, and intertwinement principle (Revina & Leung, 2019). These principles encourage student-centered, exploratory learning, progressing from informal to formal understanding.

This study also incorporates the Project Based Learning (PjBL) model, which provides students with opportunities to apply mathematical knowledge through authentic projects. Previous studies have shown that combining PMRI and PjBL enhances student engagement, contextual understanding, and critical thinking (Fisher et al., 2020; Trisnadati, 2018). PjBL follows a structured sequence: formulating guiding questions, planning and scheduling, monitoring progress, and evaluating results (Fisher et al., 2020; Himmi et al., 2025).

This study is grounded in the Hypothetical Learning Trajectory (HLT) framework, which describes students' potential learning paths based on a series of instructional activities designed to foster conceptual growth. By integrating HLT in learning, students are not only invited to understand the material but also to concept regarding the relationship between concept and their apllication in real situations (Jufri et al., 2024). Over time, the HLT contributes to the formulating a Local Instructional Theory (LIT), which reflects theoretical and empirical insights into student learning.

Overall, this study demonstrates that the integration of PMRI and PjBL approaches enhances students' understanding of statistical concepts by providing relevant real-world contexts and enabling direct application through collaborative projects. Guided by the HLT framework, these approaches not only support students' conceptual development but also foster their critical thinking skills in solving statistical problems. By tailoring the learning experience to students' needs and responses, these approaches offer potential for overcoming learning obstacles.

Thus, this study explores students' learning trajectories in understanding the concept of central tendency using the real-world context of water pH levels. Specifically, it seeks to answer the question: How to design a contextual learning trajectory to support students' understanding of central tendency concepts mean, median, and mode? This study aims to design a contextual learning trajectory that supports students' understanding of central tendency concepts mean, median, and mode.

B. METHODS

This study employed a design research methodology within a qualitative descriptive framework, which is widely recognized for its strength in developing and refining educational innovations through iterative cycles of design, implementation, and analysis (Gravemeijer & Cobb, 2013). The stages of the research are presented in Figure 1 below:

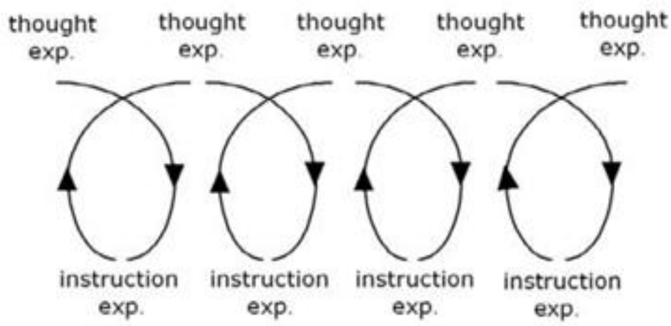


Figure 1. Design Research Cyclic Process

The stages in design research are described as follows:

1. Preliminary Design (First Phase)
   In this phase, a HLT was developed to predict students' learning paths for central tendency concepts. Instructional tasks were designed in alignment with the PMRI approach and Project-Based Learning (PjBL) model to ensure context-based, meaningful learning (Fisher et al., 2020; Zulkardi et al., 2020).
2. Design Experiment (Second Phase)
   The pilot experiment involved 6 students grade VIII students, and the teaching experiment involved 29 students from a different class at the junior high school. These experiments aimed to test and refine the HLT and observe how students engaged with the learning tasks through student worksheets. Data collected from this phase informed ongoing revisions to the instructional design.

3. Retrospective Analysis (Third Phase)
   After the experiments, a retrospective analysis was conducted to evaluate how students' understanding developed from informal to formal statistical concepts. This phase involved interpreting student responses and refining the LIT (Fauzan et al., 2018).

To gather comprehensive data, several methods were employed throughout the study. These included (1) classroom observations were conducted to capture the teaching process, monitor student engagement, and assess the implementation of the instructional design through student worksheets; (2) Video recordings and field notes were utilized for documentation, enabling the analysis of students' responses and interactions during learning activities; (4) tests were administered to assess students' understanding of central tendency concepts before and after the intervention, providing measurable insights into the effectiveness of the learning activities. To ensure the validity of the data, student worksheets and tests were reviewed and validated by two experts in statistics and one in mathematics education and revised based on their feedback.

C. RESULT AND DISCUSSION

1. Preliminary Design

The preliminary design phase began with an in-depth literature review to establish the theoretical and methodological foundations of the study. Key areas of focus included the concept of measures of central tendency (mean, median, mode), supporting mathematical ideas, the PMRI approach, the PjBL model, the real-world context of water pH, and the use of design research as a methodological framework. In addition to the theoretical analysis, classroom observations were conducted in Grade VIII.2 at SMP Negeri 34 Palembang designated teaching experiment class. Discussions with the mathematics teacher, who also served as the model teacher, were held to gather insights on classroom conditions, student characteristics, and teaching history.

The findings revealed that the dominant instructional practice was still conventional, characterized by direct explanation from the teacher, individual student practice, and the absence of any implementation of PMRI or PjBL approaches in teaching measures of central tendency. Based on this diagnosis and in alignment with science curriculum objectives, the context of water pH was selected as the starting point for instruction. This context was not only familiar and meaningful to students but also allowed for interdisciplinary integration with science education. Drawing on design research principles, a HLT was constructed to support the meaningful learning of statistical measures. The HLT framework, as defined includes three key components: (1) the learning goals to be achieved by students; (2) the sequence of learning activities designed to facilitate those goals; and (3) conjectures about students' thinking and potential learning pathways (Fauzan et al., 2018; Zulkardi et al., 2020).

In this study, four learning activities were designed as part of the HLT, namely (1) Exploratory Activity: Students measured water pH from various local sources using litmus paper. They were guided to organize and present their findings using tables, bar graphs, and pie charts. This activity is linked directly to environmental and science literacy, bridging mathematics with natural sciences; (2) Understanding Mode: Students place coloured buttons into plastic cups labelled with different pH values. The aim was for students to identify the most frequently occurring value (mode) and describe one of its properties; (3) Understanding Mean: Students were tasked with calculating the mean pH by simulating water samples using coloured buttons and computing the arithmetic mean. They were also guided to articulate one property of the mean; and (4) Understanding Median: In this final activity, students sorted water pH data to identify the median value. They were prompted to compare, predict, and make decisions based on their interpretation of the median while also stating one of its mathematical properties. Each activity was designed to move students from informal reasoning based on real-life context toward formal statistical concepts through guided reinvention and model-based reasoning, consistent with both PMRI and PjBL pedagogical principles.

2. Design Experiment

Based on the findings obtained during the preliminary design phase, the research proceeded to the development of the HLT. The HLT was constructed by the three core components outlined by Fauzan: 1) clearly defined mathematical learning goals, 2) learning activities strategically designed to facilitate the achievement of these goals, and 3) hypothetical learning processes or conjectures, which represent anticipated patterns of students' thinking, strategies, and conceptual development during instructional engagement (Fauzan et al., 2018). These conjectures are predictive tools to inform instructional decisions and adapt the learning trajectory in real-time.

To provide a detailed visualization of this design, Table 1 outlines the structure of the HLT for measures of central tendency, encompassing the statistical concepts of mean, median, and mode. Each row in the table includes specific learning objectives, contextualized instructional tasks, and the anticipated student responses and reasoning patterns, thus offering a comprehensive map of how students are expected to progress from informal understanding to formal mathematical concepts.

Table 1. Outlines the Structure of the HLT for the topic of Measures of Central Tendency

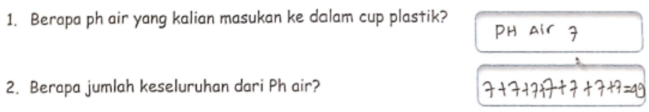
Learning Goal	Learning Activities	Hypothetical learning process /conjecture
Activity 1: 1. Present data in a table based on the "pH Water" project. 2. Present data in bar charts, line charts, and pie charts to visualize data from the "pH Water" project	1. As part of the project learning in Pjbl, students ask basic questions about how to present data, starting with the "Let's Ask" activity. 2. In groups, students plan a project by collecting water pH data from their homes or nearby areas. 3. Students create a project schedule by deciding when each part of the project will be done. 4. Students carried out the project by presenting the water pH data they collected in tables and diagrams (bar charts, line charts, and pie charts). 5. Student present the results of their group project to the class.	1. Students can ask basic questions related to how data is presented. 2. Students can plan projects by collecting water pH data using litmus paper 3. Students can create a project schedule by deciding when each part of the project will be done 4. Students can present their water pH data in a table with correct and complete information. 5. Students can present the water pH data in bar, line, and pie charts with enough information. 6. Students can present the water pH data in diagrams (bar, line, and pie), but the information is incomplete. 7. Student can present their project results and explain the conclusions they found during the "pH Water" project.

Activity 2: 1. Understand the most frequent value (mode) from observing water pH data. 2. Find the most frequent value (mode) from water pH data. 3. Explain the properties of mode.	1. Students work in groups by placing buttons with the same colour (representing the same water pH) into the prepared plastic cups. 2. Students answer questions: 1) What can you conclude about the mode? 2) What do you know about the properties of mode? 3) What can concluded about the water quality in your areas?	1. Students can group buttons by colour into cups based on the same pH value and determine which water pH appears most often. 2. Students can answer questions. 1) The mode is the value that appears most often. 2) A data set might have no mode or more than one mode and the mode is not affected by extreme values (very high or very low). 3) Students can conclude that water samples may have normal, acidic, or alkaline pH values.
Activity 3: 1. Understand the concept of mean by observing water pH data 2. Find the mean of water pH data. 3. Explain properties of mean. 4. Compare data, make predictions, and decide the best option.	1. Students place buttons with the same colour (representing the same pH) into plastic cups that match the type of water. 2. Students answer questions: 1) How much is the total pH value of the water your group collected? 2) How many data sample did your group collect? 3) Write your answer by dividing the total pH value by the number of samples. 4) What can you conclude from your calculation about the average (mean)? 5) Write the mathematical formula for calculating the mean. 6) What do you know about the properties of the mean? 7) Based on the pH data, what can you say about the water quality in your area? 3. Students compare the mode and the mean, then decide which one gives a better result for solving the problem.	1. Students place buttons with the same colour (representing the same water pH) into plastic cups that match the type of water. 2. Students can answer questions: 1) Students calculate the total pH data by adding all the values, either one by one or multiplying repeated data before adding. 2) Students determine how many data samples they have collected in their group. 3) Students write their answers by dividing the total pH value by the number of data samples. 4) Students conclude that the arithmetic mean is found by dividing the total data by the number of data points. 5) Students wrote a mathematical formula for the mean, such as $\bar{X} = \frac{X_1+X_2+X_3+X_4+\cdots+X_n}{n}$ or $\bar{X} = \frac{\sum x}{n}$, but the symbols were not written correctly. 6) Students can understand one of the properties of the arithmetic mean, such as: a. All data values must be included when calculating the mean. b. A data set only has one arithmetic mean. c. The value of the mean can be affected by very large or very small numbers. 7) Students can conclude that water quality is different in each area, some have a normal pH (6.5–8.5), some are acidic (pH below 6.5), and some are alkaline (pH above 8.5). 3. Students can compare the mode and the mean, then predict and decide which gives a better result. 4. Students cannot yet compare the mode and the mean to decide which gives a better result.

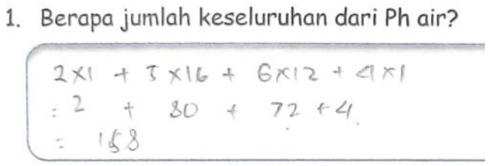
Activity 4: 1. Understand the concept of median by observing water pH data 2. Find the median of water pH data. 3. Explain the properties of the median. 4. Compare data, make predictions, and decide the best option.	1. Students find the smallest and largest water pH values. 2. Students sort the pH data from the smallest to the largest (odd data and even data) 3. Students determine the median, lower, and upper quartile from odd and even data, and write the formula. 4. Students compare the data used in activity 3 (mean) and activity 4 (median) to see their difference. 5. Students can conclude their area's water quality based on the collected data. 6. Students can compare mode, mean, and median data to make predictions and make the best decision.	1. Students can find the smallest and largest water pH values. 2. Students can sort the pH data from the smallest to the largest (odd data and even data) 3. Students can determine the median, lower, and upper quartile from odd and even data, and write the formula. Odd data $Me = data\ ke\ \frac{n+1}{2}$ Even data $Me = \frac{data\ ke\ \frac{n}{2} + data\ ke\ \frac{n+2}{2}}{2}$ 4. Students can find the median, lower, and upper quartile from odd and even data but cannot write the formula. 5. Students cannot find the median, lower, or upper quartile from either odd or even data. 6. Students can compare the data used in activity 3 (mean) and activity 4 (median) and identify one of the mean properties, such as: 1. All data values must be included in the calculation. 2. A set of data only has one mean. 3. The mean is affected by extreme values (very high or very low). 7. Students can identify one of the properties of the median, such as: a. A data set has only one median. b. To find the median, the data must be sorted from smallest to largest. c. The median is not affected by extreme values like the mean. 8. Students cannot yet compare Activity 3 (mean) and Activity 4 (median). 9. The student can conclude that water quality varies in each area based on the median value; some are normal, acidic, or alkaline. 10. Students can compare mode, mean, and median data to make predictions and choose the best decision. 11. The student cannot yet compare mode, mean, and median data to make predictions and make the right decision.

Based on the HLT outlined in Table 1, four student worksheets were designed by integrating the PMRI approach with the PJBL model. Subsequently, the research proceeded to the second phase, which included (1) a pilot experiment involving 6 students and (2) a teaching experiment involving 29 eighth-grade students at junior high school. Revisions were made to the student worksheets after the pilot experiment and before the teaching experiment. These modifications improved students' understanding of statistical concepts, particularly in interpreting the collected water pH data. The following is an example of the revisions made to the student worksheets , as illustrated in Figure 2 below.

Pilot experiment


1. Berapa ph air yang kalian masukan ke dalam cup plastik? PH Air 7
2. Berapa jumlah keseluruhan dari Ph air? 7+7+7+7+7+7+7=49

Teaching experiment


1. Berapa jumlah keseluruhan dari Ph air?
2×1 + 5×16 + 6×12 + 4×1
= 2 + 80 + 72 + 4
= 168

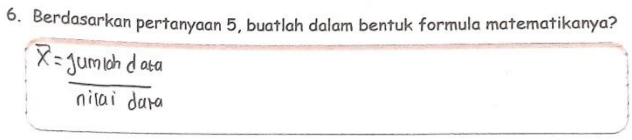
Pilot experiment
What is the pH value of the water you placed into the plastic cup? pH air 7
How many total pH data points did you collect? $7 + 7 + 7 + 7 + 7 + 7 + 7$

Teaching experiment
How many total pH data points did you collect?
$= 2 \times 1 + 5 \times 16 + 6 \times 12 + 4 \times 1$
$= 2 + 80 + 72 + 4$
$= 168$

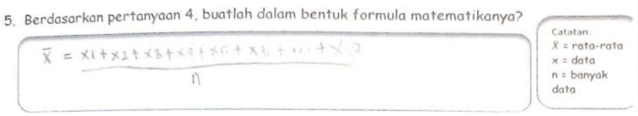
Figure 2. Revised student worksheets after the pilot experiment (1)

In Figure 2, a change was made to the guiding question related to the concept of the mean. Initially, students were asked, "What is the pH value of the water you placed into the plastic cup?" expecting them to report various pH values. However, many students responded with a single value, such as "7", and subsequently calculated the total pH data by repeating this value (e.g., 7 added seven times), indicating a misunderstanding of data collection. To address this, the question was rephrased to, "How many total pH data points did you collect?" prompting students to consider all collected data comprehensively, leading to more accurate responses like calculating the total from multiple values. The next revision involved providing a written note as a reminder to guide students in expressing the mean using a mathematical formula, as shown in the following Figure 3.

Pilot experiment


6. Berdasarkan pertanyaan 5, buatlah dalam bentuk formula matematikanya?
X̄ = jumlah data / nilai data

Teaching experiment


5. Berdasarkan pertanyaan 4, buatlah dalam bentuk formula matematikanya?
Catatan: X̄ = rata-rata, x = data, n = banyak data

Pilot experiment
Can you write the mathematical formula used to calculate the mean pH value?
(The questions are not accompanied by any guidance notes, which may make it harder for students to write the correct mathematical formula)

Teaching experiment
Can you write the mathematical formula used to calculate the mean pH value?
(The questions are accompanied by guiding notes to help students write the correct mathematical formula)

Figure 3. Revised student worksheets after the pilot experiment (2)

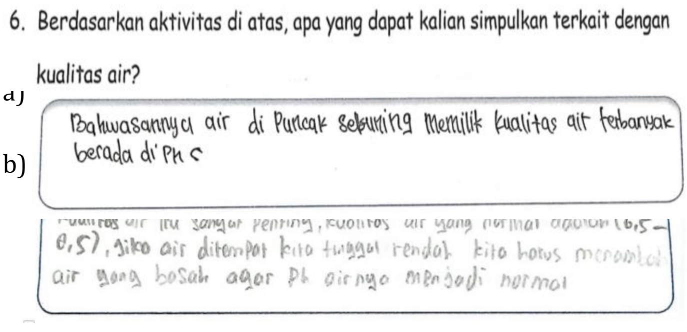
In the Figure pilot experiment, the student's answer shows an attempt to represent the mean by writing the symbol and dividing the total data value by the number of data points. However, the student uses descriptive sentences rather than proper mathematical notation. This indicates that the student did not fully comprehend the instruction, explicitly asking for the mean to be written in formula form. In contrast, the Figure in the teaching experiment demonstrates a clearer understanding of the question. With the help of a small guiding note, the student could accurately express the mean using the correct mathematical formula, appropriately adjusted to the data collected. This suggests that the guiding note significantly supported students' ability to translate conceptual understanding into proper mathematical expression. These revisions were then applied and tested in the teaching experiment to assess their effectiveness in a broader learning context.

During the experiment, the learning process generally followed the path predicted by the HLT, with most students successfully transitioning from informal reasoning to a formal approach in understanding statistical concepts such as mean, median, and mode. Students initially used intuitive strategies, such as visual estimation, to calculate the mean, and over time, they began using the correct mathematical formulas, in line with the predictions made by the HLT.

a. First Activity (context real world)
   The first activity in the learning process begins by presenting a contextual situation that encourages students to explore the quality of water used in their daily lives. Using the PjBL model, students are engaged in field investigations by measuring the pH levels of various water sources using litmus paper. This activity introduces students to scientific concepts related to water quality and promotes active participation in designing simple, real-world projects, aligning closely with the principles of the PMRI approach (Zulkardi et al., 2020). Through this activity, students are expected to collect, organize, and present data in tables, bar charts, and pie charts. The integration with the science subject further supports interdisciplinary learning, fostering connections between mathematical representations and scientific understanding.
b. Second Activity (Putting buttons into plastic cups with the same pH)
   In the second activity, students are provided with learning media consisting of colored buttons (representing different water pH levels) and plastic cups for grouping buttons of the same color. This activity is designed to help students understand the concept of mode by determining the most frequently observed water pH value based on their previous data collection. Additionally, students are introduced to the properties of the mode through this hands-on experience. Figure 4 illustrates students' engagement in this activity as documented in Student worksheets 2.

Figure 4. Student worksheets 2

After completing the activity of grouping buttons of the same color (representing identical water pH values) into plastic cups, students were guided through reflective questions provided in Student worksheets 2 (LAS). As illustrated in Figure 4, students analyzed their group data. They concluded that the water quality from all samples was unsuitable for consumption, as the pH values recorded were below the standard range for drinking water (6.5–8.5). Furthermore, based on the analysis from three student groups, most students could understand the concept of mode as the value that appears most frequently in a dataset. They successfully identified the mode from their collected data and described its characteristics. The analysis revealed that most water samples had a pH of 5, indicating that the water quality was generally poor and did not meet the minimum health standards. However, it was also noted that the two groups could not draw appropriate conclusions based on their data. This observation is further illustrated in Figure 5.



6. *Based on the activities above, what can you infer regarding the water quality?*

a) *The water at Puncak Sekuning has acidic water quality.*

b) *Water quality is important. Normal water has a pH (6,5-8,5). If the pH is below this range, the water is acidic. If the water is heated or boiled, the pH becomes neutral.*

Figure 5. Student' Responses on Mode

Figure 5a displays a student response that only mentions pH five as the most frequently occurring data point without relating this finding to the interpretation of water quality based on acceptable pH standards for drinking water (typically 6.5–8.5). Similarly, Figure 5b presents a student's statement suggesting the need to "add wet water" to normalize the pH. This indicates that students have not yet developed the ability to draw accurate and meaningful conclusions from the data. Their reasoning remains focused on surface-level observations without integrating statistical concepts, specifically, the mode with real-world interpretations such as water potability standards. This highlights the need for further scaffolding to guide students in making deeper connections between mathematical analysis and environmental context.

c. Activity Third (Putting buttons into plastic cups in accordance with data on the type of water obtained

In the third activity, students continued to use learning media such as colored buttons representing water pH and plastic cups to place the buttons according to the pH data. This activity aimed to help students understand the concept of the arithmetic mean, calculate it, and explain its properties. Through this task, students learned to recognize data patterns, compute the average pH from the collected data, and understand the meaning and importance of the average. The students' responses are shown in Figure 6 below.

(a) $\bar{X} = \frac{x}{n}$

(b) $\bar{X} = \frac{x1 + x2 + x3 + x4 + x5 + x6 + \cdots + x30}{n}$

(c) $\bar{X} = \frac{x1 + x2 + x3 + x4 + x5 + x \ldots + x21}{n}$

(d) $\bar{X} = \frac{X_1 + X_2 + X_3 + \cdots X_n}{n}$

Figure 6. Student' Responses on Mean

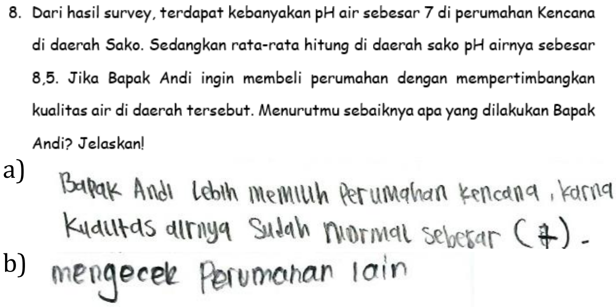
Figure 6a shows a student's answer that does not yet correctly represent the concept of the mean using a proper mathematical formula. The student only writes that the average results from dividing the total data (symbolized by x) by the number of data points (symbolized by n) without showing the step of adding all the data before dividing. This indicates that the student has not fully understood the complete structure of the mean formula. In contrast, Figures 6b and 6c show a better understanding. In Figure 6b, the student correctly writes the formula by adding all 30 data points ($x_1 + x_2 + x_3 + x_4 + \cdots + x_{30}$) and dividing by the total data (n). Similarly, in Figure 6c, the student adds 21 data points before dividing them, reflecting a good grasp of calculating the mean in a mathematical context.

Figure 6d shows a student who also writes the correct general formula ($x_1 + x_2 + x_3 + x_4 + \cdots + x_n$) divided by n. However, students have not yet applied this formula to actual data, meaning that although they understand the formula structure, they still struggle to connect it to real data in context. Several previous studies have shown that many students still have misconceptions when writing mathematical concepts in formal form. These include using incorrect symbols, writing formulas inaccurately, and showing a lack of understanding in translating their ideas into proper mathematical expressions (Ismail & Chan, 2015; Maryati & Priatna, 2018). The next goal is for students to be able to explain the properties of the arithmetic mean, as guided by the questions presented in Figure 7.

6. Apa yang terjadi jika data pH air sumur berubah bertambah sebesar 1 karena faktor suhu dan cuaca, bagaimana dengan rata-rata hitungnya, Bandingkanah rata-rata hitung sebelum ditambahkan dan sesudah ditambahkan. Apa kesimpulanmu? maka rata rata sempel air akan menjadi beda	6. *"If the pH value of well water changes by 1 due to temperature or weather, does it affect the average? Compare the mean before and after the change. What can you conclude?"* *"The average water level will be different."*

Figure 7. Student Activity on Understanding the Properties of the Arithmetic Mean

In Figure 7, students showed an understanding of the properties of the arithmetic mean. Their answers indicated that the average pH also changes if the water pH increases by 1. However, when asked to compare the mode and the mean, only two groups could explain the properties of the arithmetic mean correctly, while the other three groups still had difficulty. This can be seen in Figure 8 below.


8. Dari hasil survey, terdapat kebanyakan pH air sebesar 7 di perumahan Kencana di daerah Sako. Sedangkan rata-rata hitung di daerah sako pH airnya sebesar 8.5. Jika Bapak Andi ingin membeli perumahan dengan mempertimbangkan kualitas air di daerah tersebut. Menurutmu sebaiknya apa yang dilakukan Bapak Andi? Jelaskan!

a) Bapak Andi lebih memilih perumahan kencana, karna kualitas airnya sudah normal sebesar (7).

b) mengecek perumahan lain

8. If most of the water pH in the Kencana housing complex is 7, but the average pH in the Sako area is 8.5, and Mr. Andi wants to buy a house based on water quality, what should Mr. Andi consider? Explain your answer.

a) Mr. Andi prefers Kencana Housing, because the water quality is already in the normal range (around pH 7)

b) Mr. Andi should check the water quality in other places before choosing.

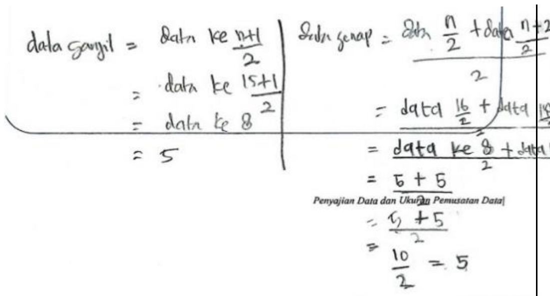
Figure 8. Activity Student About the Average Nature

Figure 8a shows student answers demonstrating a good understanding of mode and mean. This is seen from students choosing the Golden housing area because the water pH is already normal (pH 7). Meanwhile, Figure 8b shows student answers that indicate they do not fully understand the concepts of mode and mean. They checked other housing without realizing that the Kencana housing already had good water quality. This means students have not yet been able to effectively use statistical data, such as mode and mean, to make real-life decisions.

d. Activity Fourth (Sorting the data that leads students to understand median concept)

The fourth activity is designed to allow students allow students to sort data to understand the concept of the median. In this activity, students learn to find the median, explain one of its characteristics, compare data, make predictions, and draw appropriate conclusions based on data analysis. The students' answers are shown in Figure 9 below.

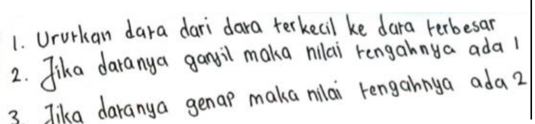
a)



$$Odd\ data = data\ to \frac{n+1}{2} = data\ to \frac{15+1}{2} = data\ to\ 8 = 5$$

$$Even\ data = \frac{data\ to \frac{n}{2} + data\ to \frac{n+1}{2}}{2} = \frac{data\ to \frac{16}{2} + data\ to\ \frac{16+1}{2}}{2} = \frac{data\ to\ 8 + data\ ke\ 9}{2} = \frac{5+5}{2} = 5$$

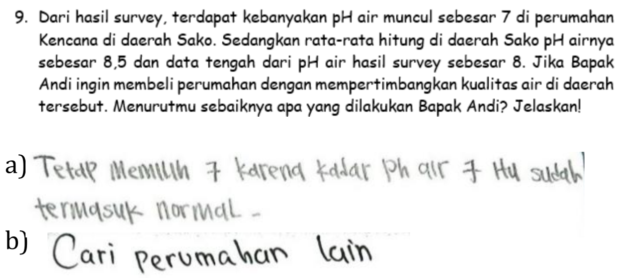
b)


1. Urutkan data dari data terkecil ke data terbesar
2. Jika datanya ganjil maka nilai tengahnya ada 1
3. Jika datanya genap maka nilai tengahnya ada 2

1. Sort the data from the smallest to the largest.
2. If the number is odd then the middle value is 1
3. If the number is even then the middle value is 2

Figure 9. Student' Responses on Median

In Figure 9a, students show a good understanding of the median. They can write the correct mathematical formulas for odd and even data and use the correct symbols to show the median's position. In Figure 9b, students only write the steps to find the median without using a formula. Even though they can sort the data and find the median value, their explanation is still descriptive. This means their understanding is still basic and has not reached the formal level. A similar finding was observed in previous studies, which reported that most students were unable to correctly perform operations related to the median (Aydin Karaca & Ay, 2025). The next goal is for students to be able to explain the properties of the median by comparing it with the concepts of mode and mean, as shown in Figure 10.


9. Dari hasil survey, terdapat kebanyakan pH air muncul sebesar 7 di perumahan Kencana di daerah Sako. Sedangkan rata-rata hitung di daerah Sako pH airnya sebesar 8,5 dan data tengah dari pH air hasil survey sebesar 8. Jika Bapak Andi ingin membeli perumahan dengan mempertimbangkan kualitas air di daerah tersebut. Menurutmu sebaiknya apa yang dilakukan Bapak Andi? Jelaskan!

a) Tetap Memilih 7 karena kadar Ph air 7 itu sudah termasuk normal.

b) Cari perumahan lain

9. What should Mr. Andi do if he wants to choose a Kencana housing based on the water quality, knowing that in the Kencana housing most water pH is 7, the mean pH in Sako is 8.5, and the middle value is 8? Please explain your answer?

a) Still choose the housing with pH 7 because a pH level of 7 is considered normal and safe for daily use.

b) Mr. Andi should check the water quality in other places before choosing.

Figure 10. Activity Student About Median Properties

Figure 10a shows that the student has a good understanding of mode, mean, and median. This is seen from their decision to choose Kencana Housing because the water quality is already standard, with a pH of 7. In contrast, Figure 10b shows that the student does not fully understand these concepts. The student looks for another housing without considering that Kencana Housing already meets water quality standards. This means the student cannot interpret or apply statistical data in real-life situations.

3. Retrospective Analysis

After completing the pilot and teaching experiments, an evaluation was conducted to assess how well the learning activities helped students understand the concepts of mean, median, and mode and improve their critical thinking skills. This was done through a retrospective analysis, comparing the Actual Learning Trajectory (ALT) with the HLT designed based on the PMRI approach, PjBL model. The learning process successfully guided students through contextual tasks, such as analyzing water pH, helping them connect abstract math concepts to real-life situations (Hikayat et al., 2020; Nova et al., 2022). This supports Landtblom's argument that real-world contexts are essential to move beyond procedural tasks and build genuine conceptual understanding (Landtblom, 2023). The PjBL model made the learning experience more engaging by involving students in real projects, allowing them to collect, analyze, and present data together. These activities encouraged them to think critically and make decisions based on their data analysis (Koparan, 2015; Siswono et al., 2018).

The HLT helped predict how students would understand the material, and the results showed that the actual learning matched the plan well (Jufri et al., 2024). Students were also asked to explain why they used a particular measure of central tendency, which helped strengthen their reasoning and reflection (Uyen et al., 2021). Therefore, learning that starts from contextual problems such as water pH, supported by collaboration between the PMRI approach, PjBL model, and HLT framework, has proven effective in deepening students' understanding of measures of central tendency and enhancing their ability to think critically and apply mathematical concepts in real-life situations.

D. CONCLUSION AND SUGGESTIONS

Integrating the HLT with the PMRI approach and PjBL has proven effective in enhancing students' understanding of central tendency concepts mean, median, and mode through contextual problems such as water pH analysis. The alignment between the Actual Learning Trajectory (ALT) and HLT indicates that the designed learning path successfully supported students' cognitive development. By engaging in real-life, collaborative projects, students practiced data representation and analysis, which are hard skills that help them process and interpret data using mathematical methods. These activities also developed critical thinking and the ability to make informed decisions based on statistical reasoning, which are soft skills essential for problem-solving and applying knowledge in real-world contexts.

This confirms that combining PMRI, PjBL, and HLT offers a meaningful and impactful strategy for improving mathematics education. To build on these results, teachers are encouraged to integrate contextual problems into their lessons, using real-world situations to help students connect abstract mathematical concepts to everyday life. By incorporating project-based learning, teachers can foster collaboration, communication, and problem-solving skills among students, helping them work together on meaningful tasks. Curriculum developers should consider designing mathematics curricula that integrate HLT and PMRI, ensuring that the learning paths are carefully structured and align with students' developmental stages.

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